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Abstract

This paper derives the optimal response of the primary budget surplus to changes in the debt-to-GDP ratio in a stochastic model of debt. Under the optimal solution the surplus reactivity to the debt-to-GDP ratio is independent of the debt ratio itself, but its size depends on economic fundamentals and on the degree of uncertainty surrounding the impact of fiscal policies. We propose two measures of fiscal resilience under the optimal control that may be used to gauge the soundness of a consolidation plan and as early warning indicators of fiscal imbalances.
Keywords: Debt-to-GDP Ratio, Optimal Control, Fiscal Consolidation, Resilience.
JEL Classification Codes: H62, H63, E63.

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1 Introduction

Since the Great Recession and the global financial crisis, dramatic fiscal developments have brought renewed attention on the issue of public debt sustainability. This issue has turned out to be of particular concern for several members of the Euro Area, where the risk of a debt overhang has kicked in. The severe degree of uncertainty surrounding the effects of fiscal actions calls for the development of new tools of analysis that allow us to study the issue of the debt-to-GDP-ratio control in a stochastic environment, and provide measures of fiscal resilience that may come in handy as early warning indicators in the surveillance of fiscal imbalances and as indicators of the soundness of fiscal policy packages. To this end this paper proposes a continuous time stochastic model of debt where a budget rule automatically triggers a correction mechanism of the primary surplus to the debt-to-GDP ratio. The problem is that of a government wishing to narrow the gap between the current level of debt ratio and a given target, while uncertainty comes through shocks that may frustrate or magnify the effects of any fiscal intervention, and thus the effective size of the primary surplus. In the attempt to close the gap between the actual debt ratio and a given target, the government faces a trade-off between the need of a strong fiscal reaction and the uncertainty that the reaction itself may generate. In particular, the government follows a fiscal rule that incorporates two components: (i) an endogenous component according to which the government reacts to the rising of public debt by increasing the primary surplus, (ii) an exogenous component reflecting budgetary shocks and capturing interdependencies between the fiscal stance and other determinants of debt accumulation, such as interest rates, GDP growth, expectations etc...

By relying on optimal control theory and applying the Hamilton-Jacobi-Bellman equation, we show that under the optimal Markov control the relationship between the primary surplus and the deviation of the debt-to-GDP ratio from its target is linear. The reactivity of the primary surplus to the debt ratio is increasing in the growth rate of GDP, but decreasing in the real interest rate bearing on debt and in the degree of uncertainty surrounding the outcome of fiscal policies. Thus the optimal correction rule prescribes a more vigourous fiscal consolidation effort under favourable economic conditions.

Given the optimal control we propose two measures of fiscal resilience in the presence of uncertainty. One measure simply refers to the time needed to reach an expected debt ratio. Clearly, the larger the time interval necessary to close the gap by a given amount, the lower the degree of fiscal resilience. Nonetheless, the time necessary to close the gap will be affected by GDP growth, the rate of return on debt, and the uncertainty surrounding the effects of fiscal policy.

The second measure of fiscal resilience refers to the probability that the public-debt ratio is on the right path towards a fiscal consolidation objective. This probability relies on the concept of harmonic measure, describing the probability distribution of the debt ratio as it hits the boundary of a given open interval.[1]

[1] The theory of harmonic measure has been extensively used in several applications, such as the corona problem and in mapping problems. It has particularly interesting applications in probability theory, especially in relation to Brownian motions. See Garnett and Marshall (2005)

The idea is that of seeing whether the debt ratio bends towards a target or not. In a way, it can be interpreted as an index of vulnerability to potential shocks that may undermine the capability to be on a stable trajectory. As far as we know we are the first to employ an approach based on a harmonic measure to construct a measure of fiscal resilience.

We argue that both the proposed measures of fiscal resilience may be fruitfully used as indicators of the goodness of a fiscal package and as pre-alert indicators of fiscal imbalances.

Our paper contributes to the literature that applies optimal control theory to public debt management in a stochastic environment. This methodology allows us to rigorously include random components in the policy action and assess the benefits and the costs of fiscal interventions under external shocks.[2] In this respect, we seek to contribute to this literature by providing a theoretical background for debt reduction policies under uncertainty based on an optimising set-up. We also show how this methodology can be used to assess fiscal resilience, meant as the capability to withstand changes under the optimal control.

The closest predecessors of our paper are those dealing with stochastic control problems of the debt-to-GDP ratio. In particular, Ferrari (2018) explores the case of a government whose objective is that of reducing the debt-to-GDP ratio through the minimization of two opposing costs, namely the expected opportunity cost of having debt on the one hand, and the expected cost from the reduction policy on the other hand. In more detail, Ferrari (2018) shows that the solution of the control problem is related to that of an auxiliary optimal stopping problem. Put it differently, dealing with the optimal stopping problem is equivalent to work out the solution of the corresponding control problem. In conclusion, the optimal policy is found to be that of keeping the debt-to-GDP ratio under an inflation-dependent ceiling. Ferrari and Rodosthenous (2018) introduce the problem of a government managing the debt-to-GDP ratio in a stochastic continuous time model where uncertainty comes through a macroeconomic risk process affecting the interest rate bearing on public debt. The exogenous risk process is modelled as N-state continuous-time Markov chain, while the government faces a trade-off between the potential benefits from high public investments and the costs deriving from having an excessive debt ratio and austerity policies. At the optimum the government would keep the debt ratio in an interval whose boundaries depend on the possible states of the Markov process. Callegaro et al. (2019) study the problem of a government aiming at reducing the debt ratio under partial information where the underlying macroeconomic conditions are not directly observed.

for a survey of the theory and applications concerning this measure.

[2]In the last decades debt sustainability analysis has evolved to account for uncertainty. This strand of literature, mostly developed at institutional level and within international organizations, explicitly accounts for the fact that fiscal solvency and debt behaviour depend on the future dynamics of economic fundamentals that are not known for sure (e.g. Berti 2013, Rozenov 2017 and Cherif and Hasanov 2018), highlights the importance of designing fiscal rules that are truly operational (see Eyraud et al. 2018), and proposes methods to quantify the fiscal stress (e.g. Balducci et al. 2011 and Pamies Sumner and Berti 2017) and the fiscal space (Ghosh et al. 2013).

Cadenillas and Huamán-Aguilar (2016) develop a stochastic debt control model to find the optimal ceiling for the government debt. As in Ferrari (2018) the government objective is that of minimising the trade-off between the opportunity cost of having debt and the cost from arising from its reduction. Cadenillas and Huamán-Aguilar (2016) obtain a closed-form solution for the optimal government debt ceiling and find that the fiscal policy will be active if the debt-to-GDP ratio is greater than the optimal debt ceiling, while a passive fiscal policy will be desirable if debt is lower than the ceiling. The model is also extended to account for the link between debt and economic growth.[3] In a subsequent paper Cadenillas and Huamán-Aguilar (2018) study the optimal debt ceiling accounting for the fact that the ability of the government to reduce its debt ratio is bounded.

Starting from these contributions we solve a stochastic control problem to find the optimal corrective action of primary surplus to changes in the debt-to-GDP ratio. The novelty of our paper is twofold.

First, in our setup the aim of the government is that of minimising the cost of having a debt higher than a given target. In pursuing this fiscal sustainability objective the government follows a reaction rule according to which the size of the corrective measures depends upon the distance between the actual debt-to-GDP ratio and its target.[4] However, the effective size of the primary surplus may be affected by shocks. Uncertainty affects the effective size of the fiscal policy and thus the effectiveness of debt-reduction policies.

Second, we assess the properties of the optimal stochastic control in terms of fiscal resilience. This is useful to single out the role of uncertainty in setting the optimal control and identify the major factors that may undermine the achievement of the target.[5]

The remainder of the paper is structured as follows. Section 2 lays out the model. Section 3 introduces and solves the optimisation problem of the government. Section 4 presents two measures of resilience of the optimal policy. Section 5 presents concluding remarks.

2 The Model Setup

A simple starting point for the formal discussion of public finances is the flow budget constraint of the government which dictates that the next period debt is

[3]In a previous contribution Huamán-Aguilar and Cadenillas (2015) propose a stochastic model for government debt control under the assumption that debt may also be issued in foreign currency. They show that for high debt aversion and exchange rate uncertainty, it is optimal to reduce the share of the debt burden denominated in foreign currency in favour of domestic currency.

[4]For a study in which the objective of the government is, instead, that of keeping the level of output closer to a reference value in the attempt of stabilizing the economy over the business cycle, see Correani et al. (2014), who use optimal control theory and apply the Hamilton-Jacobi-Bellman equation in a stochastic IS-LM model.

[5]The fiscal resilience measures we propose in this paper do not cover all the possible metrics recognised in the related literature. See Alessi et al. (2018) for a wide-ranging analysis of the resilience of European countries.

given by the current period debt minus the primary surplus (government revenues minus expenditures excluding interest payments) times a gross interest factor. Following Bohn (1998) this relationship can be easily written in terms of GDP share as follows:

$$B_{t+1} = \frac{1+r}{1+g}(B_t - S_t), \tag{1}$$

where B is the the stock of public debt as a proportion of GDP, S is the primary surplus-GDP ratio, r denotes the average effective interest, supposed constant, on government debt, and g is the average (trend) growth rate of GDP.[6]

By making use of the approximation $\frac{1+r}{1+g} \approx 1 + r - g$, equation (1) can be equivalently expressed in terms of deviations of the debt-GDP ratio from a generic debt target, say $\overline{B}$, as follows:

$$X_{t+1} = (X_t - Z_t)(1 + \alpha). \tag{2}$$

where $X_t = B_t - \overline{B}$ is the debt-GDP ratio deviation from its target, $\alpha = r - g$ and $Z_t = S_t - \alpha\overline{B}/(1 + \alpha)$ is the primary surplus (as ratio of GDP) scaled down by a constant factor. The term α then reflects market fundamentals.

The continuous time counterpart of equation (2) is as follows:

$$dX_t = \alpha X_t dt - (1 + \alpha)dv_t, \tag{3}$$

where dX_t is the differential of X_t and v_t is the cumulative primary surplus up to time t expressed as a proportion of the GDP.

We assume that the behaviour of the primary surplus is described by a debt-based reaction rule adjusted for uncertainty. In particular, we focus on a reaction rule to the debt-GDP deviation from the target of the form:

$$dv_t = \rho_t X_t dt + \sigma \rho_t X_t dW_t, \tag{4}$$

where ρ_t is the government control variable measuring the strength of the primary surplus response to the debt ratio gap X_t; σ represents the diffusion coefficient meant to transmit uncertainty to the response action of policy makers; W_t is a one-dimensional Brownian motion with zero mean and density function given by a Gaussian exponential law of the type:

$$W_t \ \sim \ \frac{e^{-\frac{y^2}{2t}}}{\sqrt{2\pi t}}. \tag{5}$$

Based on this feedback rule, the government undertakes corrective measures whose size depends upon the gap between the debt ratio and its target.[7] However, the effective size of the fiscal action may be affected by shocks. Similarly to Leeper (1991), we consider a fiscal reaction rule adjusted for a random error so as

[6]According to (1) interest payments on the outstanding debt are made at the end of period. If a government borrows at an interest rate that exceeds the growth rate of the economy, continuing surpluses will be needed to avoid an explosive debt path.

[7]The rule prescribes a response of the primary surplus ratio to changes in the debt ratio in the spirit of Bohn (1995, 1998).

to capture various non-modelled sources of uncertainty.[8] The first term in (4) pins down the structural component of the fiscal reaction function and measures the response of the government to the debt-GDP ratio deviation from its target.[9] The second term in (4) captures the uncertainty that may surround the final outcome of any fiscal intervention and is related to the macroeconomic effects of fiscal policy. This second term is a sort of 'catch-all' stochastic component reflecting the shocks that may hit the economy and any other unpredictable dynamics stemming from fiscal actions or the political decision process. Clearly, this stochastic component affects the effective size of the primary surplus ratio. However, differently from Leeper (1991), we assume that the volatility of the random component in the debt rule (4) is time varying, and its size of depends on the size of the structural component itself.[10]

This uncertainty may originate from several factors and macroeconomic interdependency mechanisms.[11] An excessive surplus correction may give rise to distributional consequences and social tensions if it entails cuts of spending on welfare such as pensions and health care. Reduction of debt through cuts of these items may be very unpopular and costly to policymakers, pushing towards a downward revision of the initial corrective plan. This change of incentives will give rise to a negative shock in (4). Besides, an ambitious fiscal consolidation plan may deteriorate economic conditions to such an extent that tax revenues decline and social spending increases, partially frustrating the initial correction. This self-defeating mechanism of the corrective measure may thus lead to a negative shock.[12] Similarly, a strong corrective fiscal intervention may undermine growth prospects, pushing private investors to cut down their investment plans, leading to a knock-on effect to the level of economic activity and thus to the debt-to-GDP ratio. Different beliefs about the type of fiscal consolidation may give rise to waves of optimism that may improve the performance of the consolidation itself or, alternatively, to waves pessimism that may magnify the contractionary effects the ongoing specific fiscal plan.[13]

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[8]Actually Leeper (1991) considers a fiscal rule consisting of a systematic response of taxation to government debt plus a random shock. For an empirical analysis on fiscal reaction function incorporating volatility of the fiscal stance, see e.g. Anzuini et al. (2017).

[9]It should be noted when the target is reached (i.e. $X_t = 0$), the rule prescribes a balanced-budget rule with primary surplus covering interest payments on debt.

[10]As in Anzuini et al. (2017) we then assume that the volatility of the random component is time varying. From an econometric point of view rule (4) reflects the existence of endogeneity. For other relevant contributions on fiscal uncertainty shocks, see Born and Pfeifer (2014) and Fernández-Villaverde et al. (2015).

[11]See also Balibek and Köksalan (2010) for a model of debt management taking into account the uncertainty concerning the future state of the economy.

[12]See DeLong et al. (2012). According to empirical evidence, fiscal multipliers are large during recessions and small when the economy operates close to potential. See Auerbach and Gorodnichenko (2012) and Corsetti et al. (2013). On the positive effects of fiscal expansion see e.g. Blanchard and Perotti (2002).

[13]The effects of fiscal actions also depend on the underlying monetary-fiscal policy regime, on expectations about future regimes and on the credibility of an announced fiscal plan. All these factors are not directly controlled by policymakers. In this respect, for a comprehensive discussion on how ”darned hard” fiscal analysis is, see Leeper (2015).

However, debt reduction may be also conducive to positive shocks. Indeed, a surplus correction may increase the confidence of private investors, so that we may observe a positive effect on the surplus ratio. Further, possible non-Keynesian effects of fiscal policy may give rise to beneficial effects on the budget balance by magnifying the effects of a fiscal intervention. A credible fiscal consolidation plan can signal future reductions of taxes and, therefore, an increase in permanent income, producing an increase in private consumption. Private investments may also respond positively, via the interest rate channel or an expected lower tax burden in the future.[14] Non-Keynesian effects of fiscal policy could then account for positive shocks. However, as shown by Botta (2018), the circumstances under which austerity may be expansionary are very fragile.

In general, the stochastic component in (4) may reflect a sort of control error in the policy implementation. The idea is that policymakers can control their instruments only up to a random error. Overall, according to (4) in the attempt to close the gap between the actual debt ratio and a given target, policymakers must strike a balance between the need of a strong action and the uncertainty that the action itself may magnify. This trade-off fades away when the debt gets closer to the target.

To scrutinise how the debt accumulation (4) works in the presence of a stochastic term, we substitute the fiscal rule (4) into equation (3) and obtain:

$$dX_t = [\alpha - (1+\alpha)\rho_t]X_t dt - \sigma(1+\alpha)\rho_t X_t dW_t. \tag{6}$$

The variable described by equation (6) is an Itô process with a unique solution, since it satisfies the two conditions for the existence and uniqueness of the solution. For more details about these conditions, see Øksendal (2003).

The stochastic integral component $\rho_t X_t dW_t$ in the debt dynamics thus represents the fact that the debt ratio can increase (or decrease) due to some factors that are beyond the control of the government. Thus, when the government is faced with a high public debt load two effects are in place. First, by means of the deterministic term in (4), the government is able to push the debt-GDP ratio down. In this case the magnitude of the reaction will depend on the specific setting of ρ_t. Second, the higher ρ_t, the more extensive the propagation mechanism of uncertainty through the stochastic term.

We now shortly discuss some of the mathematical features underlying equation (6). The intent is twofold: to clarify the mathematical notation of the text and define precisely the control variable. Since for every t we have a random control variable which the random variable X_t depends upon, we consider a complete probability space Ω with filtration $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$, where the filtration $\mathcal{F}_t$ is the one generated by the standard one-dimensional Brownian motion W_t and is augmented by $\mathbb{P}$-null sets, that is

$$\mathcal{F}_t = \sigma\Big(W_s,\ 0 \leq s \leq t\Big) \cup \Big\{A \in \mathcal{F} \,|\, \mathbb{P}(A) = 0\Big\}, \qquad \forall\, t \geq 0. \tag{7}$$

Moreover, we assume that X_0 is an integrable random variable with law π_0 and measurable with respect to $\mathcal{F}_0$ representing the initial value of the current

[14]See e.g. Giavazzi and Pagano (1990) and Alesina and Ardagna (2013).

debt-to-output ratio gap. For the sake of simplicity, we assume that at the time 0 the debt-ratio gap is deterministic, so that $X_0 = x$ is a constant with $x > 0$.

The control variable $\rho_t = \rho(t, \omega)$ is taken from a given family $\mathcal{A}$ of admissible controls:

$$\mathcal{A} := \Big\{ \rho_t = \rho(t, X_t(\omega)) \text{ for some Borel-measurable} \\ \text{bounded functions } \rho : [0, T] \times [0, \bar{x}] \longrightarrow [0, \bar{\rho}] \Big\} \tag{8}$$

and is adapted to the filtration (7).

At time t the value of the function $\rho(t, X_t)$ only depends on the state of the system at that time, thus it does not depend on the probability space ω explicitly, but only through the process X_t. Such a ρ is called *Markov control* and the corresponding process X_t becomes an Itô diffusion, in particular a Markov process (see Øksendal 2003, section 11.1). For further technical details and definitions, see Appendix A.

3 The Optimisation Problem

In this Section we consider the problem of a government aiming at keeping the current level of debt ratio as close as possible to the reference value $\overline{B}$. Moreover, the fiscal authority is assumed to have always access to the available policy tool ρ, that is the strength of the primary surplus response to the debt ratio gap. The government is assumed to be increasingly worse-off the larger the debt ratio deviations from the target. The idea is that the government faces an instantaneous loss related the rising of the public debt ratio. Notably, a large public debt may crowd out private investment undermining growth prospects.[15] In addition one of the potential effects associated with a rising public debt is that of an increase in the perceived risk that a country may default on its debt. This change in market sentiments may push an economy towards a bad equilibrium through self-fulfilling upward effects on yields and debt may become unsustainable. Moreover, since the unpleasant arithmetic of Sargent and Wallace (1981) it has been well known that it is impossible for a monetary authority to sustain low inflation in the presence of excessive public debt and profligate fiscal policy. Finally, the implementation of restrictive fiscal policies in response to an increase in the debt ratio may hinder growth, especially during a recession (see DeLong et al. 2012).

This assumption translates in a quadratic expected loss function J_T of the type:

$$J_T = \mathbf{E}\left[(X_T)^2\right] \equiv \int_\Omega (X_T)^2 \, d\mathbb{P}, \tag{9}$$

where X_T is the stochastic level of debt ratio gap at time T and $\mathbf{E}$ denotes the expectation value with respect to the probability law of X, that is with respect to

[15] There is a quite vast empirical literature which shows that there is a negative correlation between public debt and economic growth (see e.g. Reinhart and Rogoff 2010, Woo and Kumar 2015). Yet, the casual interpretation of the correlation is an open issue since there might be cases in which causation goes from low growth to high debt, rather than the other way round.

the probability measure $\mathbb{P}$. The time T is the *exit time* of the process X_t from its interval $[0, \bar{x}]$ introduced in (8), that is it yields

$$T = \inf_t \{t > 0 \text{ such that } X_t \leqslant 0\}, \tag{10}$$

with $\mathbf{E}[T] < \infty$.

As usual in the dynamic programming literature, we now let the controlled diffusion X start at time s from level $x > 0$, that is

$$\begin{cases} dX_t^{s,x} = [\alpha X_{t-s} - (1+\alpha)\rho_{t-s} X_{t-s}]\, dt - \sigma(1+\alpha)\rho_{t-s} X_{t-s} dW_{t-s}, \\ \text{sub} \qquad X_t^{s,x} = x. \end{cases} \tag{11}$$

The optimization problem now reads:

$$\phi(s, x) = \inf_{\rho \in \mathcal{A}} \mathbf{E}\left[(X_T^{s,x})^2\right], \tag{12}$$

where $\phi(s, x)$ denotes the value function. To solve the system, we use the Hamilton-Jacobi-Bellman (HJB) equation.[16]

Nevertheless, to apply the HJB method we have to preliminarily transform the mean value J_T, given in (9), into the mean value of an integral by relying on the Dynkin's formula. For more details about the HJB equation and the Dynkin's formula, see Appendix B. According to the Dynkin's formula the mean value J_T, given in (9), reads, for $X_{t-s}^{s,x} \equiv X_{t-s}$

$$\begin{aligned} J(s, x; \rho) = x^2 + \mathbf{E}\Bigg[\int_s^T \{&[\alpha X_{t-s} - (1+\alpha)\rho_{t-s} X_{t-s}](2X_{t-s}) + \\ &+ \sigma^2(1+\alpha)^2 \rho_{t-s}^2 X_{t-s}^2\}\, dt\Bigg], \end{aligned} \tag{13}$$

where T is the stopping time introduced in (10). By virtue of the invariance of the problem under time translation (that is, the homogeneous problem over time), we can rewrite $J(s, x; \rho)$, given in (13), in the form:

$$\begin{aligned} J(s, x; \rho) = x^2 + \mathbf{E}\Bigg[\int_0^{T-s} \{&[\alpha X_t - (1+\alpha)\rho_t X_t](2X_t) + \\ &+ \sigma^2(1+\alpha)^2 \rho_t^2 X_t^2\}\, dt\Bigg]. \end{aligned} \tag{14}$$

where we denote the process $(X_t^{0,x})_{t \leq T}$ with X_t. Then the optimization problem can be equivalently written in the form:

$$\begin{cases} \phi(s, x) := x^2 + \inf_{\rho_t \in \mathcal{A}} \mathbf{E}\left[\int_0^{T-s} \{[\alpha X_t - (1+\alpha)\rho_t X_t](2X_t) + \sigma^2(1+\alpha)^2 \rho_t^2 X_t^2\}\, dt\right], \\ \text{sub} \qquad dX_t = [\alpha X_t - (1+\alpha)\rho_t X_t]dt - \sigma(1+\alpha)\rho_t\, X_t\, dW_t, \qquad \text{with} \quad X_0 = x, \end{cases} \tag{15}$$

[16]For more details about the HJB methodology, see Fleming and Soner (2006) and Stengel (1986).

where the initial value $X_0 = x$ is fixed in order the equation for X_t to have an unique solution.

Note that in choosing the optimal value for ρ_t, the government will take into account two relevant elements: first, the reduction power of the primary surplus, second, the relative volatility that the fiscal action itself may transmit to the system. As already discussed, equation (6), in fact, combines these two elements in a simple way so as to create a trade-off between debt reduction and uncertainty: the higher the effort in stabilizing the debt, the more uncertainty will enter the system. As a result, the optimal solution of ρ_t is expected to strike a balance between these two opposite forces, as any effort in reducing debt is conducive to more uncertainty for the system.

By applying the HJB equation to the second term on the right hand side of the optimization problem in (15), we obtain the following variational equation for $w \equiv \rho_t$:

$$\inf_w \Big\{ [\alpha X_t - (1+\alpha) w X_t](2X_t) + \sigma^2(1+\alpha)^2 w^2 X_t^2 + \\ + \frac{\partial \phi}{\partial t} + [\alpha X_t - (1+\alpha) w X_t] \frac{\partial \phi}{\partial x} + \frac{1}{2}\left[\sigma^2(1+\alpha)^2 w^2 X_t^2\right] \frac{\partial^2 \phi}{\partial x^2} \Big\} = 0 \tag{16}$$

To find an optimal control we now derive equation (16) with respect to w and for $X_t = x$ we obtain the following equation:

$$-2(1+\alpha)x + 2\sigma^2(1+\alpha)^2 w x - (1+\alpha)\frac{\partial \phi}{\partial x} + \sigma^2(1+\alpha)^2 w x \frac{\partial^2 \phi}{\partial x^2} = 0, \tag{17}$$

from which it immediatly follows:

$$w \equiv \rho_t = \frac{2(1+\alpha)x + (1+\alpha)\dfrac{\partial \phi}{\partial x}}{\sigma^2(1+\alpha)^2 x \left(2 + \dfrac{\partial^2 \phi}{\partial x^2}\right)}. \tag{18}$$

To find a solution for equation (18) we try with a guess function with separated variables of the following type:

$$\phi(s, x) = c\, x^2\, g(s), \tag{19}$$

with $g(0)$ is a constant, that is $g(0) = K$, because the value function $\phi(s, x)$ in the optimization problem (15), as we shall see, is a multiple of x^2, that is it yields $\phi(0, x) = \tilde{K} x^2$.

By substituting (19) into (18), where $\partial\phi/\partial x = 2cxg(s)$ and $\partial^2\phi/\partial x^2 = 2cg(s)$, we obtain:

$$\hat{\rho}_t = \hat{\rho}(t, X_t(\omega)) = \hat{\rho} = \frac{1}{\sigma^2(1+\alpha)}, \tag{20}$$

that is the optimal Markov control $\hat{\rho}(t, X)$ is constant. Note that the constant control (20) is admissible as it belongs to the set $\mathcal{A}$ as in (8). Indeed a constant function is a Borel-measurable function ρ belonging to the set in (8).

Sufficient conditions ensuring that the optimization problem (15) satisfies the requirements for the optimality of the optimal Markov control solution are in Appendix C where we show that the application which associates $cx^2g(s)$ with every pair (s, x) satisfies all the conditions of the *Verification Theorem*.

From equation (20) three remarks are in order. First, the optimal correction factor is independent of the debt ratio distance from its target. This implies that at the optimum the relationship between the surplus ratio and the deviation of the debt-output ratio from its target will be linear.

Second, the optimal correction is procyclical, since it prescribes a stronger non-linear reaction in the presence of favourable economic conditions (high growth) and a lower adjustment in the case of economic downturn (low growth). However, if the real interest rate is high, other thing being equal, then (20) will imply a weaker response. This apparently counterintuitive result stems from the existing trade-off generated by a vigorous reaction to the debt ratio, namely that between the expected faster fiscal consolidation and the higher uncertainty surrounding the final outcome of the fiscal consolidation itself. The idea is that when economic conditions are already favourable for the stability of the debt ratio (i.e. low α), the government would find it optimal to undertake a strong corrective measure.

Third, when the coefficient diffusing uncertainty is high, the correction factor should correspondingly be low. The idea behind this result is that large shocks can potentially undermine or magnify the effectiveness of the fiscal effort, so that it is 'optimal' to limit the magnitude of the correction mechanism itself.

These results can be summarised by looking at the value function (12) corresponding to the optimal solution. To derive it, we have to substitute the expressions (19) and (20) into equation (16) in order to obtain the separated equation for the temporal function $g(s)$:

$$g'(s) = \left(\frac{1}{\sigma^2} - 2\alpha\right) g(s) + \frac{1}{c}\left(\frac{1}{\sigma^2} - 2\alpha\right). \tag{21}$$

If we insert the optimal control (20) into the evolution equation in (15), this equation becomes

$$dX_t = \left(\alpha - \frac{1}{\sigma^2}\right) X_t \, dt - \frac{X_t}{\sigma} \, dW_t \, ,$$

whose solution, by virtue of Itô's lemma, is

$$X_t = x e^{(\alpha - 1/(2\sigma^2))t - W_t/\sigma}. \tag{22}$$

If we substitute the solution (22) into the integral in (15), the value function, for $s = 0$, becomes

$$\phi(0, x) = x^2 + (2\alpha - 1/\sigma^2)x^2 \; \mathbf{E}\left[\int_0^T e^{(2\alpha - 1/\sigma^2)t - 2W_t/\sigma} dt\right] = \tilde{K}x^2,$$

because the mean value of a random variable is obviously a constant.

If we now consider the initial condition $g(0) = K$, the solution of the temporal ordinary differential equation (21) reads

$$g(s) = \left(K + \frac{1}{c}\right) e^{\left(1/\sigma^2 - 2\alpha\right)s} - \frac{1}{c} \, . \tag{23}$$

We then have an explicit expression of the value function $\phi(s,x)$ given by

$$\phi(s,x) = \left[(Kc+1)\, e^{\left(1/\sigma^2 - 2\alpha\right)s} - 1\right] x^2. \tag{24}$$

Under the condition $1/\sigma^2 > \alpha$, namely, fiscal solvency is expected to be satisfied, the value function is increasing in x, suggesting that the larger the initial debt ratio gap, the higher the 'loss' the government will experience at the optimum. If uncertainty is high and/or economic fundamentals are adverse to fiscal consolidation, then at the optimum the 'loss' will be lower.

Given the optimal rule (20), from equation (6) we are then able to find the adjustment process for the debt ratio gap given the realization of shocks. However, given the presence of uncertainty the following questions arise. What is the time needed to meet a fiscal target? How robust is the adjustment rule to adverse shocks? Or better, what is the probability that given the materialization of adverse shocks public debt is still on the right track towards a preset fiscal goal? In the next Section we will address these questions proposing two different approaches.

4 Measures of Fiscal Resilience

In this Section we assess the properties of the optimal policy (20) in terms of fiscal resilience, that here we interpret as the capability to stay on track towards a given fiscal consolidation objective despite the occurrence of adverse shocks. Specifically, we provide two measures of fiscal resilience: (i) the time necessary to reach an expected fiscal consolidation objective, (ii) the probability of reaching that objective. Nonetheless, each measure, pointing to the same underlying idea of resilience, provides us with different information about the ability of the government to meet its target in a stochastic environment.

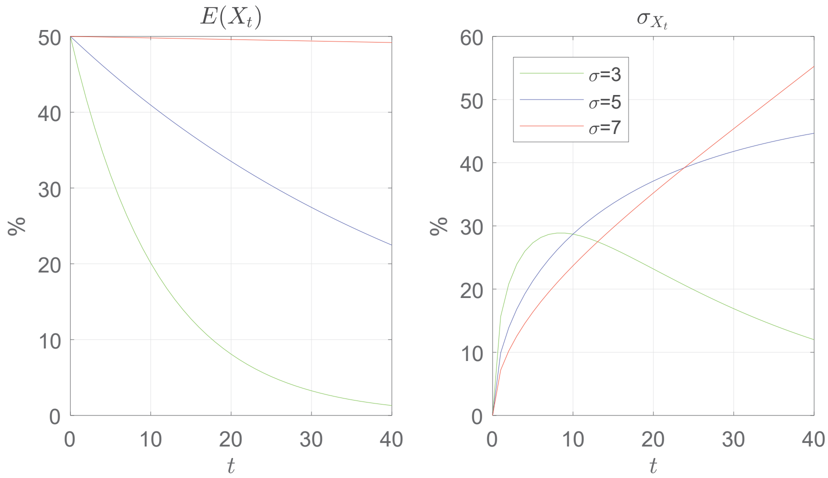
In particular, the first measure tries to pinpoint the time needed to reach a given objective that is expected to be achieved in the presence of shocks. This may be seen as the time resistance towards the objective when the system is placed under pressure. An increase in this measure then implies lower fiscal resilience to shocks.

The second measure, the probability of reaching a fiscal consolidation target, may be interpreted as the capability to absorb negative shocks without changes that may undermine the achievement of a fiscal objective. In other words, this measure captures the ability of the government to get back on track towards the consolidation objective in the presence of adverse shocks that push the debt up. It thus gauges the flexibility and the adaptive capacity of the system as a whole. As we will see, α and σ play a major role in amplifying or reducing this adaptability. An increase in this measure then indicates higher fiscal resilience to unforeseen shocks.

4.1 Time Needed to Reduce the Debt Ratio Gap

What is the time needed to reduce the debt ratio gap of a given amount? How do uncertainty and fundamentals affect the time required to meet a given target?

Figure 1: Debt Ratio Gap and Uncertainty - Theoretical Moments



Note: the figure plots the mean and the standard deviation of the debt-ratio gap X_t for different values of uncertainty, σ, given an initial debt-ratio gap $x = 50\%$ and fundamentals $\alpha = 0.02$.

These questions are relevant issues for policymakers, since the achievement of their goals is conditioned by the expected time necessary to meet them. Indeed, adverse cyclical factors and changed political conditions may considerably expand the time eventually needed to reach a given objective, prolonging the time span of a policy action. The credibility of any fiscal reform also depends on the expected time necessary to reach an established aim. The more distant in the future the achievement of the final goal, the less credible the policy action will be.

We start by substituting (20) into (6) in order to obtain the geometric Brownian motion describing the stochastic evolution of debt under the optimal policy:

$$dX_t = \left(\alpha - \frac{1}{\sigma^2}\right) X_t\, dt - \frac{1}{\sigma} X_t\, dW_t\,. \tag{25}$$

Equation (25) describes the debt ratio dynamics when the government is committed to follow the optimal correction rule. It is easy to note that, up to a random effect given by W_t, the model dynamics and, therefore, the speed of the adjustment towards a given target depend on the combination of α and σ. Specifically, for a high α, reflecting a high interest rate and/or a low GDP growth, the adjustment of the debt ratio towards the target will be slower. Similarly, a high σ, reflecting more uncertainty around the effectiveness of the fiscal reaction, slows down the adjustment process towards the debt ratio objective.

The solution of equation (25) can be obtained by using the Itô's lemma as follows:

$$X_t = x\, e^{\left(\alpha - \frac{3}{2\sigma^2}\right)t} e^{-\frac{1}{\sigma} W_t}. \tag{26}$$

From (26), recalling (5), the expected value of the debt ratio gap $\mathbf{E}[X_t]$ is

$$\mathbf{E}[X_t] = x\, e^{\left(\alpha - \frac{3}{2\sigma^2}\right)t} \int_{\mathbb{R}} e^{-w/\sigma} \frac{e^{-w^2/(2t)}}{\sqrt{2\pi t}}\, dw = x\, e^{\left(\alpha - \frac{1}{\sigma^2}\right)t}, \tag{27}$$

while the variance is

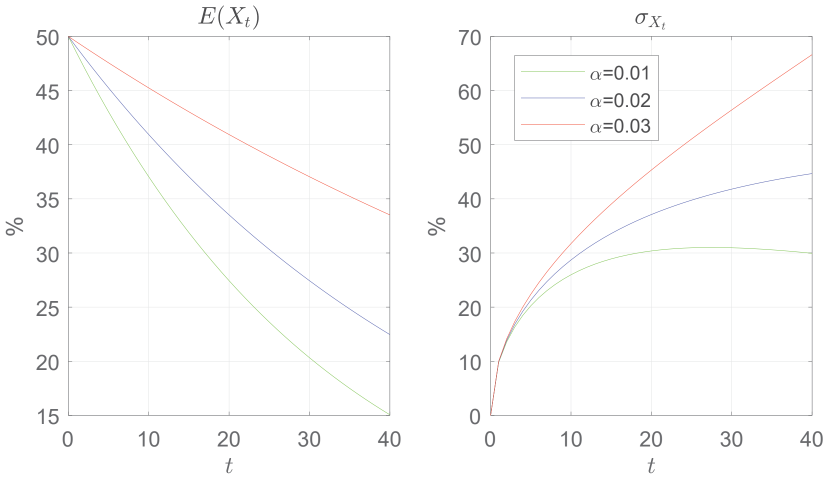
$$\mathbf{E}[X_t - \mathbf{E}[X_t]]^2 = x^2\,(e^{\frac{1}{\sigma^2}t} - 1)e^{2\left(\alpha - \frac{1}{\sigma^2}\right)t}. \tag{28}$$

From (27) the expected value of the debt gap ratio declines over time provided that condition $1/\sigma^2 > \alpha$, that is, fiscal solvency is expected to be satisfied.[17] The speed of convergence towards an expected zero-gap target is clearly increasing in α and in σ. If $1/\sigma^2 > \alpha$, then the variance of X_t will display an hump-shaped dynamics over time. To better illustrate the behaviour of moments as time changes we will make use of a numerical example.

Figure 1 presents the expected value of the debt-ratio gap $E(X_t)$ and its standard deviation σ_{X_t}, given an initial debt-ratio gap $x = 50\%$, $\alpha = 0.02$ and three different values of σ. For high uncertainty the time necessary to reach the objective will expand as a result of the fact that the government will find it 'optimal' to slow down the fiscal effort in response to the higher unpredictability of the final outcome of the policy intervention. As an example, after 10 years for $\sigma = 5$, $E(X_t)$ will be about 20 p.p. higher than what observed under more stable economic conditions, that is for $\sigma = 3$. After 40 years $E(X_t)$ is close to the zero target with $\sigma = 3$, while for $\sigma = 5$ the expected value is still 20 p.p. above the target. However, a strong reaction to the debt ratio initially generates a high variability especially when σ is lower. This is because the optimal rule (20) prescribes a strong reaction to the debt ratio gap when σ is low. At later stages, instead, the standard deviation declines faster the lower the degree of uncertainty. As long as the debt ratio declines and the gap towards the long-run objective is narrowed, the amount of uncertainty is sharply reduced. This is the result of the initial trade-off faced by the policymaker at the earlier stages of the adjustment towards the closure of the gap, discussed in Section 2. Figure 2 shows the role played by market fundamentals in determining the time path of the expected value of the debt ratio gap and of its standard deviation. We observe that, other things being equal, the higher α (reflecting adverse economic conditions such as low growth and/or high interest rate), the slower the convergence towards the objective, and thus more the time needed to meet the established objective and the higher the variability. Hence, as expected, a high α severely reduces the stabilizing properties of the rule. Overall the effects of changes in market fundamentals are magnified in the presence of high uncertainty. This can by easily explained by close inspection of equation (25), where for an increase in σ the role of market fundamentals becomes pivotal in shaping the time path of the debt ratio gap. In the presence of high uncertainty the optimal rule, in fact, implies a weaker reaction, so that the adjustment of the debt ratio towards a given target relies on market fundamentals at a greater extent. Table 1 summarizes the above findings presenting the time needed to reach different debt ratio gaps that are expected to be achieved in the presence of shocks for different values of σ and α. We consider four different

[17]Note that the mean value (27) goes to zero asymptotically, meaning that the objective is met when the time tends to infinite, and not for a finite time t. In this respect, equation (27) is assumed to be zero as long as the value of the function itself is lower than a (given) threshold very close to zero. For this reason, we can argue that the mean value (27) becomes zero in a finite time (at the exit time), although the convergence is just asymptotic.

Figure 2: Debt Ratio Gap and Fundamentals - Theoretical Moments



Note: the figure plots the mean and the standard deviation of the debt-ratio gap X_t for different values of fundamentals, α, given an initial debt-ratio gap $x = 50\%$ and uncertainty $\sigma = 5$.

expected fiscal consolidation goals and from (27) we compute the time needed to reach the objective. As before, the initial value x is set at 50%. In the more favourable scenario, with α set at 0.01 and σ at 3, the time needed to close the gap by 10 p.p. is 2 years, while in the worst scenario, with α at 0.03 and σ at 5, is 22 years. Similarly, closing the gap by 40 p.p. is feasible in 16 years under favourable conditions and in 161 years under adverse circumstances.

4.2 Probability of Reaching a Fiscal Objective

In this section we propose a fiscal resilience measure aiming at quantifying the ability to meet a debt target in a stochastic environment. We also assess how fiscal resilience may vary when the underlying economic conditions change. Since the debt ratio is constantly bounced around by a number of shocks, the reduction of debt towards a target is uncertain. Indeed, as a result of two opposing forces, the correction rule pushing debt down on the one hand, and adverse shocks that may drive debt up on the other hand, debt trajectories are surrounded by uncertainty. Thus, in a stochastic environment it becomes relevant for policymakers to measure the degree of confidence associated with the effectiveness of the fiscal action at play, namely the capability of pursuing an objective. This, in turn, describes its resilience relative to the target.

In detail, the basic setup is as follows. We assume that the government is committed to meet a target following the optimal rule (20). Let us define this target $\underline{x}$. It is then assumed that the current level of debt ratio gap x is higher than the established objective of the government $\underline{x}$. It should be noted that this target may be interpreted either as an intermediate objective of a fiscal consolidation

Table 1: Time Necessary to Meet an Expected Debt Ratio Target

$\sigma = 3$			
	$\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.03$
$x - E(X_t) = 10$	2	2	3
$x - E(X_t) = 20$	5	6	6
$x - E(X_t) = 30$	9	10	11
$x - E(X_t) = 40$	16	18	20
$\sigma = 4$			
	$\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.03$
$x - E(X_t) = 10$	4	5	7
$x - E(X_t) = 20$	10	12	16
$x - E(X_t) = 30$	17	22	28
$x - E(X_t) = 40$	31	38	50
$\sigma = 5$			
	$\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.03$
$x - E(X_t) = 10$	7	11	22
$x - E(X_t) = 20$	17	26	51
$x - E(X_t) = 30$	31	46	92
$x - E(X_t) = 40$	54	80	161

Note: the table reports the number of years necessary to close the gap by an expected given amount for different combinations of uncertainty, σ, and fundamentals, α, given an initial debt ratio gap $x = 50\%$.

plan ($\underline{x} > 0$) or as the final target itself of closing the debt ratio gap ($\underline{x} = 0$).

The stochastic component W_t in (26) may push the debt ratio gap up for some time, thus frustrating the policy effort. Besides, the deterioration of the economic conditions that comes from the fiscal correction, in turn, may increase uncertainty, so further eroding the confidence towards the objective. Let us assume that, from the government perspective, $\overline{x} > x$ is deemed to be the worst outcome as a result of very adverse shocks. Thus, at time 0 debt is assumed to be in between two extremes, namely the target and the worst outcome: $x \in D := (\underline{x}, \overline{x})$. Given these hypotheses, we address the following questions. What is the probability that, given uncertainty, at time t the debt ratio is bending towards the objective $\underline{x}$? What is the role played by fundamentals and uncertainty?

In order to construct this probability we make use of the concept of harmonic measure. Formally, a harmonic measure of X_t describes its distribution as X_t hits the boundaries of D, namely $\underline{x}$ or $\overline{x}$. More specifically, to build up such a measure we first take the debt accumulation dynamics under the optimal correction rule as in (25). By virtue of the diffusion theory related to the Itô stochastic processes, we can associate the following second-order ordinary differential equation to equation

(25) as follows:

$$\frac{x^2}{2\sigma^2} f''(x) + \left(\alpha - \frac{1}{\sigma^2}\right) x f'(x) = 0. \tag{29}$$

Let $f \in C^2(R)$ be a solution of this differential equation. Also, let $(\underline{x}, \overline{x}) \subset \mathbb{R}$ be an open interval such that $x \in (\underline{x}, \overline{x})$ and put

$$\tau = \inf\left\{ t > 0 \,:\, X_t \notin (\underline{x}, \overline{x}) \right\}, \tag{30}$$

where τ measures the first instant of time in which the debt ratio gap does not belong to the interval $(\underline{x}, \overline{x})$. We are assuming that $\tau < \infty$ almost sure with respect to the probability law Q^x by means of the Brownian motion.

Recalling the Dynkin's formula it is possible to give a formal expression for the probability that debt is bending towards the objective $\underline{x}$. If $f(\overline{x}) \neq f(\underline{x})$, by using the Dynkin's formula then the probability may be written as

$$\mu^x(\underline{x}) = \frac{f(\overline{x}) - f(x)}{f(\overline{x}) - f(\underline{x})}. \tag{31}$$

that is the harmonic measure μ of X on $\underline{x}$.[18] For a formal proof of how (31) is obtained, see Appendix D. Such a harmonic measure is the probability that, in the first instant of time τ in which the process X_τ does not belong to the fixed open interval $(\overline{x}, \underline{x})$, the process assumes the value $\underline{x}$.

From an economic point of view, the harmonic measure (31) may have a twofold interpretation. On the one hand, it may be seen as an index of confidence about how much a system is vulnerable to potential qualitative changes. In our setup, qualitative changes may be considered a state jump into the wrong direction $\overline{x}$. On the other hand, $\mu^x(\underline{x})$ may be portrayed as the government ability to bend towards a target for a given α and σ, under the optimal correction rule and in the presence of external perturbations. The first interpretation highlights the capability to withstand negative shocks, while the second one refers to the capability of meeting an objective. Nevertheless, both interpretations outline the idea of resistance against adverse conditions. This is why in terms of fiscal resilience the two interpretations are interchangeable.

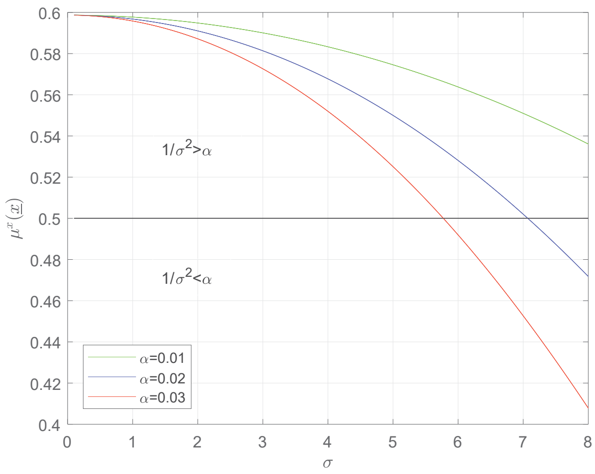
After some manipulations (see Appendix E for more details), it is possible to give an explicit expression for (31) as follows:

$$\mu^x(\underline{x}) = \frac{\overline{x}^{3-2\sigma^2\alpha} - x^{3-2\sigma^2\alpha}}{\overline{x}^{3-2\sigma^2\alpha} - \underline{x}^{3-2\sigma^2\alpha}}. \tag{32}$$

The value of $\mu^x(\underline{x})$ thus depends on the specific setting of α and σ. Figure 3 shows how it changes for different parametrization of α and σ, where we have set $x = 50\%$, $\underline{x} = 45\%$ and $\overline{x} = 55\%$. The horizontal dashed line corresponds to $\mu^x(\underline{x}) = 0.5$. It should be noted that above this line the fiscal solvency condition holds, i.e. $1/\sigma^2 > \alpha$, while it is violated below, i.e. $1/\sigma^2 < \alpha$. The three plotted curves

[18] To be sure, the corresponding harmonic measure μ of X on $\overline{x}$ can be derived as $\mu^x(\overline{x}) = 1 - \mu^x(\underline{x})$.

Figure 3: Probability of Reaching a Consolidation Target, Uncertainty, and Fundamentals



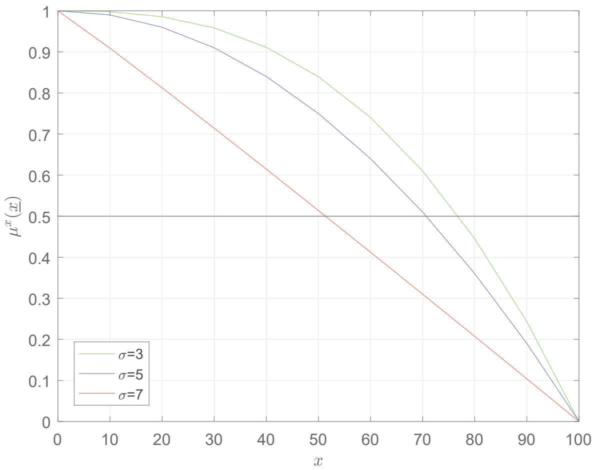
Note: the figure plots $\mu^x(\underline{x})$ for different values of uncertainty, σ, and fundamentals, α, given an initial debt-ratio gap $x = 50\%$ and interval boundaries $\overline{x} = 55\%, \underline{x} = 45\%$.

intersect the horizontal lines for values of σ, such that $1/\sigma^2 = \alpha$. We observe that a lower α increases the probability of reaching the objective as a result of more favourable economic conditions (high output growth and/or low interest rate). A higher σ, instead, injects more fiscal policy uncertainty into the system and the ability to meet the objective thus tends to inch down. This measure is also affected by the distance between the worse outcome $\overline{x}$ and the current state of debt x. The higher this distance the higher the resilience. In other terms, if the current debt x is relatively close to the objective $\underline{x}$ (that is, relatively far from $\overline{x}$) the chances to meet it will be higher than in the case in which x is far from the objective. The underlying idea is that the closeness to the objective entails that uncertainty may hardly undermine the capability to reach the objective itself. In a way, the confidence that the positive effects from the correction rule will prevail over negative shocks is higher if the objective is at hand. However, also in this case the specific parametrization of σ may affect the capability to meet the target.

Figure 4 shows how this measure of fiscal resilience varies for different values of x in the interval $\underline{x} = 0\%$ and $\overline{x} = 100\%$ and for three different parametrizations of σ, ensuring that the fiscal solvency condition (i.e. $1/\sigma^2 > \alpha$) always holds. We observe that, for given σ, $\mu^x(\underline{x})$ decreases the farer x from the objective. As expected, if the current state of debt is very far from the target, then the confidence to meet the objective will be a remote possibility and $\mu^x(\underline{x})$ will be next to zero accordingly. We observe that a higher σ pushes the probability down as a result of the limiting effect of more uncertainty.

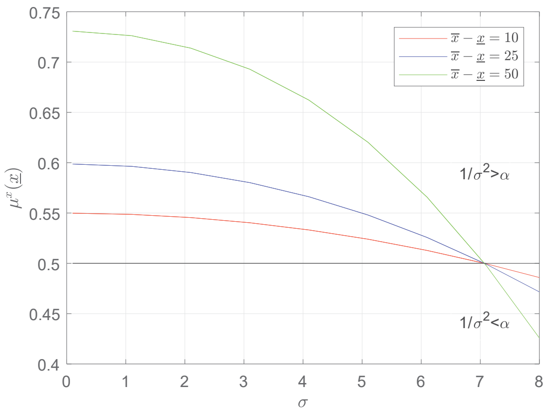
Finally, in Figure 5 we explore how fiscal resilience changes with the size of the interval $(\underline{x}, \overline{x})$ under the assumption that x is always centered in it. Clearly, for a given level of uncertainty the wider the interval, the higher $\mu^x(\underline{x})$ provided.

Figure 4: Probability of Reaching a Consolidation Target, Initial Conditions, and Uncertainty



Note: the figure plots $\mu^x(\underline{x})$ for different initial values of the debt ratio x, and of uncertainty σ, given interval boundaries $\overline{x} = 100\%$, $\underline{x} = 0$ and fundamentals, $\alpha = 0.02$.

Figure 5: Probability of Reaching a Consolidation Target, Uncertainty, and Interval Size



Note: the figure plots $\mu^x(\underline{x})$ for different values of uncertainty, σ and of interval boundaries $\overline{x} - \underline{x}$, given an initial debt-ratio gap $x = 50\%$ and fundamentals, $\alpha = 0.02$.

All the curves intersect at a point such that $\alpha = 1/\sigma^2$, which describes the loci ensuring that $\mu^x(\underline{x}) = \mu^x(\overline{x}) = 0.5$ for a given x centered in any interval. It should be noted that up to this point, starting from a very low level of uncertainty, fiscal solvency is expected to be satisfied, while afterwards the expected value of the debt ratio gap does not converge to zero. This is why $\mu^x(\underline{x})$ declines sharply the larger the size of the interval. This result can be explained by the fact that when market fundamentals are such that fiscal solvency is not expected to be met (i.e. the condition, $1/\sigma^2 > \alpha$, does not hold) a wide interval entails a lower probability that things may improve just by chance. This second measure of fiscal resilience is thus more general than that based on the time necessary to reduce the debt ratio gap, since it may be applied to cases in which the debt ratio gap is expected to expand over time and public debt is unsustainable.

5 Conclusion

This paper studies the optimal debt reduction policy in a simple stochastic model of debt by using optimal control theory and applying the Hamilton-Jacobi-Bellman equation. The government is assumed to follow a simple feedback rule according to which the primary surplus is adjusted to the deviations of the debt-to-GDP ratio from a given target. However, the government has partial control over the primary budget balance, since the final impact of any fiscal intervention is surrounded by uncertainty. In such an environment the optimal Markov control policy turns out to prescribe that the reactivity of the primary surplus to the debt ratio gap is independent of the debt-GDP ratio itself, rather it depends only on the interest rate, the growth rate of the economy and the degree of uncertainty surrounding the effects of fiscal policies. Overall, the optimal rule envisages a strong fiscal effort under favourable economic conditions. This result suggests that a simple linear rule of primary surplus adjustment to the deviation of the debt-GDP ratio from its target may be optimal, provided that the size of the adjustment coefficient is tailored to the underlying market fundamentals.

We propose two different measures of fiscal resilience under the optimal control. The first measure is simply based on the time needed to meet an expected fiscal consolidation objective. The second measure, relying on harmonic measure theory, is constructed from the probability distribution of the debt ratio as it hits the boundary of a given open interval. This measure gives us the probability that the debt ratio bends towards a target and may be then used to assess whether the debt ratio is on an explosive path or not. As expected high growth rates and low interest rates improve fiscal resilience, while a higher degree of uncertainty jeopardizes fiscal stability. We argue that these two measures could be used as simple indicators to gauge the goodness of a fiscal consolidation plan and as early warning indicators of fiscal imbalances.

In this paper we have deliberately considered a parsimonious model, yet general enough to capture several dimensions of the public debt control problem. The analysis may be extended in a number of directions. First, all the results have been obtained taking both the GDP growth and the interest rate as given. Both the fiscal conduct and the level of the outstanding debt may directly affect these variables and change the dynamics of debt accumulation. Second, the analysis may be extended to account for the interaction between monetary and fiscal policies. The underlying monetary regime may facilitate or make more difficult the optimal control of public debt, and when it comes to ensure jointly price stability and fiscal solvency, the policy trade-offs may become more severe and the optimal control problem more challenging. Finally, the measures of fiscal resilience proposed in this paper should be compared with other fiscal indicators and their behaviour should be analyzed in practice. We leave these aspects for future research.

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Declarations of Interest

None.

Appendix A

Definition 1. Given a set Ω, a *$\sigma - algebra$* $\mathcal{F}$ on Ω is a *family* $\mathcal{F}$ of subsets of Ω that fulfill the following properties:

(i) the empty set $\emptyset$ belongs to $\mathcal{F}$;

(ii) if $F \in \mathcal{F}$, then the complement $\bar{F}$ of F in Ω belongs to $\mathcal{F}$, too;

(iii) if $A_1, A_2, A_3, \dots \in \mathcal{F}$, then $A := \bigcup_{i=1}^{\infty} A_i \in \mathcal{F}$. □

Definition 2. The pair $(\Omega, \mathcal{F})$ is called a *measurable space.* □

Definition 3. A *probability measure* $\mathbb{P}$ on a measurable space $(\Omega, \mathcal{F})$ is a function $\mathbb{P} : \mathcal{F} \longrightarrow [0, 1]$ such that

(i) $\mathbb{P}(\emptyset) = 0$ and $\mathbb{P}(\Omega) = 1$;

(ii) if $A_1, A_2, A_3, \dots \in \mathcal{F}$ and $A_i \cap A_j = \emptyset, \ \forall i \neq j$, then $\mathbb{P}\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mathbb{P}(A_i)$.

□

Definition 4. The triple $(\Omega, \mathcal{F}, \mathbb{P})$ is called a *probability space.* It is called a *complete probability space* if $\mathcal{F}$ contains all subsets S of Ω with $\mathbb{P}-$*outer measure* zero, where the $\mathbb{P} -$ *outer measure*, denoted by $\mathbb{P}^*$, is defined as

$$\mathbb{P}^*(G) = \inf \left\{ \mathbb{P}(F) : F \in \mathcal{F} \text{ and } G \subset F \right\}.$$ □

Definition 5. For a given *family* $\mathcal{G}$ of subsets of Ω, the *$\sigma - algebra$* denoted by the symbol $\mathcal{F}_{\mathcal{G}}$ and defined as

$$\mathcal{F}_{\mathcal{G}} = \bigcap \left\{ \mathcal{F} \ : \ \mathcal{F} \text{ is a } \sigma - algebra \text{ of } \Omega \text{ and } \mathcal{G} \subset \mathcal{F} \right\}$$

is called the *$\sigma - algebra$ generated by* $\mathcal{G}$. □

Definition 6. If Ω is a topological space (e.g. $\Omega = \mathbb{R}^n$) equipped with the topology $\mathcal{G}$ of all open subsets of Ω, then the $\sigma - algebra$ $\mathcal{B} = \mathcal{F}_{\mathcal{G}}$ is called the *Borel $\sigma - algebra$* on Ω and the elements $B \in \mathcal{B}$ are called *Borel sets.* □

Definition 7. Given the *measurable space* $(\Omega, \mathcal{F})$, the (increasing) family $\{\mathcal{M}_t\}_{t \geq 0}$ of *$\sigma - algebras$* of Ω such that

$$\mathcal{M}_{t_1} \subset \mathcal{M}_{t_2} \subset \mathcal{F}, \qquad \forall\, 0 \leq t_1 < t_2,$$

is called a *filtration* on $(\Omega, \mathcal{F})$. □

Appendix B

Remark 1. *There exists a unique solution for the controlled equation (11) (see Øksendal (2003) for more details).*

For $\bar{X} < X_f$ let us define the exit time τ of the dynamics as

$$\tau := \inf\{t > 0 \mid X_t > \bar{X}\}. \tag{B.1}$$

Remark 2. *By virtue of well known results, the measurability of τ with respect to the σ-algebra $\mathcal{F}_t$ follows. Indeed, we have that τ is a stopping time.*

Theorem 1 (Dynkin's formula). *Let X_t be the Itô diffusion*

$$dX_t = \mu(X_t)\, dt + \sigma(X_t)\, dB_t, \quad X_0 = y, \tag{B.2}$$

and $f \in C_0^2(\mathbb{R})$. If τ is a stopping time with $\mathbf{E}[\tau] < +\infty$, then the following relationship holds:

$$\mathbf{E}[f(X_\tau)] = f(x) + \mathbf{E}\left[\int_0^\tau Lf(X_s)\, ds\right], \tag{B.3}$$

where

$$Lf(z) := \mu(z)\,\frac{df}{dz} + \frac{1}{2}\,[\sigma(z)]^2\,\frac{d^2 f}{dz^2}\,. \tag{B.4}$$

Theorem 2 (HJB equation). *Suppose that we have*

$$V(s, y) := \sup_{\rho_t \in \mathcal{A}} \mathbf{E}\left[\int_s^\tau f(X_t, \rho_t)\, dt\right], \tag{B.5}$$

with

$$\begin{cases} dX_t = \mu(X_t,\, \rho_t)\, dt + \sigma(X_t,\, \rho_t)\, dB_t, \\ X_0 = x\ . \end{cases} \tag{B.6}$$

Suppose that $V \in C^2(\mathbb{R}^+)$ satisfies

$$\mathbf{E}\left[|V(X_\alpha)| + \int_0^\alpha |L^\rho V(X_t)|\, dt\right] < +\infty, \tag{B.7}$$

for all bounded stopping times $\alpha < \tau$, for all $x \in \mathbb{R}$ and all $\rho \in \mathcal{A}$, where

$$(L^z V)(s, x) := \frac{\partial V(s, x)}{\partial s} + \mu(x, \rho)\frac{\partial V}{\partial x} + \frac{\sigma^2(x, \rho)}{2}\frac{\partial^2 V}{\partial x^2}. \tag{B.8}$$

Moreover, suppose that an optimal control ρ^* exists, then we have

$$\sup_{\rho \in \mathcal{A}} \{f(x, \rho) + (L^\rho V)(x)\} = 0, \tag{B.9}$$

and the supremum is obtained if it yields $\rho = \rho_t^*$*, that is*

$$f(x, \rho^*(t)) + (L^{\rho^*(t)} V)(x) = 0\,. \tag{B.10}$$

Theorem 2 also applies to the corresponding *minimum* problem

$$\phi(s, x) := \inf_{\rho_t \in \mathcal{A}} \mathbf{E} \left[\int_s^\tau f(X_t, \rho_t)\, dt \right]. \tag{B.11}$$

We have in fact

$$\phi(s, x) = - \sup_{\rho_t \in \mathcal{A}} \mathbf{E} \left[\int_s^\tau - f(X_t, \rho_t)\, dt \right],$$

from which, by replacing V with $-\phi$ and f with $-f$, it follows that the (B.9) in Theorem 2 becomes:

$$\inf_{\rho \in \mathcal{A}} \{ f(x, v) + (L^z \phi)(x) \} = 0. \tag{B.12}$$

For the details the reader may refer to Øksendal (2003).

Appendix C

In this Appendix we give sufficient conditions to conclude that (20) is the optimal Markov control process and (24) is the corresponding value function. The proof relies essentially on Itô's lemma as follows.

Verification Theorem. Let

$$\phi(s,x) := \inf_{\rho_t \in \mathcal{A}} \mathbf{E} \int_0^{T-s} f(t, X_t, \rho_t)\, dt,$$

with

$$dX_s = b(s, X_s, \rho(s, X_s))ds + \sigma(s, X_s, \rho(s, X_s))dW_s \quad \text{and} \quad X_0 = x,$$

be an optimisation problem. Let V be a $C^{1,2}([0,T) \times \mathbb{R}) \cap C([0,T) \times \mathbb{R})$ function and let us assume that f and V have quadratic growth, i.e. there is a constant C such that

$$|f(t,x,\rho)| + |V(t,x)| \leqslant C(|x|^2 + 1), \tag{C.1}$$

for all $(t,x,\rho) \in [0,T) \times \mathbb{R} \times \mathcal{A}$.

(i) Suppose that

$$\frac{\partial V(t,x)}{\partial t} + f(t,x,\rho) + b(t,x,\rho)\frac{\partial V(t,x)}{\partial x} + \frac{\sigma^2(t,x,\rho)}{2}\frac{\partial^2 V(t,x)}{\partial x^2} \geqslant 0 \tag{C.2}$$

on $[0,T) \times \mathbb{R}$. Then $V \leqslant \phi$ on $[0,T] \times \mathbb{R}$.

(ii) Assume further that there exists a minimizer $\hat{\rho}(t,x)$ of the function

$$\rho \to \mathcal{L}^{\rho} V(t,x) + f(t,x,\rho),$$

such that

$$\begin{aligned} 0 = \frac{\partial V(t,x)}{\partial t} + \inf_{\rho \in \mathcal{A}} \left\{ f(t,x,\rho) + b(t,x,\rho)\frac{\partial V(t,x)}{\partial x} + \frac{\sigma^2(t,x,\rho)}{2}\frac{\partial^2 V(t,x)}{\partial x^2} \right\} = \\ = \frac{\partial V(t,x)}{\partial t} + \mathcal{L}^{\hat{\rho}(t,x)} V(t,x) + f(t,x,\hat{\rho}), \end{aligned} \tag{C.3}$$

where $\mathcal{L}^{\rho} V(t,x)$ is defined as

$$\mathcal{L}^{\rho} V(t,x) := b(t,x,\rho)\frac{\partial V(t,x)}{\partial x} + \frac{\sigma^2(t,x,\rho)}{2}\frac{\partial^2 V(t,x)}{\partial x^2}. \tag{C.4}$$

Then the stochastic differential equation

$$dX_s = b(s, X_s, \hat{\rho}(s, X_s))ds + \sigma(s, X_s, \hat{\rho}(s, X_s))dW_s \tag{C.5}$$

defines a unique solution X for each given initial date $X_0 = x$ and the process $\hat{\rho} := \hat{\rho}(s, X_s)$ is a well-defined control process in $\mathcal{A}$. Then ϕ is the value function and $\hat{\rho}$ is the optimal Markov control process.

In our case from equation (15) we have

$$f(t,x,\rho) = \left[\sigma^2(1+\alpha)^2\rho^2 + 2\alpha - 2\rho(1+\alpha)\right] x^2, \tag{C.6}$$

which has quadratic growth and then it follows that there exists a positive constant such that

$$C_1 \geqslant \sup_{\rho} \left\{ |\sigma^2(1+\alpha)^2\rho^2 + 2\alpha - 2\rho(1+\alpha)| \right\} \tag{C.7}$$

and $|f(t,x,\rho)| \leqslant C_1 x^2$. Since the term in square brackets in (24) is bounded, it follows that there exists a positive constant C_2 such that $|f(t,x,\rho)| \leqslant C_1 x^2$. Then, the condition (C.1) is satisfied with a positive constant $C > C_1 + C_2 - 1$.

Further, the condition (C.2) is verified, too, because the expression

$$\frac{\partial V(t,x)}{\partial t} + f(t,x,\rho) + b(t,x,\rho)\frac{\partial V(t,x)}{\partial x} + \frac{\sigma^2(t,x,\rho)}{2}\frac{\partial^2 V(t,x)}{\partial x^2} \tag{C.8}$$

is convex with respect to ρ and thus positive or null for each $\rho \neq \hat{\rho}$.

Appendix D

In this Appendix we derive equation (31) by means of Dynkin's formula. Specifically, let

$$dX_t = r(X_t)dt + \sigma(X_t)dB_t, \tag{D.1}$$

be a 1-dimensional Itô diffusion with characteristic operator $\mathcal{A}$ and $f \in C^2(\mathbb{R})$ be a solution of the ordinary differential equation:

$$\mathcal{A}f(x) = r(x)f'(x) + \frac{\sigma^2}{2}\, f''(x) = 0, \quad x \in \mathbb{R}. \tag{D.2}$$

Let $(a, b) \subset \mathbb{R}$, with $b > a > 0$, be an open interval such that $x \in (a, b)$ and put

$$\tau \equiv \tau(a, b) = \inf\{t > 0 \text{ such that } X_t \notin (a, b)\},$$

and assume that $\tau < \infty$ a.s. with respect to the probability law of X_t. If we define

$$p \equiv P^x[X_\tau = b],$$

it follows

$$p = \frac{f(x) - f(a)}{f(b) - f(a)}\,. \tag{D.3}$$

Proof. If we consider the function $f_0 \in C_0^2(\mathbb{R})$ such that $f_0(x) \equiv f(x)$ on (a, b) and $\mathcal{A}f_0(x) = \mathcal{A}f(x) = 0$, by means of Dynkin formula we can write:

$$E^x[f(X_\tau)] = f(x) + E^x\left[\int_0^\tau \mathcal{A}f(X_s)\, ds\right] = f_0(x). \tag{D.4}$$

Since $f_0(x) \in C_0^2(\mathbb{R})$ and $X_{\tau(a,b)} \notin (a, b)$, it follows that the random variable $X_{\tau(a,b)}$ can assume the two values a and b, only. Then the mean value $E^x[f(X_\tau)]$ of $f(X_\tau)$ is given by the sum of the two products of the values $f_0(a)$ and $f_0(b)$ multiplied by the corresponding probabilities, $1 - p$ and p respectively, that is

$$f_0(x) \equiv E^x[f(X_\tau)] = f_0(a)(1 - p) + f_0(b)p. \tag{D.5}$$

From the equality between the first and the third term we obtain the final relation

$$p(b) = \frac{f(x) - f(a)}{f(b) - f(a)}, \tag{D.6}$$

and thus:

$$p(a) = \frac{f(b) - f(x)}{f(b) - f(a)}, \tag{D.7}$$

because the equalities on the boundary of the interval $f_0(a) = f(a)$ and $f_0(b) = f(b)$ hold. In the text we assume that $a = \underline{x}$ and $b = \overline{x}$ from which it follows equation (31).

Appendix E

In order to give an explicit expression to (31) we have first to solve equation (29). To this aim we transform it into a first order differential equation through the change of the variable $f'(x) = g(x)$, so that our equation now reads:

$$\frac{x^2}{2\sigma^2}\frac{dg(x)}{dx} + \left(\alpha - \frac{1}{\sigma^2}\right) x\, g(x) = 0. \tag{E.1}$$

By separating $x, g(x)$ one obtains

$$\frac{dg}{g} = \left(2 - 2\sigma^2\alpha\right)\frac{dx}{x}, \tag{E.2}$$

whose solution is

$$g(x) = f'^{2-2\sigma^2\alpha}. \tag{E.3}$$

By integration, we finally obtain the function $f(x)$:

$$f(x) = \int g(x)\, dx = C\left(\frac{x^{3-2\sigma^2\alpha}}{3 - 2\sigma^2\alpha}\right) + K. \tag{E.4}$$

We now have an explicit expression of the harmonic measure that debt hits the extremes of D as follows:

$$\mu^x(\underline{x}) = \frac{\overline{x}^{3-2\sigma^2\alpha} - x^{3-2\sigma^2\alpha}}{\overline{x}^{3-2\sigma^2\alpha} - \underline{x}^{3-2\sigma^2\alpha}}, \tag{E.5}$$

$$\mu^x(\overline{x}) = 1 - \mu^x(\underline{x}). \tag{E.6}$$

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