“REVERSE DISCRIMINATION AND EFFICIENCY IN EDUCATION”

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Reverse Discrimination and Efficiency in Education*

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Abstract

This paper shows that reverse discrimination policies can find a justification purely on efficiency grounds. We study the optimal provision of education when households belong to different groups, differing in the distribution of the potential to benefit from education among individuals, which is private information. The main result is that the high potential individuals from groups with relatively few high potential individuals should receive more education than otherwise identical individuals from groups with a more favourable distribution of these benefits.

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1 Introduction

In 1996, California approved Proposition 209, which forbids “discriminating against or giving preferential treatment to any individual or group in public employment, public education, or public contracting on the basis of race, sex, color, ethnicity or national origin”. The year before, the US Supreme Court had ruled that the Banneker programme at the University of Maryland was in violation of the Constitution; this programme provided financial assistance to blacks, and had lower ability thresholds for eligibility than the colour-blind parallel programme.

After the epochal changes brought about by the civil rights movement, sanctioned by the Supreme Court ruling that “race or ethnic background may be deemed a ‘plus’ in a particular applicant’s file”,\(^1\) the pendulum is therefore swinging back, and “reverse discrimination” legislation\(^2\) is now on the defensive, attacked both on equity and on efficiency grounds. Reverse discrimination policies, it is argued, are inequitable, as white individuals are discriminated against even though they bear no personal faults for the past suffering of individuals from other ethnic groups. They are also inefficient, because resources are not allocated to those with the highest willingness to pay or potential to benefit. Policies forbidding discrimination against disadvantaged groups are less controversial: firstly, they are clearly equitable, and, secondly, the theoretical literature has identified efficiency reasons for these policies: inefficient discrimination against some groups may emerge as an equilibrium phenomenon. Some instances of inefficient discrimination, such as an employer’s dislike for hiring members of racial minorities, are likely to be temporary, as discriminating employers lose out to profit maximising non-discriminating ones (eg Becker 1971, and Lundberg and Starz 1983). But in some cases inefficient discrimination is part of the equilibrium behaviour, and therefore is not eliminated without an explicit policy intervention.\(^3\)

\(^1\) Regents of Univ. of California v. Bakke, 438 U.S. 265, 317.
\(^2\) That is, according to the official definition used by the Clinton administration: “any effort taken to expand opportunity for women, and racial, ethnic and national origin minorities by using membership in those groups that have been subject to discrimination...” (Stephanopoulos and Edley 1996, cited in Holzer and Neumark 2000, p 488). The three main areas where reverse discrimination has found application in the US are university admissions in elite universities, favouritism in bidding for procurement and hiring and promotion, (Holzer and Neumark 2000).
\(^3\) Typical is Coate and Loury’s (1993) model of discrimination in the labour market. Individuals acquire human capital, and their productivity in employment depends on ability and human capital, which are both unobservable: ability is the realisation of a random variable
The literature analysing reverse discrimination is scarce. The recent paper by Wickelgren (2002) shows that a form of reverse discrimination may emerge as part of the equilibrium behaviour of profit maximising employers: otherwise observationally identical individuals from disadvantaged groups are treated more favourably. This happens if ability is correlated across generations, and if parental income affects educational achievement: because past discrimination lowers income, the expectation of individual’s ability, conditional on her educational achievement, is higher for individuals from disadvantaged groups, and so firms rationally expect them to perform better on the job, and pay them more.

As far as we are aware, our paper is the first to identify an efficiency rationale for an explicit policy of reverse discrimination in an utilitarian framework, that is taking the sum of individual utilities as the benchmark for optimality (moreover, in view of the functional form we posit for the individual utility functions, this is equivalent to the maximisation of total income). We study the provision of education in the presence of individuals who differ in their potential to benefit from education. Individuals can also be classified in groups according to some observable characteristic, such as race, sex, socio-economic status, and so on. The motivation of the paper rests with our assumption, crucial but empirically plausible, that groups differ in the distribution of the capacity to benefit from education. The latter depends both on innate ability and on a person’s social background, which affects their social skills and the network of formal and informal contacts and connections which can be drawn upon to obtain favourable labour market outcomes. Individuals belonging to “disadvantaged” groups, those where social skills and the social network are less developed, are less likely to have high potential to benefit from education.

Our main result, in Section 3, is that the optimal education policy is such that individuals from disadvantaged groups pay a lower tuition fee for admission to a given education level (Proposition 3), and are enrolled to higher education levels than otherwise identical individuals from advantaged households (Proposition 1). This tallies, for example, with the practice of many US universities (with the same distribution for all groups), and human capital is acquired. Employers only observe to which group an individual belongs, and use statistics about her group to infer her productivity. Coate and Loury show that there may be multiple equilibria. In one equilibrium all groups are treated equally. In another, employers believe that individuals belonging to a given group acquire less human capital and therefore pay them less: but, because of this, the rewards to, and therefore the incentives towards, human capital acquisition are weakened for individuals belonging to this group, and they will indeed acquire less human capital so that the employers’ belief turns out to be correct. Other examples, in a similar spirit, are in Loury 2002, pp 29-33. A different approach, leading to similar results, is Milgrom and Oster (1987).
to alter admission standards and financial assistance according to the ethnicity of the applicant (documented by Bowen and Bok 1998), or with the UK government’s current policy to expand access to university by applicants from less advantaged backgrounds (DfES 2003).

The result that the optimal policy is such that individuals in different groups are treated differently is a typical second best result, being due to asymmetric information: if the government did not have an information disadvantage, identical individuals from different groups would instead be treated identically. It is also a qualitative distortion from the market outcome: in the absence of public intervention, all individuals with the same potential to benefit from education acquire the same education. What determines this result? To gain some intuition, begin by noting that individuals with high potential to benefit from education (in all groups) receive more education than they would acquire privately (this is due to the externality which justifies public intervention). To induce them to acquire these higher education level, under asymmetric information, they must be offered a rent, in the form of a subsidy, financed by general taxation: they pay, in tuition fees, less than the monetary cost of their education. This subsidy has a social cost. Now note that, if there are relatively more high potential individuals in a group than in another – which, given our definition, simply means that the first group is “advantaged”–, then the aggregate cost of providing the high levels of education to individuals in that group is higher than for a group with fewer high potential individuals. Therefore, to reduce the socially costly subsidy, high potential individuals in advantaged groups are offered a lower subsidy and so choose to acquire less education than individuals with the same potential in “disadvantaged” groups. The paper is devoted to making rigorous this loosely described intuition.

2 The Model

2.1 Household characteristics

There is a continuum of households, with measure normalised to 1. Each household comprises a parent and a child. The parent chooses the household’s current consumption, the monetary transfer to the child (at the market interest rate, normalised to 0), and the investment in the child’s education. The household utility function is given by:

\[ u(b) + x, \quad u'(b) > 0, \quad u''(b) < 0, \quad \text{there exists } b^* \text{ such that } u'(b^*) = 1, \quad (1) \]
where \( b > 0 \) is the household’s current consumption and \( x \) is the (possibly expected) amount of monetary resources enjoyed by the child, given by the sum of the monetary transfer from the parent and her labour market income, \( y \). The latter is a function increasing in three arguments: her education, \( e \), a parameter \( \theta \in \mathbb{R} \), with positive support in \((\theta_0, \theta_1)\), which is different for different people and measures her potential to benefit from education, and the general education level in the economy, \( E \): \( y = y(e, \theta, E) \),\(^4\) with \( y_\theta(\cdot), y_e(\cdot) > 0 \) and \( y_E(\cdot) \geq 0 \). The variable \( \theta \) captures the fact that some people are more capable of obtaining higher labour market income for a given education level: \textit{ceteris paribus}, an individual with high \( \theta \) is one whose future income is higher. \( y_E(\cdot) \geq 0 \) implies that individuals are more productive if the general education level in the economy, \( E \), is higher,\(^5\) and justifies public intervention in the provision of education. We assume \( y(\cdot) \) also to satisfy \( y_{ee}(\cdot) < 0, \lim_{e \rightarrow \infty} y_e(\cdot) = 0 \), and \( y_{e\theta}(\cdot) > 0 \). According to the latter, given two individuals with the same education, \( e \), but different potential to benefit from education, \( \theta \), the higher \( \theta \) would benefit more from an increase in her investment in education. While this is reasonable (and tallies, for example, with the practice followed by government bodies and charitable foundations to award scholarships according to ability), it is crucial for the qualitative nature of our results (a fuller discussion is in De Fraja 2002). Finally, we make two technical assumptions on the third derivatives, \( y_{eed}(\cdot) \geq 0, y_{eed}(\cdot) \leq 0 \). These ensure that the appropriate incentive compatibility constraint is satisfied.

As well as in the potential to benefit from education, individuals also differ in race, sex, income, ethnic origin, nationality, immigrant status, religion, age,

\(^4\)This function can be derived from primitives (De Fraja 2002, pp 440–441). An example, special but nevertheless capturing the essence of our idea, is that income is \( z_H(e, E) \) with probability \( \theta \), and \( z_L(e, E) < z_H(e, E) \) with probability \( (1 - \theta) \).

\(^5\)That is, \( E \) bestows a positive externality. See Blaug 1965, especially pp 234–41, or Cohn 1975, especially pp 18–26, 127–129, 223–225 for comprehensive discussions of the potential sources of externality, West 1964, for the views of classical economists, or Lucas (1988) for a seminal formulation in the context of economic growth. An empirical estimate of the function \( y(\cdot) \) can be found in the recent paper by Moretti (2002). He calculates that a percentage point increase in the number of college graduates (corresponding here to an increase in \( E \) “increases the wages of high school drop-outs and high school graduates by 1.9% and 1.6% respectively [and] of college graduates by 0.4%” (p 3). These estimates indicates, plausibly, that \( y_{ee}(\cdot) < 0 \), though this is not necessary to our results. More generally, the existence of a positive effect of \( E \) on individual earnings is suggested by the productivity differences of low skilled workers across countries, and by the “brain drain” of highly educated workers from less developed countries to countries with higher overall level of education. Hanushek (2002, p 8 and pp 16–18) reports recent work on the topic.
sexual orientation, socio-economic status, and so on. Formally, we assume that each household belongs to one of \( n \) groups, and each group is characterised by a particular set of values for the child’s observable characteristics. Groups are labelled by the subscript \( i, i = 1, \ldots, n \). The number of households in group \( i \) is \( h_i > 0 \), with \( \sum_{i=1}^{n} h_i = 1 \).

Note that an individual’s labour market income does not depend on her observable characteristics: two equally educated individuals characterised by the same potential to benefit from education earn the same labour market income even when their skin colour or sex is different. While individuals with the same \( \theta \) are identical, groups do differ in a fundamental respect. We assume that the distribution of the capacity to benefit from education, \( \theta \), is different in different groups: in group \( i \), \( \theta \) is distributed according to the function \( \Phi_i(\theta) \) which has density \( \phi_i(\theta) = \Phi_i'(\theta) \), and monotonic hazard rate, \( \frac{d}{d\theta} \left( \frac{1-\Phi_i(\theta)}{\phi_i(\theta)} \right) \leq 0 \), for every \( \theta \in (\underline{\theta}, \overline{\theta}) \) and for every \( i = 1, \ldots, n \). We capture the differences between these functions by the hazard rate, and, for the sake of simplicity, we assume that, given any two distributions \( \Phi_i(\theta) \) and \( \Phi_j(\theta) \), the difference between the hazard rate has a constant sign over the (common) support.

**Assumption 1** Given \( i, j \in \{1, \ldots, n\} \), either \( \frac{1-\Phi_i(\theta)}{\phi_i(\theta)} \leq \frac{1-\Phi_j(\theta)}{\phi_j(\theta)} \) for every \( \theta \in (\underline{\theta}, \overline{\theta}) \), or \( \frac{1-\Phi_i(\theta)}{\phi_i(\theta)} \geq \frac{1-\Phi_j(\theta)}{\phi_j(\theta)} \) for every \( \theta \in (\underline{\theta}, \overline{\theta}) \), with strict inequality over a range.

Relaxing this assumption would simply make our conclusions less clear-cut and more verbose in their description: we would need to qualify statements by saying “for individuals with \( \theta \) up to...”. Assumption 1 determines a natural ordering: simply re-label the groups in such a way that Assumption 1 can be written, without further loss of generality, as follows.

**Assumption 1 (a)** For every \( i \in \{2, \ldots, n\} \), \( \frac{1-\Phi_{i-1}(\theta)}{\phi_{i-1}(\theta)} \leq \frac{1-\Phi_i(\theta)}{\phi_i(\theta)} \) for every \( \theta \in (\underline{\theta}, \overline{\theta}) \), with strict inequality over a range.

**Lemma 1** Assumption 1(a) implies that \( \Phi_i(\theta) \) first order stochastically dominates \( \Phi_{i-1}(\theta) \).

**Proof.**

\[
\frac{1-\Phi_{i-1}(\theta)}{\phi_{i-1}(\theta)} \leq \frac{1-\Phi_i(\theta)}{\phi_i(\theta)} \quad \text{for every } \theta \in (\underline{\theta}, \overline{\theta}) \text{ with strict inequality over a range implies:}

\[
\int_{-\infty}^{\theta} \frac{\phi_{i-1}(x)}{1-\Phi_{i-1}(x)} dx \geq \int_{-\infty}^{\theta} \frac{\phi_i(x)}{1-\Phi_i(x)} dx
\]

\[
(2)
\]

Recall that, given two distribution functions \( \Phi_i(\theta) \) and \( \Phi_j(\theta) \) with common support, \( \Phi_i(\theta) \) first order stochastically dominates \( \Phi_j(\theta) \) if \( \Phi_i(\theta) \leq \Phi_j(\theta) \) for every \( \theta \) in the support, with strict equality over a range.
for every $\theta \in (\underline{\theta}, \bar{\theta})$, with strict inequality for $\theta$ above a certain value. Next use the equality

$$-\ln (1 - \Phi_i (\theta)) = - \int_{-\infty}^{\theta} \frac{d\ln (1 - \Phi_i (x))}{dx} dx = \int_{-\infty}^{\theta} \frac{\phi_i (x)}{1 - \Phi_i (x)} dx,$$

to write (2) as $-\ln (1 - \Phi_{i-1} (\theta)) \geq -\ln (1 - \Phi_i (\theta))$, implying $\Phi_{i-1} (\theta) \geq \Phi_i (\theta)$ with strict inequality over a range, and establishing the Lemma.

That is, group 1 has the highest proportion of individuals with low potential to benefit from education, and group $n$ the lowest. It is therefore natural to label group 1 as the most “disadvantaged”; this term, though perhaps in general charged, describes accurately the specific set-up of our model: the expected labour market income – the expectation being taken over $\theta$ – of an individual with a given education in a group with a low index is lower than for an individual with the same education from groups with higher indices.

2.2 The source of differences in the distribution of $\theta$.

The interpretation of the variable measuring an individual’s capacity to benefit from education is potentially very controversial. We have carefully avoided the term “ability”, normally used to describe variables of this kind.\footnote{Even when it is not necessarily linked to intelligence: ability is “everything that contributes to the child’s income potential, is in the child at the time he takes his education decision, and cannot be purchased on the market” (Rubinstein and Tsiddon 1998, p. 19).} This is because we do not subscribe to the view that the distribution of cognitive “ability” varies among racial (or socio-economic) groups, as argued, for example, by the controversial book by Herrnstein and Murray (1994). We therefore devote this brief section to showing that the potential to benefit from education may differ across groups even when the distribution of innate abilities does not.

Assume that $\theta$ is determined by the sum of two variables, $\alpha$ and $c$, that is, $\theta = g (\alpha + c) > 0$ satisfying $g' (\alpha + c) > 0$.\footnote{The restriction to additivity between $\alpha$ and $c$ – which we need in Lemma 2 – is mild, given that both variables lack a natural measurement scale and so can be normalised to fit the required additive formulation.} $\alpha \in \mathbb{R}$ is the child’s innate ability, which clearly affects labour market success. $c \in \mathbb{R}$ captures the role of the social background. This matters both because family, peer, and cultural pressures hone skills which affect employability and capacity to earn, and because it determines the network of connections and contacts which a person will be able to draw upon after entering the labour market: it matters who you know, as well as who you are. Within each group, both $\alpha$ and $c$ can differ among
individuals. On the contrary, the distribution of innate ability is the same in all groups, given by $\Lambda (\alpha)$ with density $\lambda (\alpha) = \Lambda ' (\alpha)$, and monotonic hazard rate, $\frac{d}{d \alpha} \left( \frac{1 - \Lambda (\alpha)}{\Lambda (\alpha)} \right) \leq 0$. The distribution of the variable measuring the quality of the social background is different for different groups. Even though explicit discrimination is (increasingly) outlawed, catchphrases such as the “old boys network”, “the glass ceiling”, or “discrimination in contact” (Loury 2002, pp. 95-96) do reflect the observation that certain groups in society enjoy a narrower range of opportunity than others. Formally, in group $i$, $c$ varies according to the distribution function $\gamma_i (c)$ which has density $\phi_i (c) = \frac{d}{dc} \gamma_i (c)$, and, monotonic hazard rate, $\frac{d}{dc} \left( \frac{1 - \Gamma_i (c)}{\gamma_i (c)} \right) \leq 0$, for every $i = 1, ..., n$.\footnote{See however Yinger (1998), Darity and Mason (1998) and Ladd (1998) for evidence of discrimination in consumer, labour, and credit markets.}

Taken together, the distribution functions for the parameters $c$ and $\alpha$ generate a distribution function of the potential to benefit from education, $\mu \equiv \mu (\theta)$. Denoting it, for group $i$, by $\Phi_i (\theta)$, we have

$$
\Phi_i (\theta) = \Pr (g (\alpha + c) \leq \theta) = \int_{-\infty}^{\theta} \int_{-\infty}^{g^{-1}(\theta) - \alpha} \gamma_i (c) d\lambda (\alpha) d\alpha \\
= \int_{-\infty}^{\theta} \Gamma_i (g^{-1}(\theta) - \alpha) \lambda (\alpha) d\alpha,
$$

with $\Phi_i (\theta) = 0$, $\Phi_i (\theta) = 1$, and density $\phi_i (\theta) = \Phi_i ' (\theta)$. Note that, by virtue of the results in Miravete (2002), since both $\Lambda (\alpha)$ and $\Gamma_i (c)$ have a monotonic hazard rate, so does $\Phi_i (\theta)$, for every $i = 1, ..., n$. The result of this section shows that the ordering of the groups assumed in Assumption 1(a) can in fact be derived from the natural assumption that groups can be ordered according to the distribution of the quality of the social background. That is, the properties of the functions $\gamma_i (c)$ transfer to the functions $\Phi_i (\theta)$.

**Lemma 2** For every $i = 1, ..., n$, let $\Phi_i (\theta)$ be given by (3). For every $i \in \{2, ..., n\}$, let $\frac{1 - \Gamma_i (c)}{\gamma_i (c)} \leq \frac{1 - \Gamma_i (c)}{\gamma_i (c)}$ for every $c \in \mathbb{R}$, with strict inequality over a range. Then, for every $i = 2, ..., n$: $\frac{1 - \Phi_{i-1} (\theta)}{\phi_{i-1} (\theta)} \leq \frac{1 - \Phi_i (\theta)}{\phi_i (\theta)}$, with strict inequality over a range.

**Proof.** We can write (3) as:

$$
\Phi_i (\theta) = \Gamma_i \ast \lambda (g^{-1}(\theta)),
$$

where “$\ast$” denotes the convolution of $\Gamma_i$ and $\lambda$ (see Lang 1983, p 355). Now take a

\footnote{Both $\Lambda$ and $\Gamma_i$, $i = 1, ..., n$, have full support in $\mathbb{R}$. Relaxing this assumption would add complication and no insight.}
Fourier transform of $\frac{1-\Phi_t(\theta)}{\phi_t(\theta)}$, denoted by “$^\wedge$” (see Lang p. 363): 

\[
\left(\frac{1-\Phi_t(\theta)}{\phi_t(\theta)}\right)^\wedge = (1 - \Phi_t)^\wedge \ast \frac{1}{\gamma_i}^\wedge (g^{-1}(\theta)) = (1 - \Gamma_i)^\wedge \ast \left(\frac{1}{\gamma_i}\right)^\wedge \ast \left(\frac{1}{\chi}\right)^\wedge (g^{-1}(\theta)) \\
= (1 - \Gamma_i)^\wedge \ast \left(\frac{1}{\gamma_i}\right)^\wedge \ast \left(\frac{1}{\chi}\right)^\wedge (g^{-1}(\theta)) = (1 - \Gamma_i)^\wedge \ast \left(\frac{1}{\gamma_i}\right)^\wedge \ast \left(\frac{1}{\chi}\right)^\wedge (g^{-1}(\theta)) \\
= (1 - \Gamma_i)^\wedge \ast \left(\frac{1}{\gamma_i}\right)^\wedge \ast \hat{id} (g^{-1}(\theta)) = \left(1 - \frac{\Gamma_i}{\gamma_i}\right)^\wedge \ast \hat{id} (g^{-1}(\theta)).
\]

The first equality follows from Theorem 5.2 in Lang (1983, p 367), the second from (3), the third and the fourth from Lang (1983, p 365), the last line from Theorem 5.2 in Lang (1983), where $\Gamma_i$ is interpreted as a function of $g^{-1}(\theta)$, and where $\hat{id}$ is the identity in $\mathbb{R}$. This establishes the Lemma.

That is, if groups can be ordered by the hazard rate, and therefore, by Lemma 1, by first order stochastic dominance on the distribution of the variable $c$, then the same is true for the variable $\theta$.

### 2.3 Decision in the absence of government intervention

In the absence of public intervention, a household’s budget constraint is given by $Y = ke + b + t$, where $Y$ is the household income, $t$ the intergenerational transfer, and $k > 0$ the unit cost of privately provided education. A household’s optimisation problem is therefore:

\[
\max_{e,t} u(Y - ke - t) + y(\theta, e, E) + t. \tag{4}
\]

Let $e^S(\theta; k)$ be the value of $e$ satisfying $k = y_e(\theta, e, E)$, and let $\underline{Y}$ be the lowest household income. We assume that

\[
\underline{Y} \geq b^* + ke^S(\theta; k). \tag{5}
\]

This is an important assumption; it implies that no household is liquidity constrained. This can be either because all households can afford the level of education they wish to acquire, or because there are perfect capital markets. Clearly neither is realistic, but the point of the paper is that reverse discrimination policies may be required by efficiency considerations even when other imperfections that affect different groups differently, such as capital market imperfections, have been eliminated by appropriate policy intervention.\footnote{The case where $t$ is constrained to be non-negative and there are sufficiently poor households gives rise to situations where optimality requires differential treatment of individuals with different household income and is analysed in De Fraja (2002).}
It is immediate to verify that in the absence of public provision, a household where the child has potential to benefit from education $\theta$ chooses education level given by $e = e^S(\theta; k)$, and intergenerational transfer given by $Y - b r - ke^S(\theta; k)$ (note that, by (5), if the households who spend the most on education, those where the child has potential $\mu$ can afford its cost, then so can all other households). The investment in education is carried out to the point where its marginal benefit (the increase in future income) equals its marginal cost, $k$.\footnote{Given that, by construction, both the marginal benefits and the marginal costs are the same for all groups, so is the education level acquired by individuals of the same $\theta$. Individuals with higher $\theta$ acquire more education (this follows from $\frac{de(\cdot)}{d\theta} = -\frac{ue(\cdot)}{ye(\cdot)} > 0$). Therefore, on average, disadvantaged groups acquire less education: this is because, on average, they have fewer high $\theta$ individuals than more advantaged groups.}

The total household income is then distributed in such a way that the marginal benefit is 1 for both generations.

Note that household income has no effect on the amount invested in education, but simply affects the intergenerational transfer. This is because a person's benefit of education does not depend on her parents' income. Income is, of course, one of the characteristics that distinguishes groups: being born in a well-off household makes one more likely to have a high value of $\theta$ (see Solon 1999 for a survey of the papers studying income correlation across generations). Household utility, denoted by $P(\theta, E)$, is given by

$$P(\theta, E) = u(b^*) + y(\theta, e^S(\theta; k), E) + Y - b^* - ke^S(\theta; k),$$

and is increasing in $\theta$: $P_\theta(\theta, E) = y_\theta(\theta, e^S(\theta; k), E) > 0$. That is, a household where the child has better labour market opportunity is better off.

\section{Results}

\subsection{The problem of a utilitarian government}

The government maximises the utilitarian welfare function given by the unweighted sum of the utility of all households in the economy. The absence of any weighting of utility according to the position in the utility distribution rules out any possible redistributive preference in favour of some of the groups in the government objective function and therefore ensures that the optimality of reverse discrimination policies derived in Propositions 2 and 3 is due exclusively to efficiency considerations. Indeed, it is worth pointing out that here maximisation of total utility is equivalent to the maximisation of total income, because parents are interested in the maximisation of the child’s income and because all
households consume the same amount, \( b^* \), given in (1) and therefore the policy we derive maximises the monetary return of the investment in education.

To achieve its goal, the government selects the education – beyond a certain minimum, given by the compulsory level – to be received by each individual and the associated tuition fee. Both can be made conditional on the observable characteristics of the household, and on the child’s potential to benefit from education, \( \theta \), which however is not observable. Let therefore \( e_i(\theta) \) denote the education level offered to a household belonging to group \( i \) where the child has potential \( \theta \), and \( f_i(\theta) \) the tuition fee charged to this household. Different amounts may of course be charged to individuals in different groups for the same education, but the government will need to respect a number of constraints, derived in what follows.

We begin by noticing that, given a policy \( \{e_i(\theta), f_i(\theta)\} \), the household in group \( i \) where the daughter has opportunity \( \theta \) will, as before, choose a transfer \( t \) to maximise \( u(Y - f_i(\theta) - t) + y(\theta, e_i(\theta), E) + t \). This implies \( u'(Y - f_i(\theta) - t) = 1 \), so that the optimal transfer \( t^* \) is given by:

\[
t^* = Y - f_i(\theta) - b^*.
\] (6)

Therefore, if \( U_i(\theta) = u(Y - f_i(\theta) - t^*) + y(\theta, e_i(\theta), E) + t^* \) is the utility of a household in group \( i \) where the daughter has opportunity \( \theta \) who accepts the government offer, we can use (6) to write \( U_i(\theta) \) as:

\[
U_i(\theta) = u(b^*) + y(\theta, e_i(\theta), E) + Y - f_i(\theta) - b^*, \quad i = 1, \ldots, n, \quad \theta \in [\underline{\theta}, \overline{\theta}].
\] (7)

(7) holds provided that \( f_i(\theta) - b^* \leq Y + y(\theta, e_i(\theta), E) \): households can afford to pay the tuition fee charged by the government without having to reduce their current consumption below \( b^* \). We do not impose this as an explicit constraint, but verify that it holds at the solution. Next note that the government cannot make a household accept a combination of education and tuition fee which makes the household worse-off than it would be if it opted out of public provision by choosing private education. This is the first constraint:

\[
U_i(\theta) \geq P(\theta, E), \quad i = 1, \ldots, n, \quad \theta \in [\underline{\theta}, \overline{\theta}].
\] (8)

A second constraint follows from the fact that households have an information advantage vis-à-vis the government: unlike the household group, the child’s potential to benefit from education is private information. This implies, in the standard fashion, that the government must ensure that the chosen policy
satisfies the incentive compatibility constraint, which in this case implies:

\[
\frac{dU_i(\theta)}{d\theta} = y_\theta(\theta, e_i(\theta), E), \quad \frac{de_i(\theta)}{d\theta} \geq 0, \quad i = 1, \ldots, n, \quad \theta \in [\underline{\theta}, \bar{\theta}]. \tag{9}
\]

The government must also of course satisfy a budget constraint. In view of the externality, we assume that the government subsidises education by a fixed amount \(T > 0\). This, plus the tuition fees charged, must be sufficient to pay for the total education.\(^{14}\)

\[
T + \sum_{i=1}^{n} h_i \int_{\underline{\theta}}^{\bar{\theta}} f_i(\theta) \phi_i(\theta) d\theta - \sum_{i=1}^{n} h_i \int_{\underline{\theta}}^{\bar{\theta}} ke_i(\theta) \phi_i(\theta) d\theta \geq 0.
\]

Deriving \(f_i(\theta)\) from (7) and re-arranging, the above becomes:

\[
u(b^*) + Y - b^* + T + \sum_{i=1}^{n} h_i \int_{\underline{\theta}}^{\bar{\theta}} [y(\theta, e_i(\theta), E) - U_i(\theta) - ke_i(\theta)] \phi_i(\theta) d\theta \geq 0.
\]

Finally, the overall level of education in the economy, denoted by \(E\), is subject to a definitional constraint:

\[
\sum_{i=1}^{n} h_i \int_{\underline{\theta}}^{\bar{\theta}} e_i(\theta) \phi_i(\theta) d\theta = E. \tag{11}
\]

We can now state formally the optimisation problem faced by the government:

\[
\max_{E, \{e_i(\theta), U_i(\theta)\}_{i=1}^{n}} \sum_{i=1}^{n} h_i \int_{\underline{\theta}}^{\bar{\theta}} U_i(\theta) \phi_i(\theta) d\theta, \quad \text{subject to (8)-(11).} \tag{12}
\]

\(^{13}\)(9) follows from a straightforward application of the revelation principle: the government must ensure that each household prefers “its” combination of education level and associated payment to all other combinations available to households in the same group. If a household where the child has potential \(\theta\) chooses the combination designed for households of type \(\bar{\theta}\), its utility would be given by:

\[
\psi(\theta, \bar{\theta}) = u(b^*) + y(\theta, e_i(\bar{\theta}), E) + Y - f_i(\bar{\theta}) - b^*,
\]

and so the first order condition for choice of \(\bar{\theta}\) is

\[
\frac{\partial \psi(\theta, \bar{\theta})}{\partial \bar{\theta}} = y_\theta(\theta, e_i(\bar{\theta}), E) \frac{de_i(\bar{\theta})}{d\theta} - \frac{df_i(\bar{\theta})}{d\theta} = 0. \tag{\*)\]

Truthful reporting implies that the first order condition is satisfied at \(\bar{\theta} = \theta\). From (7):

\[
\frac{dU(\theta)}{d\theta} = y_\theta(\theta, e_i(\theta), E) + y_\theta(\theta, e_i(\theta), E) \frac{de_i(\theta)}{d\theta} - \frac{df_i(\theta)}{d\theta}.
\]

Substitute the above and \(\bar{\theta} = \theta\) into (*) to obtain the first part of (9). The derivation of the second part of (9) is also standard, and can be found in De Fraja (2002, p 460).

\(^{14}\)In De Fraja (2002), the subsidy \(T\) and the associated taxes are derived as part of the government optimisation problem. The qualitative features of the optimal education policy are the same as those derived here with the subsidy to the education sector exogenously set to \(T > 0\). An alternative approach leading to the same results is to assume an exogenously given value for the shadow cost of public funds, as in Laffont and Tirole (1993).
Towards solving (12), let $\bar{\theta}$ and $\phi$ be the lagrange multipliers associated to constraints (10) and (11), respectively. That is, $\bar{\theta}$ is the shadow cost of public funds, and $\phi$ measures the marginal benefit of additional education in the economy. For given $k_0$, let $e^G_i(\theta; \beta, k_0)$ be defined as the solution in $e$ of:

$$y_e(\theta, e, E) - k_0 = \frac{\beta - 1}{\beta} \frac{\Phi_i(\theta)}{\phi_i(\theta)} y_{e\theta}(\theta, e, E),$$

and, for given $\beta > 0$, $\phi > 0$, and $k$, let

$$k^* = k - \frac{\sigma}{\beta}.$$

Finally, let $\theta^*_i(\beta, k_0)$ be defined as the solution in $\theta$ of:

$$e^S(\theta; k) = e^G_i(\theta; \beta, k_0),$$

if a solution exists, and by $\theta^*_i(\beta, k_0) = \bar{\theta}$ if $e^S(\theta; k) < e^G_i(\theta; \beta, k_0)$. (13) is the standard optimality condition under asymmetric information when the cost of education is $k_0$: the LHS is the difference between the marginal benefit and the marginal (social) cost for type $\theta$; the RHS is a positive correction factor, which is 0 for $\theta = \bar{\theta}$ (efficiency at the top), or when the shadow cost of public funds is 1, and is positive otherwise (see Laffont and Tirole 1993, p. 65).

3.2 The optimal education policy.

We can begin to determine the optimal policy for the government. The first proposition describes the provision of education for individuals with different potential to benefit from education within a given group.

**Proposition 1** The education policy solving (12), $e^*_i(\theta)$, is given by

$$e^*_i(\theta) = \begin{cases} e^S(\theta; k) & \text{for } \theta < \theta^*_i(\beta, k^*) \\ e^G_i(\theta; \beta, k^*) & \text{for } \theta \geq \theta^*_i(\beta, k^*) \end{cases}.$$  

Moreover, the values of the multipliers at the solution are such that $\beta > 1$ and $\sigma = \sum_{i=1}^n h_i \int_{\theta^*_i(\beta, k^*)}^{\bar{\theta}} y_E(\theta, e^G_i(\theta; \beta, k^*), E) \phi_i(\theta) d\theta$.

**Proof.** (Sketch) The proof is a simplified version of the proof of Proposition 3 in De Fraja (2002). We report here the main steps, and devote an appendix (available on request from the author, and at www-users.york.ac.uk/~gd4/curres.htm#aaee) to the technical details.

The proof begins by showing that the “participation constraint” (8) must be binding for some households: there must necessarily be someone who is indifferent between state and private provision.
Lemma A1. For every \( i = 1, \ldots, n \), it cannot be that \( U_i(\theta) > P(\theta, E) \) for every \( \theta \in [\underline{\theta}, \overline{\theta}] \).

In the second step, it is shown that, if there are households whose utility is strictly above their reservation level, then all households with higher \( \theta \) in the same group also enjoy utility strictly above their reservation level.

Lemma A2. For every \( i = 1, \ldots, n \), let there exist \( \hat{\theta}_i \in [\underline{\theta}, \overline{\theta}] \) and \( \delta > 0 \) with \( U_i(\theta) > P(\theta, E) \) for \( \theta \in (\hat{\theta}_i, \hat{\theta}_i + \delta) \). Then \( U_i(\theta) > P(\theta, E) \) for (almost) every \( \theta \in (\hat{\theta}_i, \overline{\theta}] \).

That is, by Lemma A2, in each group there is a cut-off level of \( \theta, \hat{\theta}_i \in [\underline{\theta}, \overline{\theta}] \) such that households with \( \theta \) below this cut-off only receive their reservation utility, and households with higher \( \theta \) receive strictly more. The next lemma is the core of the proof: it draws the implication of Lemma A2 for the education received by the daughters in the two groups of households.

Lemma A3. \( e_i^\ast(\theta) = e^S(\theta; k) \) for \( \theta < \hat{\theta}_i \), \( e_i^\ast(\theta) = e_i^C(\theta; \beta, k^\ast) \) for \( \theta \geq \hat{\theta}_i \).

In words, this establishes that individuals below the cut-off point derived in Lemma A2 receive the same education that they would acquire privately, whereas individuals with \( \theta \) above this cut-off receive more education: they receive the second best education level when the cost is \( k^\ast \).

We now show that for (at least) some groups, there is a positive measure of households who are in the group above the cut-off \( \theta \) obtained in Lemma 2, that is, who receive strictly more utility that they obtain from private provision.

Lemma A4. There exists \( i \in \{1, \ldots, n\} \) such that \( \hat{\theta}_i < \overline{\theta} \).

This is a consequence of the positive transfer to the education sector, \( T > 0 \).

Next, \( e_i^\ast(\theta) \) is continuous, and therefore that \( \theta_i^\ast \) is in fact equal to \( \hat{\theta}_i \) and given by the condition \( e^S(\theta_i^\ast; k) = e_i^C(\theta_i^\ast; \beta, k^\ast) \).

Lemma A5. For every \( i = 1, \ldots, n \), \( e_i^\ast(\theta) \) is continuous.

The next lemma shows that there are individuals above the cut off point.

Lemma A6. For every \( i = 1, \ldots, n \), there exists \( a > 0 \) such that \( e_i^\ast(\theta) = e_i^C(\theta; \beta, k^\ast) \) for (almost) every \( \theta \in [\overline{\theta} - a, \overline{\theta}] \).

We have shown (Lemma A3) that households are divided into two subsets, those who receive the same education they would acquire privately, and those who receive more, and that the latter set has positive measure (Lemma A6). For individuals in the latter subset Lemma A3 determines \( e_i^\ast(\theta) \).

This establishes the main body of the proposition. There are a few “loose ends” to tidy up. Begin with the value of \( \sigma \): this follows immediately, using (7.120) in Leonard and van Long (1992), p 255, and differentiating the Lagrangean of problem (12) with respect to \( E \). We also need to show that \( e_i^\ast(\theta) \) is increasing in \( \theta \). Clearly, \( e^S(\theta_i^\ast; k) \) is increasing in \( \theta \). When \( \theta \geq \theta_i^\ast \), totally differentiate (13) and rearrange:
In view of the technical assumptions on the third derivative, the above is positive, establishing that $e^*_i(\theta)$ is increasing.

The proof is based on the assumption that $\beta > 1$. We now show that this must hold. By contradiction, if $\beta = 1$, then all households receive the same education, given by:

$$y_e(\theta, e^S(\theta, k - \sigma), E) = k - \sigma \quad i = 1, \ldots, n \quad \theta \in [\underline{\theta}, \overline{\theta}].$$

Notice that an increase in $e$ increases $E$, which in turn increases $e$; since $\lim_{e \to \infty} y_e(\cdot) = 0$, the benefit of education eventually falls below its cost and society cannot finance a large enough education level with a fixed budget.

The final thing to establish is that $f_i(\theta)$ is set at a level which allows everybody the optimal rate of consumption $b^*$. Clearly this is the case for the households who receive the same education as they would purchase privately. But just as clearly, for the other households, who have more utility than they would receive privately: if it were not the case, it would be possible to increase the value of the objective function by reducing the education provided to these households, and increasing their current consumption.

To interpret the Proposition, begin by noting that $k^*$ is the social cost of education. This is given by the monetary cost, $k$, reduced by the social benefit, $\sigma$, in turn reduced to take the shadow cost of public funds, $\beta$, into account. If $y_E(\cdot) = 0$, then $\sigma = 0$ and $e^*_i(\theta) = e^S(\theta; k)$, for every $i = 1, \ldots, n$, for every $\theta \in [\underline{\theta}, \overline{\theta}]$, and $T$ is transferred in cash to the household sector: in words, the optimal policy is “do nothing”, let the market operate undisturbed. This is natural: with no external effects of education, and no capital market imperfections, private provision is efficient, and the government simply duplicates it. If instead $y_E(\cdot) > 0$ for at least some $e > e^G_i(\theta; \beta, k^*)$, then $\sigma > 0$, and therefore $k^* < k$. This implies that the individuals with the highest opportunity (those for whom $\theta = \overline{\theta}$) receive strictly more education under public provision than they would purchase privately: see (13) when $\theta = \overline{\theta}$. Since $e^*_i(\theta)$ is continuous, this is also true for individuals whose $\theta$ is “close” to $\overline{\theta}$. On the other hand, individuals for whom $\theta$ is sufficiently low receive the same education they would receive if they acquired education privately.

The intuition for the particular shape for the relationship between opportunity and education for individuals in a given group is standard. Optimality requires that the education level decrease sharply as $\theta$ decreases. This is to provide a sufficient disincentive for those who have high potential to benefit from education to mimic the behaviour of those with low potential: since the former benefit more from education than the latter (as $y_{e, \theta}(\cdot) > 0$), it is more expensive for high $\theta$ individuals to give up education for a given reduction in
the tuition fee. This, however, cannot be pushed too far: for individuals with very low $\theta$, the education level $e^G_i(\theta; \beta, k^*)$ becomes too low, and they would opt out of public provision (De Fraja 2002, p 452).

The next proposition is the main result of this paper. It states that the optimal education policy is such that individuals from disadvantaged groups receive more education than individuals with the same potential to benefit from education in more advantaged groups.

**Proposition 2** Let Assumption 1(a) hold, then $e^G_{i-1}(\theta) > e^G_i(\theta)$, for every $i = 2, \ldots, n$, for every $\theta \in (\theta^G_{i-1}(\beta, k^*), \theta^G_i)$. 

This is a striking result. It provides a rationale for reverse discrimination purely on efficiency ground, with no appeal to equity or redistributive reasons whatsoever. *Individuals from a disadvantaged group are treated, other things equal, more favourably, precisely because of their appurtenance to the group.*

Figure 1 illustrates Proposition 2; it depicts, for three groups, the education received as a function of an individual’s potential to benefit from education. The solid (respectively, dotted, respectively, dashed) line depicts the education level received by individuals coming from the most advantaged (respectively, an averagely advantaged, respectively, the most disadvantaged) households. The
three lines meet at $\theta = \bar{\theta}$: individuals with the highest potential to benefit all receive the same education, irrespective of their group, given by the level where the marginal benefit of the investment in education equals the marginal social cost ("efficiency at the top": see (13), which shows that $y_\epsilon \left( \theta, c^*_i (\bar{\theta}), E \right) = k^*$ for every $i = 1, \ldots, n$). The education received by lower $\theta$ individuals declines as $\theta$ decreases, but, as illustrated in Figure 1, it declines less rapidly for disadvantaged groups: therefore, for sufficiently high $\theta$ (above the intersection of the higher of the two curves with the private education level) individuals from disadvantaged groups receive more education than individuals from advantaged groups with the same potential to benefit from education.

A different interpretative angle for the result in Proposition 2 is obtained if we draw a horizontal line from a given point on the vertical axis. The corresponding points on the horizontal axis give the potential to benefit of individuals who receive the same education level, that is, those who are enrolled in the same "school class" or "degree course" for the different groups. According to Proposition 2, the optimal policy is such that in a given class (below the very top, and above the lower levels of education), individuals from disadvantaged group have lower labour market potential than their classmates from more advantaged groups. This implies that an empirical analysis of labour market outcomes in the presence of the optimal education policy would show that the (post-education) labour market income of individuals from disadvantaged groups is lower than the labour market income of their classmates from more advantaged groups. This of course corresponds to the empirical findings by Herrnstein and Murray (1994, p 323), who however interpret it as an indictment of the US higher education policies.\footnote{Note moreover that their analysis has been criticised on methodological grounds: Neal and Johnson (1996), Cavallo et al. (1997), Cawley et al. (1997).} When $\theta$ is interpreted as ability, and when, as Herrnstein and Murray argue, ability is differently distributed in the various ethnic groups, this also tallies with their claim that in the elite universities the average ability of students from disadvantaged backgrounds is lower than for other groups (1994, p 472).\footnote{Other "utility costs", such as the perpetuation of racial stereotypes implied by the observation of lower achievement by individuals in certain groups (Murray 1994), or the sense of despair felt by minority students who find themselves in environments where they are unable to compete (D’Souza 1991) could be explicitly taken into account in the individual utility functions. To do so would dampen, without eliminating altogether, the differences in provision for individuals in different groups. Moreover, the wide-ranging investigation by Bowen and Bok (1998, especially pp 191-217) fails to unearth convincing evidence of these costs.}
We can now turn to the proof of Proposition 2. It hinges on the following Lemma, which also has independent interest.

**Lemma 3** At the solution of problem (12),
\[
e_i^G (\theta; \beta, k^* ) \geq e_j^G (\theta; \beta, k^* ) \quad \text{according to} \quad \frac{1 - \Phi_i (\theta)}{\phi_i (\theta)} \leq \frac{1 - \Phi_j (\theta)}{\phi_j (\theta)}. \quad (14)
\]

**Proof.** Let \( i > j \). To lighten notation, write \( e_i = e_i^G (\theta; \beta, k^* ) \) and \( e_j = e_j^G (\theta; \beta, k^* ) \).

Relationship (13) for the two groups considered becomes
\[
y_e (\theta, e_i, E) - k^* = \frac{\beta - 1}{\beta} \frac{1 - \Phi_i (\theta)}{\phi_i (\theta)} y_{\theta \theta} (\theta, e_i, E),
\]
\[
y_e (\theta, e_j, E) - k^* = \frac{\beta - 1}{\beta} \frac{1 - \Phi_j (\theta)}{\phi_j (\theta)} y_{\theta \theta} (\theta, e_j, E).
\]

Subtract the second from the first, and add and subtract \( \frac{\beta - 1}{\beta} \frac{1 - \Phi_i (\theta)}{\phi_i (\theta)} y_{\theta \theta} (\theta, e_j, E) \):
\[
y_e (\theta, e_i, E) - y_e (\theta, e_j, E) = \frac{\beta - 1}{\beta} \left[ \frac{1 - \Phi_i (\theta)}{\phi_i (\theta)} y_{\theta \theta} (\theta, e_i, E) - y_{\theta \theta} (\theta, e_j, E) \right] + \left[ \frac{1 - \Phi_i (\theta)}{\phi_i (\theta)} - \frac{1 - \Phi_j (\theta)}{\phi_j (\theta)} \right] y_{\theta \theta} (\theta, e_j, E).
\]

Applying the mean value theorem, there exist \( \hat{e} \) and \( \tilde{e} \) such that the above can be written as:
\[
y_{ee} (\theta, \hat{e}, E) (e_i - e_j) = \frac{\beta - 1}{\beta} \left[ \frac{1 - \Phi_i (\theta)}{\phi_i (\theta)} y_{\theta \theta} (\theta, \hat{e}, E) (e_i - e_j) + \left[ \frac{1 - \Phi_i (\theta)}{\phi_i (\theta)} - \frac{1 - \Phi_j (\theta)}{\phi_j (\theta)} \right] y_{\theta \theta} (\theta, e_j, E) \right],
\]
or:
\[
\left[ \frac{\beta - 1}{\beta} y_{ee} (\theta, \tilde{e}, E) - \frac{1 - \Phi_i (\theta)}{\phi_i (\theta)} y_{\theta \theta} (\theta, \tilde{e}, E) \right] (e_i - e_j) = \left\{ \frac{1 - \Phi_i (\theta)}{\phi_i (\theta)} - \frac{1 - \Phi_j (\theta)}{\phi_j (\theta)} \right\} y_{\theta \theta} (\theta, e_j, E).
\]

We have \( y_{ee} (\theta, \hat{e}, E) < 0 \) and \( y_{\theta \theta} (\theta, \hat{e}, E) > 0 \), implying that the term in the square brackets in (15) is negative.\(^{17}\) Therefore the sign of \( e_i - e_j \) is the opposite of the sign of the term in the curly brackets. This establishes the Lemma. \( \square \)

To complete the proof of Proposition 2 simply invoke Assumption 1(a).

By Lemma 3, given two individuals with the same potential to benefit from education belonging to different groups, which of the two should receive more education depends exclusively on the distribution of labour market opportunities in the groups to which these individuals belong.

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\(^{17}\) Lest be thought that the result depend crucially on the sign of the third cross derivative \( y_{ee \theta} (\cdot) \); note that, for the Lemma to hold, it is necessary and sufficient that the term in the square brackets on the LHS of (15) is negative. But as can be seen from the proof of Proposition 1, the negativity of this term is a necessary condition for \( e_i^* (\theta) \) to be increasing, which is a constraint to problem (12). In other words, in order for problem (12) to have the solution identified in Proposition 1, the term in the square bracket in (15) must be negative.
3.3 Interpretation

While the result in Proposition 2 may appear surprising, the intuition underlying it is relatively natural. To present it as clearly as possible, we investigate some further features of the optimal policy. These have independent interest. The first result illustrates that, with symmetric information, all individuals with the same potential to benefit receive the same education.

**Corollary 1** Suppose the government can costlessly observe $\theta$. Then the optimal policy satisfies:

$$e_i^* (\theta) = e^S (\theta; k^*) \quad \text{for every } i = 1, \ldots, n \quad \theta \in [\underline{\theta}, \overline{\theta}].$$

**Proof.** The government problem in this case is obtained from problem (12) by eliminating constraint (9), and the result follows immediately from the proof of Proposition 1, by setting the Pontryagin multiplier $\mu_i (\theta)$ identically to 0.

That is, with no information asymmetry, everybody receives education up to the point where their private benefit, $y_e (\cdot)$, equals the – endogenously determined – social marginal cost, $k^*$.

The next result derives the individual rent. Denote by $U_i^* (\theta)$ the utility obtained at the solution of problem (12) by a household in group $i$ where the child has potential $\mu$ to benefit from education.

**Corollary 2** Let Assumption 1(a) hold; then $U_{i-1}^* (\theta) > U_i^* (\theta)$ for every $i = 2, \ldots, n$, and for every $\theta \in (\theta_{i-1}^* (\beta, k^*), \overline{\theta}]$.

**Proof.** Note that $\theta_{i-1}^* < \theta_i^*$ (omitting the argument $(\beta, k^*)$), and, therefore, for $\theta \in (\theta_{i-1}^*, \theta_i^*)$ we have $U_{i-1}^* (\theta) > U_i^* (\theta)$, and, in particular, $U_{i-1}^* (\theta_i^*) > U_i^* (\theta_i^*)$. Next consider $\theta \in (\theta_i^*, \overline{\theta})$. Here we have: $\frac{dU_{i-1}^* (\theta)}{d\theta} = y_0 (\theta, e_{i-1}^* (\theta), \bar{E}) > y_0 (\theta, e_i^* (\theta), E)$ = $\frac{dU_i^* (\theta)}{d\theta}$: the inequality follows from the fact that $e_{i-1}^* (\theta) > e_i^* (\theta)$ and $y_{\epsilon \theta} (\theta, e, E) > 0$. But clearly, $U_{i-1}^* (\theta_i^*) > U_i^* (\theta_i^*)$ and $\frac{dU_{i-1}^* (\theta)}{d\theta} > \frac{dU_i^* (\theta)}{d\theta}$ imply the corollary.

That is, at the optimal policy, households in disadvantaged groups receive, ceteris paribus, more utility.

Figure 2 illustrates the corollary: it depicts the utility received by individuals in three groups of households. As before, the solid (respectively, dotted, respectively, dashed) line depicts the utility of individuals coming from the most advantaged (respectively, with an average advantage, respectively, the most disadvantaged) households. Households in group $i$ receive utility equal to what they can obtain privately up to $\theta_i^* (\beta, k^*)$ and higher utility if their children have labour market opportunity above that level.
We can now present the intuition for Proposition 2. Note, to begin with, that the households where the child has high potential to benefit from education receive more utility than the utility they would receive from private provision, depicted in Figure 2 as the thin line $P(\theta, E)$. This additional utility, and the incentive to acquire more education than the private level, is provided in the form of subsidised education: the tuition fee charged to a type $\theta$ individual from group $i$, $f_i(\theta)$, is less than the cost of her education, $k e_i(\theta)$ (see De Fraja (2002), Proposition 4, p 463). But, to the extent that the government budget constraint is binding, this subsidy has a social cost, and therefore the government attempts to reduce it, by reducing sharply the education offered to individuals with progressively lower $\theta$, as described in the discussion following Proposition 2. Note that this does not happen in conditions of symmetric information, (Corollary 1), or when the shadow cost of public funds is 1 (let $\beta = 1$ in (13)): in both these cases, rent extraction is not a concern, and the education received by each individual is independent of her group. With asymmetric information and costly public funds, however, there is a trade-off between the social cost of the rent received by high $\theta$ individuals and the social cost of distorting the education received by lower $\theta$ individuals. But now, and here is the crux of the argument, notice that the relative weight of
these costs in the social welfare function is determined by the relative number of individuals of different potential to benefit from education. More precisely. Take a given level of $\theta < \overline{\theta}$, say $\theta_0$; take a small interval $(\theta_0 - \varepsilon, \theta_0 + \varepsilon)$: there are $\Phi_i(\theta_0 + \varepsilon) - \Phi_i(\theta_0 - \varepsilon) \approx 2\varepsilon \Phi_i(\theta_0)$ individuals with this value of $\theta$ in this group, and $1 - \Phi_i(\theta_0)$ individuals with $\theta$ above $\theta_0$. Therefore, the number of individuals with $\theta$ above $\theta_0$ per individual with $\theta = \theta_0$, is (proportional to) precisely the hazard rate, $\frac{1 - \Phi_i(\theta_0)}{\Phi_i(\theta_0)}$. Because of Assumption 1, the hazard rate is smaller in a disadvantaged group than in an advantaged group. But this implies that there are fewer individuals with $\theta$ above $\theta_0$ in groups with a lower hazard rate, and therefore, in aggregate, it is relatively less costly to give more rent to the high $\theta$ individuals in a group with a lower hazard rate in order to reduce the social loss given by the reduction in education below the social optimum for type $\theta_0$ individuals in that group. And this is exactly what Proposition 2 says. Conversely, if a group has a high hazard rate, then it has more above $\theta_0$ individuals per $\theta_0$ individual, and the social saving obtained by reducing their informational rent becomes relatively more attractive than the social cost incurred by reducing the education level offered to lower $\theta$ individuals.

3.4 Financial contribution

It is now fairly simple to derive the relationship between the level of education received and the tuition fee paid. Within a group, the analysis is essentially identical to the standard model of incentives under asymmetric information (Laffont and Tirole 1993, pp 69-70), which shows that the optimal policy can be implemented by offering all households in a given group an appropriately designed menu of contracts. In our model, this means an appropriately designed schedule of fees for the possible education levels. The dotted line in Figure 3 illustrates the situation for group $i$: the locus $F_i(e)$ denotes the fee paid by individuals in group $i$ for choosing the education level $e$. It is straightforward to show (Laffont and Tirole 1993, pp 69-70) that if $F_i(e_i^*(\theta)) = f_i^*(\theta) \equiv y(\theta, e_i^*(\theta), E) - U_i(\theta) + Y + u(b^*) - b^*$, for every $\theta \in [\underline{\theta}, \overline{\theta}]$, then households of type $\theta$ in group $i$ choose the combination $\{e_i^*(\theta), f_i^*(\theta)\}$, and that the locus is increasing and concave, as shown. More central to the topic of this paper is the relationship between the schedules for different groups. Proposition 3 establishes that households in disadvantaged groups are required to pay a lower fee for the same education, as depicted in Figure 3.

**Proposition 3** Let $F_i(e)$ and $F_j(e)$ be the fee schedule available to households
in groups $i$ and $j$ respectively, with $i < j$. Then $F_i(e) \leq F_j(e)$, with strict inequality for $e > e_i^*(\theta_i^*)$.

**Proof.** We need to consider three regions, $e \leq e_i^*(\theta_i^*)$, $e \in (e_i^*(\theta_i^*), e_j^*(\theta_j^*))$, and $e > e_j^*(\theta_j^*)$. In the first $\theta < \theta_i^*$, and, $e_i^*(\theta) = e_j^*(\theta) = e^S(\theta; k)$ and $U_i(\theta) = U_j(\theta)$. Here therefore $F_i(e) = F_j(e)$. Consider the third region. Here both groups receive an education level strictly above what they would receive privately (in the second region this is the case only for the advantaged group, and the argument is essentially the same, if notationally slightly more complicated). Take $\hat{\theta}_i$ and $\hat{\theta}_j$ be the corresponding values of $\theta$: $\hat{\theta}_i = e_i^*(\hat{\theta}_i) = e_j^*(\hat{\theta}_j)$. We want to show that $f_i^*(\hat{\theta}_i) < f_j^*(\hat{\theta}_j)$.

Substitute the values of $f_i^*(\hat{\theta}_i)$ and $f_j^*(\hat{\theta}_j)$ from (7), to obtain:

$$f_j^*(\hat{\theta}_j) - f_i^*(\hat{\theta}_i) = [y(\hat{\theta}_j, \hat{\theta}_i, E) - y(\hat{\theta}_i, \hat{\theta}_j, E)] - [U_j(\hat{\theta}_j) - U_i(\hat{\theta}_i)].$$

(16)

The term in the first square brackets can be written as $\int_{\hat{\theta}_i}^{\hat{\theta}_j} y_0(\theta, \hat{\theta}_j, E)d\theta$, and, adding and subtracting $U_j(\hat{\theta}_i)$, (16) can be written as:

$$f_j^*(\hat{\theta}_j) - f_i^*(\hat{\theta}_i) = \int_{\hat{\theta}_i}^{\hat{\theta}_j} y_0(\theta, \hat{\theta}_j, E)d\theta - \left[ U_j(\hat{\theta}_j) - U_i(\hat{\theta}_i) \right] - \left[ U_j(\hat{\theta}_i) - U_i(\hat{\theta}_i) \right].$$

Note that the second square brackets on the RHS of the above is:

$$\int_{\hat{\theta}_i}^{\hat{\theta}_j} y_0(\theta, e_j^*(\hat{\theta}_j), E)d\theta - \int_{\hat{\theta}_i}^{\hat{\theta}_j} y_0(\theta, e_j^*(\hat{\theta}_i), E)d\theta = \int_{\hat{\theta}_i}^{\hat{\theta}_j} y_0(\theta, e_j^*(\hat{\theta}_i), E)d\theta,$$

and that, since $\hat{\theta} = e_j^*(\hat{\theta}_j)$, the RHS of (16) can be written as:

$$\left[ \int_{\hat{\theta}_i}^{\hat{\theta}_j} y_0(\theta, e_j^*(\hat{\theta}_j), E)d\theta + \int_{\hat{\theta}_i}^{\hat{\theta}_j} y_0(\theta, e_j^*(\hat{\theta}_i), E)d\theta \right] + \left[ U_i(\hat{\theta}_i) - U_j(\hat{\theta}_i) \right].$$


The term in the first square bracket is positive because \( \hat{\theta}_j > \tilde{\theta} \) for \( \tilde{\theta} \in (\hat{\theta}_i, \hat{\theta}_j) \), and \( y_{\theta_e}(\cdot) > 0 \), the second is positive by Corollary 2. This establishes Proposition 3.

We end the paper with a remark, which tempers somehow the reverse discrimination flavour of the paper. While it is clear that an individual from a disadvantaged group makes a bigger claim on the total budget of the education sector, \( T \), than an otherwise identical individual from a more advantaged group, it is ambiguous whether a disadvantage group as a whole also has a bigger claim than a more advantaged group. Formally, this can be seen by noting that the total rent of group \( i \) is given by \( \int_{\hat{\theta}}^{\tilde{\theta}} [U_i(\hat{\theta}) - P(\hat{\theta}, E)] \phi_i(\hat{\theta}) d\hat{\theta} \), and that the difference between the value of this expression for different groups cannot be determined in general.

4 Conclusion

The paper derives the optimal education policy in the presence of groups which differ according to the distributions of individuals’ potential to benefit from education. Because of asymmetric information, the first best policy cannot be implemented. The second best optimal policy is an instance of reverse discrimination: it favours individuals from disadvantaged backgrounds, who need a lower potential to benefit to receive a given education level and who pay a lower fee for the same education than individuals from more advantaged backgrounds. Implementation of the policy can be achieved in practice through financial assistance programmes which are differentiated according to the group of appurtenance of applicants. We stress the efficiency viewpoint of the paper, which derives these policies not from a sense of justice or fairness, or from the desire of righting past wrongs, but from a dispassionate calculation of society’s costs and benefits, using the viewpoint (standard in normative analysis in public economics) of a utilitarian welfare function. Our result holds irrespectively of the reason why one group is disadvantaged. Indeed, paradoxically, if differences in labour market opportunity between groups are due to differences in the distribution of innate ability between individuals in various groups (be they genetically or environmentally determined) which are unavoidable and will not be changed by conscious policy intervention, then the bias in education policy favouring disadvantaged groups illustrated in Propositions 2 and 3 should also be persistent. If, on the other hand, differences in the distribution of labour market opportunity can be reduced by other forms of intervention, the opti-
mal education policy would tend to become “group blind”: the three curves in Figures 1, 2, and 3 would tend to converge to a single one, in line, for example, with the view expressed by US Supreme Court Justice O’Connor, who “expect[s] that 25 years from now, the use of racial preferences will no longer be necessary”.

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References


