Wealth Accumulation and Growth in a Specific-Factors Model of Trade and Finance

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Abstract

This paper investigates the allocative properties of an OLG specific-factors model of capital accumulation with land as a productive asset and perfect capital mobility. The analysis focuses on wealth formation and economic development within an articulated portfolio structure and different labor market regimes. The key-finding of the analysis is that exogenous shocks that do not affect human wealth and/or the propensity to save leave consumption, financial wealth and labor hours unchanged. In these cases, capital formation is driven by the static effects that exogenous disturbances exert on the production structure and sectoral labor. Disturbances that alter human wealth and/or

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the subjective discount rate affect financial wealth and consumption as they involve an intergenerational redistribution of resources that modifies individual and aggregate saving. In these cases, while wealth accumulation is driven by the redistribution across generations, capital formation is the result of the consequences of the shocks on the production structure, financial wealth and the firm cost of capital. Aggregate manhours are solely affected by changes in the world interest rate and the rate of time preference. Our results differ substantially from those obtained by Jones (1971), Samuelson (1971) and Eaton (1987 and 1988). Finally, we show that the consideration of a labor market with structural unemployment, due to incentive-wage considerations of the shirking type, does not affect qualitatively the results obtained with competitive wages and no unemployment, except for the world interest rate and the rate of time discount shocks.

*JEL classification:* F41, F43, O41;

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1 Introduction

The specific-factors model of international trade, developed by Jones (1971) and Samuelson (1971), represents, once formulated in a dynamic version, the natural environment for investigating the distinct role that reproducible and non-reproducible productive assets play in the intertemporal allocation of resources. In fact, physical capital and unimproved land, the usual specific-factors considered, represent alternative vehicles for holding wealth. Moreover, specific-factor supplies and values are strictly linked to wealth accumulation; in particular, saving decisions make the supply of capital endogenous, but they solely affect the land market value, being the supply of land fixed.\footnote{The simultaneous role of land as a fixed factor of production and an asset has been initially analyzed within an intertemporal optimizing model by Feldstein (1977). However, the original idea of the interaction between land rent and capital accumulation dates back at least to Ricardo (1817).}

The analysis of a specific-factors economy in an intertemporal context has been carried out by Eaton (1987), who considers factor-asset specificity within a life-cycle model of capital accumulation under financial autharky. In such dynamic setting, the predictions of the simple Jones-Samuelson model in terms of relative commodity prices and factor endowments are enriched and sometime modified. The consideration of capital and land as stores of value adds to the static effect of exogenous shifts an asset-valuation effect, deriving from the change in the land value, that affects the amount of saving devoted to capital formation; the change in capital stock exerts ripercussions on factor prices, factor rewards and the pattern of production.

Financial capital immobility, however, is at odds with the reality for many advanced economies that have an unrestricted access to the world financial market, whose agents hold in their portfolios foreign assets in addition to
domestic assets, and are exposed to the reiterated waves of financial globalization.

Some recent articles have developed specific-factors models within a perfectly integrated financial world. See, for example, Roldos (1991), Brock and Turnovsky (1993), and Kose (2002). These studies, addressing different issues within a representative-agent small open economy—like the growth effects of tariffs and the international generation-propagation of the business cycle—do not consider land as an asset, but simply as a fixed factor of production. Therefore, they do not consider the transmission mechanism of exogenous impulses on the market value of the fixed asset and hence their feed-backs on saving canaled towards capital formation and the accumulation of net foreign assets.

The purpose of this paper is to investigate the allocative properties of an overlapping-generations specific-factors model of capital accumulation with land as a productive asset, perfect capital mobility and endogenous labor.

The analysis focuses on wealth formation and economic development within an articulated portfolio structure, characterized by two productive assets, one reproducible and one non-reproducible, and one non-productive asset. Furthermore, two different labor market structures are at the center stage of the analysis: one with competitive wages and no unemployment, one with incentive-wages and structural unemployment.²

²A specific-factors model of capital accumulation in a financially globalized economy is sketched in the case of an inelastic labor supply by Eaton (1988), but its general allocative properties and the exogenous shock implications are not investigated, being the analysis mainly conducted within a one-sector model with a fixed asset. Regarding a specific-factor economy, Eaton (1988) studies only the effects of parametric changes in foreign assets, within a financially semi-integrated world economy, and in the relative commodity prices, within an economy with perfect capital mobility.
We analyze the quasi-opposite case to the static Jones-Samuelson one (where the capital endowment is fixed and the price of capital is flexible), since physical capital is endogenous, while the price of capital is fixed at a world level because of the small open economy.\(^3\)

The key-finding of the analysis is that exogenous shocks that do not affect human wealth and the saving rate—like the terms of trade and factor endowment shifts, as well as technological shocks in the land-using sector—leave consumption, financial wealth and aggregate manhours unchanged. In these cases, capital formation is driven by the static effects that the exogenous disturbances exert on the production structure and in particular on labor used in the capital-using sector.

Disturbances that alter human wealth—like exogenous shifts regarding capital, labor and the interest rate, as well as technological shocks in the capital-using sector—and the propensity to save affect financial wealth and consumption as they involve an intergenerational redistribution of resources that modifies individual and aggregate saving. In such cases, while wealth formation is driven by the intergenerational forces operating through the human wealth channel, capital formation is the result of changes in the production structure, financial wealth and the firm cost of capital. The land-valuation effect along with capital formation determines the consequence of exogenous

\(^3\)The fixed price of capital exerts strong implications on the distributional structure of the specific-factors setup. The distributional aspects of the specific-factors model are frozen also in Roldos (1992), where an infinite-lived monetary economy with financial capital immobility is studied. Apart from the monetary considerations, which add long-run inflation to the list of exogenous shifts, the relevant point of the Roldos (1992) analysis is that the predictions of an immortal specific-factors economy are more clear-cut than those of an OLG economy since the rate of time preference pins down the stock of capital and, by fixing the capital price, plasters the distributional features of the economy.
shocks on net foreign assets holdings. Only disturbances that hit the world interest rate and the rate of time preference do change aggregate labor hours.

Our results differ substantially from those obtained by Jones (1971), Samuelson (1971) and Eaton (1987 and 1988).

Finally, we show that the consideration of a labor market with structural unemployment, due to incentive-wage considerations of the shirking type, may qualitatively affect the results obtained with competitive wages and no unemployment only when exogenous changes in the world interest rate and the rate of time discount take place.

The structure of the paper is as follows. Section 2 sets out the neoclassical model and investigates the its allocative characteristics as well as the steady state effects of several shifts. Section 3 presents the incentive-wage economy and studies its properties of comparative statics. Section 4 concludes.

2 Neoclassical economy

2.1 The model

Let us consider a real small open economy that produces two goods, \( X \) and \( Y \), and operates in a world of perfect capital mobility. The two sectors of production, which present a Jones-Samuelson physiognomy, are competitive and use standard neoclassical constant returns to scale production functions.\(^4\)

Good \( X \), assumed to be the numeraire, is obtained by using physical capital \( K \), which is sector-specific, and labor \( L_X \), which is perfectly mobile across sectors, namely \( X = F(K, L_X) = L_X f\left(\frac{K}{L_X}\right) \), where \( f(\ ) \) is the sectoral output-labor ratio, and \( f' > 0, f'' < 0 \). Good \( Y \), whose price measured in

\(^4\)See Jones (1971), Samuelson (1971) and Mussa (1974).
terms of the numeraire is $\tilde{p}$ fixed at world level, is produced by employing unimproved land $T$ and sectoral labor $L_Y$, i.e. $Y = H(T, L_Y)$.

First-order conditions for maximum profit in the two sectors entail

\[ f'(\frac{K}{L_X}) = r^*(1 + \tau_K) \]  
\[ f(\frac{K}{L_X}) - \frac{K}{L_X} f'(\frac{K}{L_X}) = v(1 + \tau_L) \]  
\[ \tilde{p} H_T(T, L_Y) = R \]  
\[ \tilde{p} H_L(T, L_Y) = v(1 + \tau_L), \]

where $r^*$ is the given world interest rate (equal to the domestic interest rate as perfect capital mobility has been assumed), $\tau_K$ the ad valorem tax rate on capital, $v$ real wage, $\tau_L$ the ad valorem tax rate on labor, and $R$ is the land reward.\(^5\) Full wage flexibility and perfect sectoral mobility of labor ensure that both sectors face identical wages.

On the demographic-side, this economy is peopled by Blanchard-Yaari households having uncertain lifetimes with no bequest motives, facing a constant mortality rate $\theta$ and supplying labor endogenously.\(^6\) Population, composed of chronologically disconnected cohorts continuously entering the economy, is assumed to remain constant and hence is normalized to one.

Assuming that the individual utility is logarithmic in consumption, $c$, and leisure, $l - l$ (where $\tilde{l}$ represents time endowment and $l$ labor hours

\(^5\)Capital and labor are assumed to be taxed, while land is untaxed. See footnote 16 below for a discussion of the effects of a tax on land.

\(^6\)See Yaari (1965), Blanchard (1985) and Phelps (1994, ch. 16). We depart from Eaton (1987) and (1988), where instead two-period Samuelson-Diamond demographics are employed.
(supplied), at each instant $t$ a consumer born at time $s$ solves the following problem

$$\max \int_t^\infty \left\{ \alpha \ln c(s,j) + (1 - \alpha) \ln \left[ \hat{l} - l(s,j) \right] \right\} \exp[-(\theta + \rho)(j - t)] dj,$$

subject to the instantaneous budget constraint

$$\frac{dw(s,t)}{dt} = (r^* + \theta)w(s,t) + v(t)l(s,t) - c(s,t),$$

and the solvency condition precluding Ponzi schemes

$$\lim_{j \to \infty} w(j,t) \exp[-(r^* + \theta)(j - t)] = 0,$$

where $w(s,t)$ denotes nonhuman wealth of a consumer born at time $s$, $\rho$ the rate of time preference (exogenous), and $\alpha$ a positive preference parameter.

The optimality conditions for the individual problem are

$$c(s,t) = \alpha(\theta + \rho)[w(s,t) + h(s,t)]$$

$$\hat{l} - l(s,t) = \frac{(1 - \alpha)c(s,t)}{\alpha v(t)}$$

$$\frac{dc(s,t)}{dt} = (r^* - \rho)c(s,t),$$

where $h(s,t)$ is the consumer’s human wealth, given by

$$h(s,t) = \int_t^\infty [v(j) \hat{l}] \exp[-(r^* + \theta)(j - t)] dj.$$

The demand-side of the model can be expressed in aggregate terms as

$$C = \alpha(\theta + \rho)(W + H) \quad \text{(2a)}$$
\[
\tilde{L} - L = \frac{(1 - \alpha)C}{\alpha \nu} \tag{2b}
\]
\[
\dot{H} = (r^* + \theta)H - v \tilde{L} \tag{2c}
\]
\[
C + \dot{W} = r^*W + vL, \tag{2d}
\]
where the time index has been omitted and capital letters denote aggregate variables of the corresponding individual variables. By using relationships (2), the Blanchard-Yaari law of consumption dynamics is obtained
\[
\dot{C} = (r^* - \rho)C - \alpha \theta(\theta + \rho)W. \tag{2a'}
\]
Nonhuman wealth is composed of three perfectly substitutable assets, i.e. physical capital \(K\), unimproved land \(T\) and net foreign assets, \(B\);\(^7\) that is \(W = K + qT + B\), where \(q\) is the price of land. As the stock of nonhuman wealth is assumed to be strictly positive, the steady state equilibrium requires that \(r^* > \rho\) from (2a').

Perfect asset substitutability requires that the rates of return from holding each asset must be equal when expressed in terms of the same numeraire
\[
r^* = \frac{R}{q} + \frac{\dot{q}}{q}, \tag{3}
\]
where perfect foresight has been assumed.

\(^7\)We are assuming that capital and land are entirely owned by domestic residents, who are free to borrow and lend abroad. It could be alternatively assumed, without altering the equilibrium, that the stock of capital and land are partly owned by domestic residents and partly by foreigners (see Eaton, 1988).
The economy is endowed with a fixed quantity of non-reproducible land $\tilde{T}$, fully used in the land-using sector. Labor market equilibrium requires that the amount of labor employed by firms in the two sectors of production must equal aggregate labor supplied by households; that is

$$L_X + L_Y = L. \quad (4)$$

The government collects revenues from taxing capital and labor, and spends them unproductively. Therefore, the government budget constraint is given by

$$\tau K r^* + \tau LL = G, \quad (5)$$

where $G$ represents unproductive government spending. The budget is kept continuously balanced through the endogenous adjustment of $G$.\footnote{The case of a compensatory finance based on consumption taxation would leave our findings unchanged. Moreover, we have deliberately avoided to consider lump-sum tax financing as changes in lump-sum taxes would cause a redistribution of income across generations, modifying aggregate saving and the stock of nonhuman wealth, and obscuring the direct implications of exogenous shocks on the resource allocation.}

Finally, the current account, i.e. the trade balance plus the interest income earned of net foreign asset, gives the rate of accumulation of $B$

$$\dot{B} = X + \tilde{p} Y - C - \dot{K} - G + r^* B. \quad (6)$$

The complete macroeconomic model, obtained by combining the optimality conditions for firms and households with the market clearing conditions, the government budget constraint, and the relevant equations of accumulation, is saddle-point stable as shown in the Appendix.
2.2 Comparative statics

The analysis focuses on the long-run allocative properties of our OLG specific-factors economy. It is worth emphasizing some mechanical features of the steady state in order to facilitate the understanding of the comparative statics effects of exogenous shifts.

First, the marginal productivity of capital is tied down by the given cost of capital for firms, \( r^*(1 + \tau_K) \). This implies that (1a) uniquely determines capital intensity in the \( X \)-sector; that is

\[
\bar{K} = \kappa \bar{r}^*(1 + \tau_K), \quad \kappa < 0,
\]

(7a)

where overbar variables denote steady state values and \( \kappa(\cdot) = f'^{-1}(\cdot) \). Equation (7a) establishes, for a given \( r^*(1 + \tau_K) \), a positive relationship between capital stock and labor employed in the capital-using sector. An increase in the cost of capital for firms, due to either higher \( r^* \) or \( \tau_K \), lowers capital intensity as it reduces the demand for capital.

Second, by using (1b) and (7a), the wage rate can be expressed as

\[
\bar{v} = \omega \bar{r}^*(1 + \tau_K)
\]

(7b)

\[
\frac{1}{1 + \tau_L}
\]

where \( \omega(\cdot) = f[\kappa(\cdot)] - \kappa(\cdot)f'[\kappa(\cdot)] \). A rise in either the world interest rate or the capital tax rate, by shrinking capital intensity, drives the real wage down. A higher labor taxation leaves the firm labor cost unchanged, but lowers household wage.

Third, the reduced form for labor employed in the land-using sector, obtained by solving \( \tilde{p} H_L(\tilde{T}, \tilde{L}_Y) = \omega \bar{r}^*(1 + \tau_K) \), is
\( \bar{L}_Y = \Lambda[\bar{p}, \bar{T}, r^*(1 + \tau_K)], \quad \Lambda_{\bar{p}} > 0, \quad \Lambda_{\bar{T}} > 0, \quad \Lambda_{r^*(1+\tau_K)} > 0. \) \hspace{1cm} (7c)

A rise in the terms of trade (or in the land endowment) stimulates labor demand in the land-using sector and hence increases sectoral labor. A higher cost of capital, by reducing the wage rate, induces firms to hire more labor in the \( Y \)-sector.

Define \( \bar{y}^W \) as income from nonhuman wealth, given by the sum of income on wealth and the actuarial premium on wealth received by households from insurance competitive companies; that is, \( \bar{y}^W = (r^* + \theta) \bar{W} \). \hspace{1cm} (8d)

Taking into account relationships (7) and the income from wealth definition, the economy can be summarized by the system

\[
\bar{K} = \kappa[r^*(1 + \tau_K)] \left\{ \bar{L} - \Lambda[\bar{p}, \bar{T}, r^*(1 + \tau_K)] \right\} \quad (8a)
\]

\[
\bar{L} - \bar{L} = \frac{(1 - \alpha) \bar{C}}{\alpha \bar{b}} \quad (8b)
\]

\[
\bar{C} = \frac{\alpha \theta (\theta + \rho)}{(r^* - \rho)(r^* + \theta)} \bar{y}^W \quad (8c)
\]

\[
\bar{C} = \frac{r^*}{(r^* + \theta)} \bar{y}^W + \bar{v} \bar{L}. \quad (8d)
\]

\( \bar{L}_X \) is determined from (7a), once (8a) is used for capital stock.

\( \Lambda_{\bar{p}} = -\frac{H_L}{\bar{p} H_{LL}} > 0, \quad \Lambda_{\bar{T}} = \frac{L_Y}{\bar{T}} > 0 \), and \( \Lambda_{r^*(1+\tau_K)} = -\frac{\kappa [r^*(1 + \tau_K)]}{\bar{p} H_{LL}} > 0 \).

\( \bar{y}^W \) is done with the scope of facilitating the comparison of the neoclassical economy with the incentive-wage one presented below.
Furthermore, in order to understand how aggregate labor hours are determined, we can proceed as follows.\(^{11}\) Substituting \(\tilde{C}\) from (8d) into (8b), and rearranging, we obtain

\[
\frac{\bar{L}}{L} = \alpha - \frac{(1 - \alpha) r^* \bar{y}^W}{(r^* + \theta) \bar{v}L}.
\]  

Equation (9) gives the labor supply in terms of nonwage-income-to-wage ratio. An increase in \(\bar{y}^W\) lowers manhours worked because it raises, through (8d), the consumption-to-wage ratio and hence, through (8b), leisure. Equation (9) is represented by the \(LS\) schedule in Fig. 1. The \(LS\) schedule is shifted downward by a rise in \(r^*\).

Plugging (8d) into (8c) yields

\[
\frac{\bar{L}}{L} = \Theta(r^*, \rho) \frac{\bar{y}^W}{\bar{v}L}, \quad \Theta_{r^*} < 0, \quad \Theta_\rho > 0,
\]  

where \(\Theta(r^*, \rho) = \frac{[\alpha \theta (\theta + \rho) - r^* (r^* - \rho)]}{(r^* - \rho)(r^* + \theta)}\).\(^{12}\) Equation (10) describes the combinations of \(\frac{\bar{L}}{L}\) and \(\frac{\bar{y}^W}{\bar{v}L}\) compatible with the Blanchard-Yaari asset market equilibrium (as described by the arbitrage condition between consumption and wealth returns). This relationship is represented by the \(BY\) schedule in Fig. 1. Intuitively, an increase in the ratio of income-from-wealth-to-wage induces higher consumption-to-wage ratio; since \(\frac{\bar{y}^W}{\bar{v}L}\) increases in absolute terms less than the \(\frac{\tilde{C}}{\bar{v}L}\), a compensatory rise in manhours is needed in order

\(^{11}\)See Petrucci and Phelps (2004).

\(^{12}\)Note that the condition \(\alpha \theta (\theta + \rho) > r^* (r^* - \rho)\) guarantees saddle-point stability of the steady state (see the Appendix).
to satisfy the consumer budget constraint (8a). The BY curve is shifted downward by a rise in $r^*$ or a fall in $\rho$.

[Insert Fig. 1]

The intersection between the LS and BY schedules determines labor hours and the nonwage-income-to-wage ratio. Since the exogenous shifts that impinge on (9) and (10) only regard $r^*$ and $\rho$, labor hours and the income-from-wealth-to-wage ratio are invariant with respect to any other exogenous disturbance. Once the consequences of exogenous shocks on $\bar{L}$ and $\bar{y}_W / \bar{\sigma}_L$ are identified, the effects on $\bar{K}$ and $\bar{C}$ and can be desumed by using (8a) and (8d), respectively.

Moreover, the reduced forms for the land reward and the land value are given by\textsuperscript{13}

$$
\bar{R} = R[\bar{p}, r^*(1 + \tau_K)], \quad R^- > 0, \quad R_{r^*(1+\tau_K)} > 0; \quad (11a)
$$

$$
\bar{q} = \frac{R[\bar{p}, r^*(1 + \tau_K)]}{r^*} = q[\bar{p}, r^*(1 + \tau_K)], \quad q^- > 0, \quad q_{r^*} > 0, \quad q_{\tau_K} > 0. \quad (11b)
$$

Finally, the stock of net foreign assets can be computed, once $\bar{y}_W$ and $\bar{K}$ are determined, through the following expression

\textsuperscript{13}Equations (11) are obtained by using (1c), (3) and (7). The effects of exogenous shifts on the land reward and the land market value are:

$$
R^- = H_T + \frac{H_L \bar{L}_Y}{T}, \quad R_{r^*(1+\tau_K)} = \frac{\kappa(r^*, \tau_K) \bar{L}_Y}{T}, \quad (11c)
$$

and

$$
q^- = \frac{R^-}{r^*} > 0, \quad q_{r^*} = \frac{1}{r^*} \left( \frac{\bar{K}}{\bar{L}_X} r^*(1 + \tau_K) \bar{L}_Y \right), \quad q_{\tau_K} = \frac{R_{\tau_K}}{r^*} > 0. \quad (11d)
$$
\[ B = \frac{y^W}{(r^* + \theta)} - K - q[\tilde{p}, r^*(1 + \tau_K)]^T. \] (11c)

We can now study the comparative statics.

2.2.1 The terms of trade

Consider the effects of a rise in the terms of trade \( \tilde{p} \).\(^{14}\) Since the terms of trade do not enter (9) and (10), \( \bar{L} \) and \( \frac{y^W}{\bar{r}L} \) remain unchanged. Therefore, nonhuman wealth and consumption are invariant, since the household wage is constant from (7b). As the increase in \( \tilde{p} \) expands \( \bar{L}_Y \) from (7c), a reduction of labor used in the capital-using sector occurs so as to leave total labor hours constant. The contraction of \( \bar{L}_X \) brings about a fall in capital stock.

Non-land input prices are constant, while the land reward is driven up since land and labor are Edgeworth complementary. Thus, the market value of land is increased. As capital stock falls and the land value increases, the stock of foreign assets may either rise or fall.\(^{15}\)

2.2.2 Land endowment

A rise in \( \tilde{T} \) reproduces qualitatively most of the macroeconomic effects of a rise in \( \tilde{p} \).\(^{16}\) The land endowment shock, however, does not affect the marginal

\(^{14}\) A terms of trade shock can be assimilated qualitatively to either a technological shock that affects the land-using sector and/or to a \( Y \) output tax hike.

\(^{15}\) The net foreign assets multiplier is given by

\[ \frac{d\tilde{B}}{d\tilde{p}} = \frac{-\tilde{p}}{r^*} H_T - H_L \left\{ \frac{\kappa}{\tilde{p}} \frac{r^*(1 + \tau)}{r^*} + \frac{\tilde{L}_Y}{r^*} \right\}. \]

\(^{16}\) If land rent taxation were considered, a rise in the land tax would, under a government spending compensatory finance, be neutral for the resource allocation and the incidence analysis. The sole effects of the land tax would be a fall in the land value and a rise in the net foreign asset holdings. The same effect is obtained by Eaton (1988) under the
productivity of land and the land market value. Hence, in this case, net foreign assets unambiguously rise.

2.2.3 Capital shift

Since the experiment of a pure parametric change in the reproducible specific-factor cannot be performed, being capital endogenously accumulated, we study alternatively the effect of a capital-promoting shock, like a reduction in the capital tax rate.\(^{17}\)

A fall in \(\tau_K\), accompanied by a compensatory contraction of government spending, does not change labor hours and the income-from-wealth-to-wage ratio. The reduction in the capital tax rate raises capital intensity and the wage rate, through (7a) and (7b), respectively. The higher wage rate pulls nonhuman wealth up alongside with consumption. Labor in the land-using sector is reduced because of the higher wage rate, while labor used for producing \(X\) rises. The increase in capital intensity implies that capital stock expands proportionally more than \(\bar{L}_X\). The cost of capital for firms, the land yield and the land price fall. Net foreign assets may go up or down.

The consequences of \(\tau_K\) on income from wealth and consumption have an intergenerational motivation as they derive from the effect exerted on nonhuman wealth. In fact, the decline in rise in \(\tau_K\) brings about an increase in human wealth,\(^{18}\) which redistributes income from the older generations, who consume more and save less, to the younger generations, who consume less and save more. This mechanism leads to higher aggregate saving, which

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\(^{17}\) This shock is qualitatively equivalent to either a technological change that affects the capital-using sector and/or to a \(X\) output tax shock.

\(^{18}\) From (2c), long-run human wealth is given by \(\bar{H} = \frac{\omega(r^*, \tau_K) \bar{L}}{(1 + \tau_L)(r^* + \theta)}\).
in turn expands the stock of financial wealth and consumption.

2.2.4 Labor shift

Being labor supply endogenous, we explore the effect of a labor stimulative policy with the scope of investigating something-like the effects of an exogenous shift that affects the mobile factor.\textsuperscript{19} A reduction in the labor tax rate $\tau_L$ exerts no effects on labor hours and the nonwage-income-to-wage ratio. The firm cost of labor is unchanged; this implies that the take-home wage is pulled up being the labor tax rate lower. Nonhuman wealth and consumption are therefore increased. Since $\tilde{L}$ and $\tilde{L}_Y$ are invariant, labor used in the capital-using sector and hence capital stock remain constant. The price of capital, the land reward and the price of land are untouched by the shock as well. A rise in net foreign assets occurs.

2.2.5 Saving shift

In order to fully understand how this specific-factor economy works, we study, as in Eaton (1987), the consequences of a saving-stimulative shock.\textsuperscript{20} Our

\textsuperscript{19}Another possible labor shift that can be considered is a change in the aggregate time endowment. An increase in $\tilde{L}$ can be associated with a more efficient use of the time endowment, due, for example, to a reduction of travelling-time costs in traffic de-congested cities or technical changes (like the ICT revolution) that allows to work at home. An increase in $\tilde{L}$ raises manhours supplied and nonhuman wealth, given that the wage rate is constant. Consumption expands. Since labor used to produce $Y$ is unchanged, the rise of aggregate labor is entirely matched by an increase of labor employed in the capital-using sector, which results in a higher capital stock. Input prices and the land value are not touched by the disturbance, while net foreign assets fall.

\textsuperscript{20}When financial capital immobility is considered, as in Eaton (1987), this shock is equivalent to a capital stock stimulus; in our context, instead, such a shock differs from a
experiment considers an increase in thrift derived from a reduction in the rate of time preference.\footnote{A reduction of either the mortality rate $\theta$, i.e. a longer life-time span for consumers, or a nonwage income tax rate would generate the same qualitative consequences of a fall in $\rho$. For the effects of the latter shock within one-sector OLG small open economy, see Nielsen and Sorensen (1991).}

A fall in $\rho$ shifts the $BY$ schedule downward; see Fig. 1. We move from the initial equilibrium at point A to point A’. The effect is to reduce aggregate labor and increase income from nonhuman wealth to wage ratio. This implies that $\bar{y}^W$ increases as $\bar{y}$ is given. Consumption rises proportionally less than income from wealth. Labor in the capital-using sector reflects the reduction in $\bar{L}$, since $\bar{L}_Y$ is constant. Consequently also a reduction in $\bar{K}$ takes place. Factor prices and the land value do not move. The rise in nonhuman wealth derives entirely from the accumulation of net foreign assets, which represents the only way-out for channelling the higher saving.\footnote{If labor supply were inelastic, i.e. $\bar{L}=\bar{L}$, the decline in $\rho$ would be always stimulative for nonhuman wealth and consumption, but neutral for capital formation and the sectoral allocation of labor. If a hike in consumption taxation (accompanied by an increase in government spending) were implemented with the aim of stimulating saving, its effects would be neutral for the macroeconomic equilibrium, except for the consumption level, which would fall, leaving consumption expenditure constant.}

\subsection*{2.2.6 World interest rate}

A world interest rate shock may be seen as a composite shock that results from the simultaneous change in $\rho$, in the opposite direction, and $\tau_K$, in the same direction.

A rise in $r^*$ shifts both the $LS$ and $BY$ schedules downward; the $BY$ schedule downward; see Fig. 1. We move from the initial equilibrium at point A to point A’. The effect is to reduce aggregate labor and increase income from nonhuman wealth to wage ratio. This implies that $\bar{y}^W$ increases as $\bar{y}$ is given. Consumption rises proportionally less than income from wealth. Labor in the capital-using sector reflects the reduction in $\bar{L}$, since $\bar{L}_Y$ is constant. Consequently also a reduction in $\bar{K}$ takes place. Factor prices and the land value do not move. The rise in nonhuman wealth derives entirely from the accumulation of net foreign assets, which represents the only way-out for channelling the higher saving.\footnote{A reduction of either the mortality rate $\theta$, i.e. a longer life-time span for consumers, or a nonwage income tax rate would generate the same qualitative consequences of a fall in $\rho$. For the effects of the latter shock within one-sector OLG small open economy, see Nielsen and Sorensen (1991).}
schedule shifts down by more. \( \tilde{L} \) falls and \( \tilde{v}_L \) rises. Capital intensity in the \( X \)-sector shrinks as capital is more expensive for firms. Since the wage rate is reduced, income from nonhuman wealth moves ambiguously; also the effect on consumption is ambiguous.\(^{23}\)

Labor in the land-using sector is stimulated by the lower firm labor cost, while labor in the capital-using sector is reduced; \( \tilde{K} \) falls proportionally more than \( \tilde{L}_X \). The marginal productivity of land is pulled up, while the land value as well as net foreign assets moves unclearly.

# 3 Incentive-wage economy

## 3.1 The model

The neoclassical model does not explain equilibrium unemployment as changes in labor are only due to variations of manhours and wages adjust to equate labor supply and demand. In order to investigate the role played by the natural rate of unemployment in the allocation of resources within the specific-factors economy under consideration, we use the incentive-wage theory, based on the assumption of the shirking behavior of workers, developed by Calvo (1979), Solow (1979), and Shapiro and Stiglitz (1984).

We adapt to our case the two-sector model developed by Phelps (1994, ch. 9). The production function for good \( X \) is given by \( X = F(K, \varepsilon N_X) = \varepsilon N_X f(k) \), where \( F(\ , \ ) \) is linearly homogeneous, \( \varepsilon \) is a continuous variable that represents the efficiency of a single worker in the firm, \( N_X \) is the number of workers employed in the \( X \)-sector, \( f(k) \) is the output per unit of labor

\(^{23}\)The lower the mortality rate \( \theta \), the more likely the negative multipliers for nonwage income and consumption.
expressed in efficiency units, \( k = \frac{K}{\varepsilon N_X} \) represents the efficiency-adjusted capital-labor ratio, \( f' > 0 \), and \( f'' < 0 \). The production of good \( Y \) uses the constant-return-to-scale production function \( Y = H(T, \varepsilon N_Y) \), where \( N_Y \) is the number of employees in sector \( Y \). The workers’ effort is the same in the two-sectors.

Following Phelps (1994), we use the function \( \varepsilon = \varepsilon(\frac{z}{v}, \frac{y^W}{v}) \) (where \( \varepsilon_i < 0, \varepsilon_{ij} < 0, \) for \( i, j = 1, 2 \)) to describe the employee’s effort; \( z \) is the expected income obtainable elsewhere if the worker is fired, \( v \) gives the wage per employee paid in the firm and \( y^W \) is the average nonwage income of workers, taken as a ratio to the worker population (whose size is unity).\(^{24}\)

The first-order conditions for maximum profit in the two sectors are\(^{25}\)

\[
 f'(k) = r^*(1 + \tau_K) \tag{12a}
\]

\[
 \varepsilon [f(k) - kf'(k)] = \varepsilon \tilde{p} H_N(T, \varepsilon N_Y) = v(1 + \tau_L) \tag{12b}
\]

\[
 -[f(k) - kf'(k)] \left( \varepsilon_1 \frac{z}{v} + \varepsilon_2 \frac{y^W}{v} \right) = - \tilde{p} H_N \left( \varepsilon_1 \frac{z}{v} + \varepsilon_2 \frac{y^W}{v} \right) = v(1 + \tau_L) \tag{12c}
\]

\[
 \tilde{p} H_T(T, \varepsilon N_Y) = R. \tag{12d}
\]

The workers’ expected income can be expressed as \( z = Nv \) if population and the labor force are normalized to one (see Calvo, 1979, and Salop, 1979).\(^{24}\) As in Phelps (1994), the propensity to shirk is assumed to be homogeneous of degree zero in \( z, v \) and \( y^W \).

\(^{25}\) The concavity of the production function and the assumed signs of the second derivatives of the effort function ensure that the second-order conditions of the firm’s optimality problem are satisfied.
Combining (12b) and (12c) and using \( z = Nv \), we obtain the “modified Solow condition”; that is

\[
-\left( \frac{\varepsilon_1}{\varepsilon} N + \frac{\varepsilon_2 y^W}{v} \right) = 1.
\]  

(12c’)

According to (12c’), the sum of the partial elasticities of the effort function, taken in absolute value, must be equal to one.

Equation (12c’) can be solved for \( N \) as follows

\[
N = \Gamma\left(\frac{y^W}{v}\right), \quad \Gamma' < 0,
\]

(13)

where \( \Gamma' = -\frac{\left(2\varepsilon_2 + \varepsilon_{12} \hat{N} + \varepsilon_{22} \frac{\bar{y}^W}{\bar{v}}\right)}{(\pi / \bar{y}^W)^2 \left(2\varepsilon_1 + \varepsilon_{11} \hat{N} + \frac{\bar{y}^W}{\bar{v}^{\rho \varepsilon_{12}}}\right)} > 0. \) Equation (13) represents the incentive-wage equation in implicit form. It implicitly gives the optimal wage that firms wish to pay for any level of \( N \) and \( y^W \).

Employment in the two sectors equals total employment in the economy

\[
N_X + N_Y = N.
\]

(14)

The rest of the model is the same as in the neoclassical economy, once \( L \) is replaced by \( N \).

In the long-run equilibrium, aggregate employment and the nonwage-income-to-wage ratio are determined by using (13) along with the Blanchard-Yaari asset market equilibrium condition, given by

\[
\hat{N} = \Theta(r^*, \rho) \frac{\bar{y}^W}{\bar{v}}, \quad \Theta_{r^*} < 0, \quad \Theta_\rho > 0.\]

(15)

The rest of the long-run economy is described by

\[\text{The function } \Theta(\cdot, \cdot) \text{ has been defined above.}\]
\[ \tilde{K} = \tilde{\varepsilon} \kappa [r^*(1 + \tau_K)] (\tilde{N} - \tilde{N}_Y), \quad \kappa' < 0, \quad (16a) \]

\[ \tilde{p} H_N(T, \tilde{\varepsilon} \tilde{N}_Y) = \omega [r^*(1 + \tau_K)], \quad \omega' < 0 \quad (16b) \]

\[ \bar{\varepsilon} (1 + \tau_L) = \tilde{\varepsilon} \omega [r^*(1 + \tau_K)] \quad (16c) \]

\[ \bar{C} = \frac{r^*}{(r^* + \theta)} \tilde{y}^W + \bar{\sigma} \tilde{N}, \quad (16d) \]

where \( \tilde{\varepsilon} = \varepsilon \left( \frac{\tilde{y}^W}{\tilde{\sigma}} \right) \) and \( \bar{B} = \frac{\tilde{y}^W}{(r^* + \theta)} - \tilde{K} - q[\tilde{p}, r^*(1 + \tau_K)] \tilde{T} \).

### 3.2 Comparative statics

In the long-run, employment and income-from-wealth-to-wage ratio are determined by (13) and (15). Therefore, \( \bar{N} \) and \( \frac{\bar{y}^W}{\bar{\sigma}} \) are solely influenced by changes in \( \rho \) and \( r^* \), in a way that is qualitatively the same as in the neoclassical economy.

Shocks to the terms of trade, land endowment and tax rates leave aggregate employment, the income-from-wealth-to-wage ratio, and therefore the effort of employees unchanged. As the factor demand system that governs the effects of these shocks is basically the same as in the competitive-wage economy (since \( \tilde{\varepsilon} \) is given), no qualitative changes are obtained with respect to the neoclassical case, once \( \tilde{L}, \tilde{L}_X \) and \( \tilde{L}_Y \) are replaced by \( \tilde{N}, \tilde{N}_X \) and \( \tilde{N}_Y \). Hence, for the \( \tilde{p}, \tilde{T}, \tau_K \) and \( \tau_L \) shocks, it is not worth repeating the comparative statics analysis.

---

27 In the system (16), the following expressions have been used \( \kappa(\cdot) = f'^{-1}(\cdot), \omega(\cdot) = f[\kappa(\cdot)] - \kappa(\cdot)f'[\kappa(\cdot)]. \)
Disturbances that influence $\rho$ and $r^*$, instead, by affecting (13) and (15), alter the workers’ effort and therefore produce effects on the system that are not immediate and deserve some analysis.

3.2.1 Saving shift

A reduction in $\rho$ shifts the $BY$ schedule downward. We move from point A to point A’ in Fig. 2. The effect is to reduce aggregate employment and increase income-from-wealth-to-wage ratio. The employee effort is pulled up by the lower $\bar{N}$, but is simultaneously driven down by the higher $\bar{y}^W/\bar{b}$. The net effect on $\bar{\varepsilon}$ is therefore ambiguous. However, aggregate employment expressed in efficiency units $\bar{\varepsilon}\bar{N}$ declines.\(^{28}\) Since $\bar{\varepsilon}\bar{N}_Y$ is constant from (16b), $\bar{\varepsilon}\bar{N}_X$ falls; capital stock is also driven down as capital intensity in efficiency units is fixed.

The unclear effect on $\bar{\varepsilon}$ implies that the wage rate, income from wealth, consumption, sectoral employment, and net foreign assets may rise or fall. The land reward and the land value instead do not move.

Let us see what happens if $\bar{\varepsilon}$ rises.\(^{29}\) Real wage, consumption, nonhuman wealth and net foreign assets increase. $\bar{y}^W$ increases proportionally more than $\bar{v}$. $\bar{N}_Y$ and $\bar{N}_X$ are both reduced.

If instead $\bar{\varepsilon}$ declines, the wage per employee falls from (12b). Consumption and nonhuman wealth may rise or decline. Employment in the $Y$-sector increases, while employment in the capital-using sector declines.

\[^{28}\text{Its multiplier is given by: } \frac{d(\bar{\varepsilon}\bar{N})}{d\rho} = \frac{\bar{N}^2 \varepsilon_2 \Theta_\rho}{\Theta^2} > 0.\]

\[^{29}\text{This case occurs if the effect of the labor market prospects on the effort function is relatively stronger in magnitude than the effect of nonwage income.}\]
3.2.2 World interest rate

Fig. 2 can be used to describe the effects of a rise in $r^*$. $\tilde{N}$ falls and $\frac{\hat{y}^W}{\hat{y}L}$ rises; employee’s effort moves unclearly, but $\hat{e}\tilde{N}$ is decreased. Capital intensity is reduced by the higher cost of capital.

Labor employed in the land-using sector expressed in efficiency units is stimulated from (16b), while labor cost in efficiency units in the capital-using sector is reduced. The wage rate per employee most probably falls, while income from wealth and consumption may rise or fall. The marginal productivity of land is pulled up, while the land market value as well as net foreign assets moves ambiguously.

4 Conclusions

This paper has investigated the comparative statics properties of a non-altruistic life-cycle economy with specific-factors and perfect capital mobility.

Two special features are incorporated into the model: the disconnection of heterogeneous generations, on the one hand, and the interaction of productive (capital and land) and unproductive (net foreign assets) financial assets, on the other hand.

An additional element of relevance considered in the analysis is the role of the different labor market structures. Two types of labor market have been studied: one with competitive wages and no unemployment, one with incentive-wages and structural unemployment, due to the shirking behavior of workers.

We depart substantially from the findings contemplated by the static specific-factors model of Jones (1971) and Samuelson (1971) and the dynamic
specific-factors model of financial autharky developed by Eaton (1987).

The terms of trade, land endowment, human wealth, and the rate of subjective time discount play a crucial role for the allocation resources; they influence wealth and capital formation in differentiated ways. Shocks that do not affect human wealth and the propensity to save, like the terms of trade and land endowment shifts, exert no intergenerational consequences on the economy, but only change inputs and factor prices according to the static input demand system. Disturbances that impact on nonhuman wealth -like \( \tau_L, \tau_K, \) and \( r^* \) shifts- and \( \rho \), instead, exerts intergenerational effects on the system by altering the distribution of income across generations with different propensity to save. Such redistributive mechanism leads to a change in aggregate saving, which in turn alters the stock of nonhuman wealth and consumption. The effects on factor price and factor use descent from these aggregate consequences.

The consideration of a specific-factors economy with structural unemployment does not change the consequences of exogenous disturbances obtained in the neoclassical economy, except for the subjective discount rate and the world interest rate shocks, where some differences are possible. In fact, in these cases, the induced effect on workers’ effort-efficiency may alter the results for sectoral employment and the income-from-wealth-to-wage ratio.

One of the main objective of the paper has been to study wealth accumulation and economic growth within an open economy with asset-factor specificity and the interest rate fixed at a world level.

The analysis has shown that wealth and capital formation obey to different rules of macroeconomic determination. Within a neoclassical economy, long-run financial wealth is determined by human wealth (i.e. the wage rate, which is influenced by \( \tau_K, \tau_L \) and \( r^* \)) and aggregate labor hours (which are
influenced by $\rho$ and $r^*$). The reduced form for nonhuman wealth is

$$\bar{W} = W(\tau_K, \tau_L, \rho, r^*).$$

The terms of trade and land endowment exert no effects on nonhuman wealth.

Capital stock is instead determined by capital intensity (which depends on $\tau_K$ and $r^*$), aggregate labor hours and manhours in the land-using sector (determined in turn by $\tilde{p}, \tilde{T}, \tau_K$ and $r^*$); its long-run reduced form is given by

$$\tilde{K} = K(\tilde{p}, \tilde{T}, \tau_K, \rho, r^*).$$

Labor taxation is neutral for capital stock. Among the several exogenous shocks investigated, only capital taxation exerts similar long-run effects in qualitative terms on nonhuman wealth and capital stock.

Aggregate labor hours or employment levels are solely affected by some wealth-stimulating shocks, like the world interest rate and the rate of time discount.

Within an incentive-wage economy, the effects of $\rho$ and $r^*$ on nonhuman wealth and capital stock may change because of the ambiguous effects they exert on the employee propensity to shirk.
References


APPENDIX

Neoclassical economy: Analysis of stability

Once the relationships (7) and (11a), which are also valid in the short-run, are used, the short-run model can be written as

\[\dot{C} = (r^* - \rho)C - \alpha \theta (\theta + \rho)(K + q \tilde{T} + B)\]  \hfill (A.1a)

\[\dot{q} = r^* q - \tilde{R}\]  \hfill (A.1b)

\[K + \dot{B} = \bar{\nu} L + \tilde{R} \tilde{T} + r^*(K + B) - C\]  \hfill (A.1c)

\[\tilde{L} - L = \frac{(1 - \alpha)C}{\alpha \bar{v}}\]  \hfill (A.1d)

\[K = \bar{k} (L - \bar{L}_Y),\]  \hfill (A.1e)

where \(\bar{k} = \frac{\bar{K}}{\bar{L}_X} = \kappa, \bar{v}, \bar{L}_Y\) and \(\tilde{R}\) are given by equations (7) and (11a). Government spending is solved residually from the government budget constraint (5).

Equation (A.1b) is an unstable differential equation in \(q\), which yields a finite long-run land value if and only if

\[\bar{q} = \frac{\tilde{R}}{r^*}.\]  \hfill (A.2)
The land value price is constant overtime and hence no capital gains-losses occur from holding wealth in the form of land. The dynamics of $q$ is degenerate in the sense that the land market value is always in the steady state and jumps instantaneously from one equilibrium to another one, if some exogenous shocks occur, with no transitional dynamics.

Equations (A.1d) and (A.1e) can be solved, once linearized around the steady state, for $L$ and $K$ in terms of the dynamic variable $C$ to yield

$$L = l(C), \quad l' < 0 \quad (A.3a)$$

$$K = k(C), \quad k' < 0, \quad (A.3b)$$

where $l' = -\frac{(1 - \alpha)C}{\alpha \bar{v}} < 0$ and $k' = -\frac{1}{\bar{k}} l' < 0$.

Substituting the values of $L$ and $K$ from (A.3a) and (A.3b) into equations (A.1a) and (A.1c) and incorporating (A.2), the model can be reduced to the following system of differential equations linearized around the steady state

$$
\begin{bmatrix}
\dot{C} \\
\dot{Z}
\end{bmatrix} =
\begin{bmatrix}
r^* - \rho & -\alpha \theta (\theta + \rho) \\
\bar{w} l' - 1 & r^*
\end{bmatrix}
\begin{bmatrix}
C - C_s \\
Z - Z_s
\end{bmatrix}
$$

(A.4)

where $Z = K + B$ represents non-land wealth.

The transition matrix must have one positive eigenvalue, associated with the jump variables $C$, and one negative eigenvalue, associated with the predetermined variable $Z$.\footnote{Since $C$ adjusts on impact, $K$ (hence $L$) jumps instantaneously as well, provided we assume, as in Mundell (1957) and Obstfeld (1989), that capital is instantaneously and costlessly mobile across borders. By considering non-land wealth $Z = K + B$ a predetermined variable, we are implicitly assuming that, as $K$ moves repentingly, $B$ adjusts instantaneously as well, but in an opposite direction so as to leave $Z$ unchanged on impact.}
The determinant of the above Jacobian must be negative as a necessary and sufficient condition for saddle-point stability. This condition is satisfied if following inequality holds

\[ \alpha \theta (\theta + \rho) - r^*(r^* - \rho) > 0. \]

It is not difficult to show that the incentive-wage economy of Section 3 requires the satisfaction of the above condition for exhibiting saddle-point stability.