Uncertainty Averse Bank Runners

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Abstract

In the framework of a Diamond-Dybvig-Peck-Shell banking model, in which a broad class of feasible contractual arrangements is allowed and which admits a run equilibrium, we stress the assumption that depositors are uncertain of their position in the queue when expecting a run. The formalization of the depositor’s attitude towards this form of uncertainty is inspired by the multiple prior maxmin expected utility (MEU) theory axiomatized by Gilboa and Schmeidler (1989). We prove that there exists a positive measure set of subjective prior beliefs, obtained from the minimization over the set of admissible priors, for which the bank run equilibrium disappears. The implication is that ‘suspension schemes’ are valuable since, in addition to the improvement in risk-sharing among agents (Wallace (1990)), they may undermine panic-driven bank runs.

Keywords: Uncertainty, Multi-Prior Beliefs, Suspension Schemes, Panic-Driven Bank Runs.

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1 Introduction

Starting from the seminal Diamond and Dybvig’s (1983) paper (D-D henceforth), a stream of literature has developed which looks at bank runs as a phenomenon originating from a coordination failure driven by an extrinsic random variable, namely a ‘sunspot’. One of the most recent and important contributions to the topic is Peck and Shell (2003) which, along the lines of the ‘classical’ D-D, designs a model admitting a multiplicity of equilibria and further develops the issue of the selection among them. A significant departure of this framework from D-D, tracing back to Wallace (1988), is the broadening of the set of feasible contractual arrangements from the ‘simple contracting’ (so-called by Green and Lin (2000)) considered by D-D to a class of banking mechanisms that allow for suspension schemes.

In what follows we will refer in particular to Peck and Shell (2003), whose model is modified in order to encompass a depositor with uncertain beliefs on her position in the queue in the case a bank run occurs. Uncertainty here is to be intended in the sense, first given by Knight (1921), that the information of each depositor is too vague to be represented by a probability distribution. Hence it is to be distinguished from the quantifiable uncertainty, usually referred to as ‘risk’. It is assumed that, when each depositor expects a (rarely observed) run to occur, she is no longer able to evaluate correctly the probability distribution of her position number in the queue. The rationale for introducing such an assumption can be justified as follows: since the bank, in finding the optimal contract, is allowed to assign different pay-offs across depositors as a function
of their place in line, when a run occurs each depositor might feel penalized, in terms of insufficient information about her personal "running skills", with respect to her 'competitors' to gain the relatively highest pay-offs. In particular, if partial suspension of convertibility characterizes the optimal mechanism (Wallace (1990)), the patient depositor might reasonably be afraid of being located among the last positions in the queue and, as we will clarify below, might be eventually discouraged from running.

Our formalization of the depositor's attitude towards uncertainty is inspired by the multiple prior maxmin expected utility (MEU) theory axiomatized by Gilboa and Schmeidler (1989). This approach is a well-established generalization of the subjective expected utility (SEU) theory, which can accommodate the choice behavior of Ellsberg-type situations, in which individuals are not able to estimate reliably probabilities. In representing subjective beliefs, the MEU decision rule suggests to replace the 'classical' single prior with a closed and convex set of priors (multi-prior beliefs). The agent is said to be uncertainty averse if this set is not a singleton. The choice among alternative acts is made by computing the minimum expected utility over this set of priors for each of them (acts) and then by singling out the one associated with the highest computed value. The application of this decision rule to our framework leads to assume a depositor who maximizes her expected pay-off with respect to the binary choice -whether or not to deposit -, while selecting the worst probability distribution (over her position in the queue) among all the admissible ones. As we will show

\[\text{\footnotesize\textsuperscript{1}}\text{Several applications of this decision rule have been elaborated over the last few years. We recall, among the others, Epstein and Wang (1994), and Hansen and Sargent (2000).}\]
in the next Section, since in a mechanism design approach pay-offs generally vary as a function of the position number, uncertainty aversion may alter the agent’s withdrawal strategy. Our proposition states that, in a MEU approach to decision-making under uncertainty, there always exists a set of minimizing prior beliefs which makes the bank run equilibrium disappear. Interestingly, this result is obtained independently of the bank’s solution to the problem of choosing the optimal mechanism. Consequently we suggest that ‘suspension schemes’ are worthy, not only because they improve risk-sharing (Wallace (1990)), but also because they may undermine panic-driven bank runs in a potentially general class of frameworks.

2 Aversion to Uncertainty and Propensity to Run

The banking model developed in Peck and Shell (2003) is characterized by aggregate uncertainty on the distribution of the agent’s type and by the observance of the so-called sequential service constraint (which forces the bank to deal with customers sequentially). There are three periods and \( N \) potential depositors, \( \alpha \) being the number of impatients and \( N - \alpha \) that of patients. Each of them is endowed with \( y \) units of consumption in period 0 regardless of type. Impatient agents evaluate utility of period 1 only, through a function \( u(c^1) \), while patient agents, who are allowed to costlessly store consumption across periods, evaluate utility of both periods 1 and 2 through the function \( v(c^1 + c^2) \), where \( c^1 \) and \( c^2 \)
represent respectively consumption received in period 1 and 2. Both functions are assumed to be strictly increasing and concave, twice continuously differentiable and with the relative risk aversion coefficients strictly higher than one for each positive $x$. The bank, whose target is to maximize the ex-ante expected utility of consumers, knows the probability distribution over all possible realizations of types $f(\alpha)$ for $\alpha = 0, 1, ..., N$ and, as usual, is not able to recognize the agent’s type. As regards technology, 1 unit of consumption invested in period 0 yields $R$ units in period 2 and 1 unit in period 1. As a consequence of the technology and preference assumptions, in autarchy patient depositors strictly prefer to consume in period 2.

In this framework it is essential to distinguish between pre- and post-deposit game. In the latter consumers are assumed to have already deposited their endowments and, after having learnt their type (at the beginning of period 1), must only decide whether to withdraw in period 1 or in period 2. The pre-deposit game also encompasses the agent’s choice between deposit and autarchy: that choice is not trivial since, for example, the agent would decide not to deposit if she knew that a bank run would occur with probability 1.

The best solution in Peck and Shell (2003) is obtained by maximizing total welfare, defined as the sum of the utilities of the two types weighted with the probabilities of all possible realizations, subject to the resource constraint and to an incentive compatibility constraint (ICC) stating that patient depositors, in comparing the expected pay-off associated with the ‘truth telling’ strategy (withdrawing in period 2) with that associated with the strategy of ‘lying’ (with-
drawing in period 1), must prefer to tell the truth. The solution to the problem reveals that, even if the ICC holds, the economy may be subject to a bank run. The no-bankrun condition (NBC) that would be violated in this case can be written as follows:

\[
\frac{1}{N} \sum_{z=1}^{N} v(c^1(z)) \leq v \left( Ny - \sum_{z=1}^{N-1} c^1(z) \right) R \]  

(NBC)

This condition states that, even though the patient depositor had the belief that any other agent would be running, she would be however interested in waiting until period 2.

Our departure from this framework is concerned with the probability distribution with which each agent is assumed to be endowed in order to evaluate her position in the queue. In Peck and Shell (2003) the agent evaluates each place in line as equally likely independently of whether or not a bank run is expected. Conversely, for the reasons stated in the introduction we allow probabilities to vary across position numbers whenever depositors believe that an unusual event such as a run is about to occur. Following the Gilboa and Schmeidler’s (1989) MEU theory, we further assume that, when a bank run is expected:

1. the agent’s subjective belief about her own position in the queue is modeled as a set of additive probability measures (multiple prior belief);

2. the agent’s choice behavior is represented as a maxmin strategy, which drives her to maximize her utility with respect to the binary choice between deposit contract and autarchy and, at the same time, to find the additive prob-
ability distribution (over her position number) which minimizes the pay-off associated with withdrawing in period 1.

We can now state the following:

**Proposition 1** For the post-deposit game there exists a positive measure set of minimizing priors which makes the bank run equilibrium disappear.

**Proof.** In the NBC the standard probability distribution over all positions in the queue can be described as:

\[ q_z = \frac{1}{N} \quad \forall z = 1, ..., N \]

where \( z \) stands for the position and \( q_z \) for the probability of being the \( z \)-th in the queue. Now replace it with the following set of priors:

\[ \tilde{q}_z = [0 + \varepsilon, 1 - \varepsilon] \quad \text{for} \quad \varepsilon > 0 \quad \text{and} \quad \forall z = 1, ..., N, \quad (1) \]

and suppose -w.l.o.g., as it will be argued below - that ‘weak’ PSC characterizes the optimal solution:

\[ c_1(1) \geq c_1(2) \geq ... \geq c_1(N - 1) \geq Ny - \sum_{z=1}^{N-1} c_1(z). \]

The relation above identifies two possible cases:

1. The optimal solution is:

\[ c_1(1) = c_1(2) = ... = c_1(N - 1) = Ny - \sum_{z=1}^{N-1} c_1(z) \quad (2) \]

In this case the minimizing distribution is anyone among all possible additive distributions belonging to the set defined in (1). Then the NBC becomes:

\[ v \left( Ny - \sum_{z=1}^{N-1} c_1(z) \right) < v \left[ \left( Ny - \sum_{z=1}^{N-1} c_1(z) \right) R \right], \]

which is always satisfied \( \forall R > 0 \) and no bank run can occur. Notice that (2) corresponds to the ‘autarchic solution’.
2. In the optimal solution, at least one pay-off is strictly greater than the others. Suppose (w. l. o. g.) that:

\[ c^1(1) \geq c^1(2) \geq \ldots \geq c^1(N-1) > Ny - \sum_{z=1}^{N-1} c_1(z). \]

In this case the minimizing prior with respect to (1) would be:

\[ q = \varepsilon \forall z = 1, \ldots, N-1; q_{N} = 1 - (N-1)\varepsilon \]

and the NBC becomes:

\[ \sum_{z=1}^{N-1} \varepsilon v(c^1(z)) + [1 - (N-1)\varepsilon] v \left( Ny - \sum_{z=1}^{N-1} c_1(z) \right) < v \left( Ny - \sum_{z=1}^{N-1} c_1(z) \right) R \]

We argue that, \( \forall R > 0 \), there is at least an \( \varepsilon > 0 \) that satisfies the condition stated above. The threshold value of \( \varepsilon \) below which the bank run disappears is:

\[ 0 < \varepsilon = \frac{v \left( Ny - \sum_{z=1}^{N-1} c_1(z) \right) R - v \left( Ny - \sum_{z=1}^{N-1} c_1(z) \right)}{\sum_{z=1}^{N-1} v(c^1(z)) - (N-1)v \left( Ny - \sum_{z=1}^{N-1} c_1(z) \right)} \]

Notice also that the assumption of PSC has been made w.l.o.g. Indeed suppose that the pay-off associated with the last position is not the minimum because there exists:

\[ v(c^1(i)) < v \left( Ny - \sum_{z=1}^{N-1} c_1(z) \right) \text{ for some } i \in [1, N-1]; \]

then it will also be:

\[ v(c^1(i)) < v \left( Ny - \sum_{z=1}^{N-1} c_1(z) \right) < v \left( Ny - \sum_{z=1}^{N-1} c_1(z) \right) R \]

and the reasoning of the proof can be repeated identically. ■

References


