Insurance and Financial Hedging of Oil Pollution Risks

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Abstract

The current international regime that regulates the maritime oil transport calls for financial contributions by oil firms once an oil spill has occurred. Their percentage of contribution to the International Oil Pollution Compensation Fund does only depend on their level of activity. In this paper, we show that this compensation regime would be more efficient if contributing oil companies adopt financial strategies relying on hedging oil pollution risks. Indeed, the optimal coverage contract is such that standard insurance is useful to manage small and medium oil spills, while investments on financial markets help to cover huge oil spills, less frequent but much more catastrophic for Society. We also show that prevention of oil spills increase when insurance is bundled with a financing hedging strategy. This positive effect on prevention is still enhanced when firms have the opportunity to send signals about their risk-reducing activities to potential investors.

Key-Words: oil spill, legislation, insurance, capital markets, prevention, catastrophe.

JEL Classification: D80, G22, Q25.
1 Introduction

The maritime transport of oil is regulated by the 1992 Civil Liability Convention in most countries of the world\(^1\), except mainly for the United States, which have their own Convention\(^2\). In this paper, we focus on the compensation system implemented when an oil spill is registered in the territorial sea of any member of the 1992 Civil Liability Convention. Since oil spills can create severe damages to the environment but also to the human activities near the coast, they may induce huge claims, which cannot be covered without a compensation system adapted to those catastrophic losses. Hence, we will show that the current International regime would benefit from a reorganization involving both standard insurance and financial hedging.

The 1992 International Oil Pollution Compensation Fund (1992 IOPC Fund) participates in the compensation of victims of an oil spill if the payment already granted by the insurer of the owner of the tanker\(^3\) is not sufficient. The contributions of oil firms to the Fund are proportional to the quantity of oil received in a year and they are due each time an oil spill has occurred in the territorial waters of a member, whatever the flag of the tanker and whatever the citizenship of the oil firm. Hence the IOPC Fund enables to compensate victims even if the owner of the tanker is not a citizen of a member state and empirical facts show that the IOPC Fund seems to be rather efficient in minimizing the time between the oil spill event and the effective compensation of victims. However, funds are levied at random dates and expenses are not smoothed through time. Hence we will show that within the current international regime, oil firms would benefit from capital markets by resorting to appropriate financial instruments. Financial mechanisms

\[^1\] 81 states ratified the 1992 Civil Liability Convention on 20 November 2002.
\[^3\] Under the 1992 Civil Liability Convention, only the owner of the tanker is held financially liable for the catastrophe. The convention obliges him to buy pollution insurance, provided by P&I Clubs which are non-profit making mutual insurance associations. These mutual groups offer insurance depending on the size of the boat and not directly on the damages that may be induced by a wreck. Hence, insurance may be limited, as it was the case for the Erika’s wreck on the French coast in December 1999 (7% of the total available funds).
improve and complete the hedging provided by insurance policies which prove to be insufficient to cover alone huge risks related to oil spills.

The oil industry has several interests in using hedging mechanisms to make the current compensation regime more efficient. Without adequate risk management, oil firms loose some efficiency in their activities and the cost induced by this inefficiency is lost for the victims’ compensation. In particular, the 1992 IOPC Fund, as it works, does not rest on the risk transfer principle. By defining contributions on the basis of the aggregate risk of the pool, the mutuality principle (Borch (1962), Wilson (1968)) is applied. Nevertheless, the aggregate risk is still variable because of the possible huge consequences of an incident and because of the limited number of contributing members in the Fund. Consequently, the mutuality principle is no longer sufficient to spread all the risk on the oil firms. We show that capital markets seem to be able to solve the issue of diversification and also to mitigate transaction costs. Indeed Doherty (2000) gives several arguments that make insurance profitable for firms, and that enhance the fact that insurance mechanisms have to be completed by some investment in capital markets when dealing with large risks. Froot (2001) provides also different reasons why markets are more efficient than insurers in global risk reductions. One important point is that securitization may reduce transaction costs such as administrative fees or costs related to agency issues. In the same spirit as in Doherty and Dionne (1993), Schlesinger (1999), Doherty and Schlesinger (2002) and in Mahul (2002), we show that insurance bundled with a financial hedging strategy dominates a situation with only standard insurance. However, our economic context is rather different from these studies. In our framework, each individual bears a percentage of the aggregate risk of the pool (here, the IOPC Fund) and an individual risk of bad reputation that is positively correlated to the aggregate risk and non insurable. Up to now, the litterature focused essentially on risks that can be split into idiosyncratic

\footnote{A large part of total contribution is done by a small number of oil companies. This can be explained by the concentration of the oil sector and the exoneration of contributions of companies belonging to member states receiving less than 150,000 tons of oil a year. Indeed, the application of the mutuality principle, which rests on the law of large numbers, is less effective in this context.}
risk, specific to the individual and easily insurable, and a systematic risk, independent from the idiosyncratic one. Losses induced by reputation constitute a central variable in our model; reputation and its impact on firms’ value has become a major concern for firms involved in environmentally risky activities for the environment as shown by Lanoie et al. (1998). This issue is even more crucial for oil companies in the wake of an huge oil spill. Another important point of our analysis deals with prevention, which is not considered in the previous models. We show that financial hedging may give additional incentives to oil firms to invest in prevention. This result is important when focusing on the current discussions that are held at the European Commission about the evolution of the financing of the IOPC Fund. Especially, it is argued that an increase of individual contributions may improve the safeness of boats chartered by oil firms and allows it fully compensate victims of infrequent huge oil spills\(^5\). Our main aim in this paper is different. We will show that prevention against maritime oil pollution may indirectly be enhanced if oil industry firms apply adequate financial strategies. These strategies imply to hedge oil spill incidents by resorting simultaneously to insurance policies and investment on capital markets.

The paper is organized as follows. The second section focuses on the current regime of the IOPC Fund. In a basis model, we introduce standard insurance mechanisms and we define the optimal insurance contract an oil firm (or the Fund) can buy to an insurer. It entails a deductible with coinsurance for all losses higher than the deductible. In the third section, we show that financial hedging may be a good way to cover the residual risk still retained by oil firms (or by the pool) after insurance. When incorporating this point in the insurance contract, the risk premium asked by the insurer decreases and more standard insurance becomes available for small and medium oil spills, while

\(^5\)In May 2003, under the auspices of the International Maritime Organization, a protocol which introduces a third tier of compensation has been adopted. Its aim is to increase significantly levels of compensation if compensation available through the Civil Liability Convention and the IOPC Fund should prove to be insufficient. Note that the signature of this new protocol is not compulsory and may de facto exclude poorer countries because of the high levels of contribution in case of an incident.
capital markets are useful for hedging huge oil spills. Another important point is that financial markets may provide incentives to invest in prevention by allowing firms to give positive signals to potential investors. Section four concludes the paper and discusses the implications of our results. All proofs are given in Appendix.

2 The 1992 IOPC Fund Regime

We start this section by presenting the main features of the current legislation. A formalization of the current risk management system is provided in a second paragraph.

2.1 Oil Firms and Risk Mutualization

Since 24 May 2002, International maritime transport (except for the United States) is exclusively regulated by the 1992 Civil Liability Convention (CLC in the course) and by the 1992 International Oil Pollution Compensation Fund Convention (IOPC Fund).

The Fund is financed by contributions of the oil industry of member states receiving more than 150,000 tons of oil per year after sea transport. The contribution of each company is proportional to the annual tonnage received and is directly payable to the Fund. One important point is that contributions, decided each year by the Assembly of the Fund, cover administrative costs and estimated compensation payments for passed pollutions. More precisely, for a given oil spill each oil firm pays an ex post indemnity equal to a percentage of the loss. Hence the oil spill can be considered as an aggregate loss of the pool, which is shared across its members.

This International compensation regime seems to be rather efficient: It has improved the protection of sea environment against oil pollution by inducing a decrease of the number of huge oil spills in the last two decades. Also, it facilitates claims settlement

\[\text{\textsuperscript{6}} \text{Actually, the first Civil Liability Convention is dated from 1969 and the Fund was created in 1971. Both were amended in 1992. For details, see the companion paper Schmitt and Spaeter (2003).}\]

\[\text{\textsuperscript{7}} \text{The number of large oil spills (spilling more than 700 tons) was 7.3 per year on average during the 1990s compared to 24.2 during the 1970s. (source : ITOPF Handbook 2003-2004) However, the}\]
for victims of pollution and it has increased compensation available for them. Although claims for damage to the ecosystem are not admissible, compensation is granted to a wide range of costs (clean-up operations, property damages, economic losses, ...). Besides, contributions borne by the oil industry are really fair compared to the revenues induced by oil activities\(^8\).

Nevertheless this regime also shows its limits regarding the total compensation available for victims\(^9\) and the incentives to enhance environmental prevention through the chartering of safety boats. Indeed, while the shipowner is solely held liable through the Civil Liability Convention, the whole oil industry participates in compensations through the IOPC Fund Convention: no direct compensation between the owner of the oil escaped from the boat and victims can be established. From a theoretical point of view, Ringleb and Wiggins (1990) show that such considerations may lead firms to subcontract risky activities (here the maritime transport of oil) and that those firms have no sufficient incentives to charter boats with high levels of quality.

Besides, even if the shipowner is held liable by the Convention, he is often protected by the corporate limited liability rule which benefits mostly low market value firms as shown by Schmitt and Spaeter (2002). Consequently, risk-reducing activities may still be worsened.

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\(^8\)From 1996 to 2001, the annual contributions represented at most 0.05% of the price per ton of crude oil received.

\(^9\)Only partial compensation was available to victims after the wrecks of Nakhodka (1997), Erika (1999) and Prestige (2002). In the case of Erika, the percentage of compensation was the highest one among these three catastrophes: About 80% of the aggregate loss estimated by the experts of the IOPC Fund.
2.2 The Model

In a first stage, we consider the current situation of the IOPC Fund, where no insurance is available.

Consider $n$ oil firms located in states that are members of the Fund. We denote $\tilde{x}_i$ the risk that a given boat chartered by Firm $i$ for the transport of its oil is wrecked. The random loss $\tilde{x}_i$ borne by Society takes the strictly positive value $x_i$ with probability $p_i$ and equals zero with probability $(1 - p_i)$. Probability $p_i$ of incident is affected by the level of prevention $e_i$ decided by the oil firm that means, here, by the safeness of the chartered boat: $p_i = p(e_i)$ with $p'(e_i) < 0$. The cost of prevention is defined as $c(e_i) = e_i$: $e_i$ increases as the safeness of the ship chartered by the oil firm increases\(^{10}\). Finally the aggregate risk of the Fund is $\tilde{X} = \sum_{i=1}^{n} \tilde{x}_i$ with values\(^{11}\) in $[0, T]$ and with distribution function $F(X/e)$, where $e$ is the vector of all individual investments in prevention. An increase in the level of individual prevention improves the distribution in the sense of the first order stochastic dominance, but at a decreasing rate: $F_{e_i} > 0$, $F_{e_i,e_i} \leq 0, \forall X \in [0, T]$ and $F_{e_i}(0/e) = F_{e_i}(T/e) = 0$.

As it works currently, each time an accident is registered in the territorial waters, the Fund calls for contributions by each oil firm. The percentage denoted $\alpha_i$ is applied to the level of the aggregate risk $X$ of the pool, up to a maximum value $\tilde{X}$. Funds of the IOPC Fund are limited and firms benefit from limited liability. In this system, which characteristics are similar to the ones of the mutuality principle, the firm does not bear all the risk directly linked to the boats she charters since it is spread across all members of the Fund. But, if the firm is the owner of the oil spilled on the coasts of a given state, she will suffer from an effect of bad reputation. Hence we assume that each

\(^{10}\)Implicitly, we suppose that the charterer, i.e. the oil firm, controls the quality of the boat and the competence of the crew. This may look as a bold hypothesis knowing that the maritime oil transport is largely subcontracted to shipowners. However, charterers take advantage of low quality ships as these permit inexpensive carriage of their cargoes. Furthermore, charterers can get a precise information on the safeness of a boat through the classification society used to check it.

\(^{11}\)Here, $T$ is simply equal to the sum of the strictly positive values of all oil spill variables: $T = \sum_{i=1}^{n} x_i$. 

incident will have a negative reputational effect on all members of the Fund and also an additional individual negative effect on the firm owner of the oil spilled. Formally, the random variable describing the total reputational effect is denoted \(-g(X, \tilde{x}_i)\) with \(0 < g_X < g_{x_i}\) and \(g_{XX} < 0\). The preferences of the firm are represented by a Von Neumann Morgenstern utility function \(u(\cdot)\) and she owns an initial non random wealth \(w_i\).

The oil firm has to choose the level of quality \(e_i\) of the boat to be chartered that maximizes her expected net welfare:

\[
\max_{e_i} R = \int_0^T \left( u(w_i - \alpha_i X - g(X, \tilde{x}_i)) - e_i \right) f(X/e) dX + \int_{\tilde{x}} \left( u(w_i - \alpha_i \tilde{X} - g(X, \tilde{x}_i)) - e_i \right) f(X/e) dX,
\]

where \(g(X, \tilde{x}_i)\) is the expected value of the reputational effect evaluated with respect to \(\tilde{x}_i\):

\[
g(X, \tilde{x}_i) = p(e_i)g(X, x_i) + (1 - p(e_i))g(X, 0), \quad \forall X \in [0, T]
\]

**Lemma 1** With the notations \(w_f = w_i - \alpha_i X - g(X, \tilde{x}_i)\), \(\tilde{w}_f = w_i - \alpha_i \tilde{X} - g(X, \tilde{x}_i)\), \(g_{e_i} = g_{e_i}(X, \tilde{x}_i)\), \(g_X = g_X(X, \tilde{x}_i)\) and for given prevention levels of the other oil firms, the optimal level of prevention \(e_i^*\) satisfies the following first order condition:

\[
1 = -\int_0^{\tilde{x}} g_{e_i} u'(w_f) f(X/e) dX - \int_{\tilde{x}}^T g_{e_i} u'(\tilde{w}_f) f(X/e) dX + \int_0^X (\alpha_i + g_X) u'(w_f) F_{e_i}(X/e) dX + \int_{\tilde{x}}^T g_X u'(\tilde{w}_f) F_{e_i}(X/e) dX
\]

The left term of Equality (3) is the expected marginal cost of prevention. From our assumptions, this amount is certain and equal to one. The right-hand-side term is the expected marginal benefit of prevention. First, chartering safer boats will reduce
the risk of bad reputation (first and second term) because the probability for Firm $i$ to be directly involved in a wreck (probability $p_i$) declines when $e_i$ increases. Second, prevention has also a positive impact on the aggregate risk of the Fund since it improves its distribution. Firm $i$ will benefit from an additional reduction in the bad reputation due, this time, to the reduction of the aggregate risk of the pool (third and fourth term). Lastly, the presence of $\alpha_i$ in the third member of the right-hand-side term represents the direct benefit of prevention: increasing prevention reduces the risk $\alpha_i \tilde{X}$ borne by Firm $i$.

This first order condition will be useful in the course for comparing the different levels of prevention obtained when the firm has successively access to standard insurance and to a joint contract that displays standard insurance and financial hedging.

From a practical point of view, notice that, following the wreck of Erika in 1999 near the French coast, the French government demanded an increase in the funds available for compensation through the IOPC Fund. In our model, such a decision fits with an increase in $\tilde{X}$. Proposition 1 hereafter informs us about the effect of such a decision on the level of prevention decided by the oil firm.

**Proposition 1** (i) The effect on prevention of a variation in the maximum amount of loss covered by the Fund is given by

$$\frac{de_i}{d\tilde{X}} = \frac{\alpha_i}{R_{e_i}} \left[ \int_{\tilde{X}}^{T} g_{e_i} u''(w_f) f(X/e) dX + u'(w_i - \alpha_i \tilde{X}) - g(\tilde{X}, \tilde{x}_i)) F_{e_i}(X/e) \right]$$

$$- \int_{\tilde{X}}^{T} g_X. u''(w_f) F_{e_i}(X/e) dX,$$

with $R_{e_i}$ the derivative of $R_{e_i}$ given by (3) with respect to $e_i$.

(ii) An increase of $\tilde{X}$ induces an increase in the level of prevention.

Having to pay more for large accident is similar for the firm to bearing more risk. Thus the marginal benefit of prevention increases, while the monetary marginal cost
of prevention remains unchanged: The price of chartering safety boats is not affected. Finally, the oil firm has incentives to increase the level of preventive investment $e_i$.

From a theoretical point of view, increasing the level of contribution by oil firms to the Fund may be a good way to improve prevention. However, this fragilizes the risk mutuality effect since more aggregate loss is borne by each individual firm. Furthermore, small firms may have some difficulties to fulfill their commitments if their contributions become too high. In what follows, we focus on standard insurance as an alternative to an increase of individual contributions and we look at the optimal contract between an insurer and an oil firm, knowing that the risk to be dealt with is a catastrophe risk. Standard insurance can be bought by each individual firm or by the pool. Here we choose the first alternative.

Now, assume that the oil firm can transfer part of her risk $\alpha_i X$ to an insurer. The compensation function is denoted $C(\alpha_i X)$. The insurance premium is $Q = (1 + \lambda)E[C(\alpha_i X)]$ where $\lambda$ represents the administrative costs of the insurer plus the risk premium per unit of transferred risk and $E$ the expectation operator over $X$. The Von Neumann Morgenstern utility function of the insurer is denoted $v(.)$ with $v'(.) > 0$ and $v''(.) \leq 0$ and $W$ is his initial wealth. Notice that the compensation function is defined over $[0, T]$ and not over $[0, \hat{X}]$.

The maximization program of the oil firm subject to the participation constraint of the insurer becomes\footnote{Function $1_{(.)}$ is the indicator function, taking value one if the condition into brackets is satisfied, zero otherwise.}:

$$\max_{C} R^C = \int_{0}^{T} \left( u(w_i - \alpha_i X.1_{\{X \leq \hat{X}\}} - \alpha_i \hat{X}.1_{\{X > \hat{X}\}} + C(\alpha_i X) - Q - g(X, \hat{X}_i)) - e_i f(X/e)dXight)$$

subject to \( \int_{0}^{T} v(W + Q - (1 + \lambda)C(\alpha_i X))f(X/e)dX \geq v(W) \) (5)

We use optimal control to solve this maximization program. The random variable $X$ plays the role of time, $C(\alpha_i X)$ is the control variable while the state variable is
\[ z(X) = \int_0^X v(W + Q - (1 + \lambda)C(\alpha_i t)) f(t/e) dt. \] Its evolution is described by the system:

\[
\begin{align*}
\dot{z}(X) &= v(W + Q - (1 + \lambda)C(\alpha_i X)) f(X/e) \\
z(0) &= 0 \\
z(T) &= v(W)
\end{align*}
\]

The Hamiltonian of Program (5) is

\[
H(X) = \left( u(w^C_f(X, \hat{X})) - \epsilon_i + \mu(X)v(W + Q - (1 + \lambda)C(\alpha_i X)) \right) f(X/e),
\]

with \( \mu \) the Lagrange function and \( w^C_f(X, \hat{X}) = w_i - \alpha_i X.1_{\{X \leq \hat{X}\}} - \alpha_i \hat{X}.1_{\{X > \hat{X}\}} + C(\alpha_i X) - Q - g(X, \hat{x}_i) \). The contract \( C^* \) that maximizes \( H \) is presented in Proposition 2 hereafter.

**Proposition 2**

(i) The optimal insurance contract displays a positive deductible when administrative costs are increasing in the level of indemnities. Marginal compensations for damages beyond the level of deductible but lower than \( \hat{X} \) are given by the equation

\[
C^0(\alpha_i X) = \frac{(1 + \frac{\alpha_i}{\alpha_i}) R_u}{R_u + (1 + \lambda)R_v},
\]

with \( R_u \) and \( R_v \) the absolute risk aversion ratios of, respectively, the insured and the insurer. For damages higher than \( \hat{X} \), marginal indemnities are given by

\[
\hat{C}^0(\alpha_i X) = \frac{\frac{\alpha_i}{\alpha_i} R_u}{R_u + (1 + \lambda)R_v}.
\]

(ii) The optimal contract presents a disappearing deductible for losses lower than \( \hat{X} \) if the insurer is risk-neutral and an upper limit for losses beyond a level \( \bar{X} \), with \( \hat{X} < \bar{X} \).

If the insurer is risk averse and asks for a high risk premium in order to insure the large risk, the coverage displays a coinsurance rate less than one for damages beyond the deductible and an upper limit of coverage.
Equation (7) is close to the one of Raviv (1979) obtained in a model with one insurable risk and to that obtained by Gollier (1996) with background risk. Actually, in our model, the risk of bad reputation is uninsurable and positively correlated to the insurable risk (we have $g_X > 0$). Thus we should expect that the insured firm accepts to pay for a higher coverage of the first risk in order to protect herself against her background risk if she is prudent in the sense of Kimball (1990). This result is obtained if we assume that the insurer is risk neutral, but prudence is not necessary. Indeed both risks are perfectly correlated and it is as if the insured firm would bear an individual “aggregate” risk, $\alpha_i X + g(X, \vec{X}_i)$, which cannot be completely insured. Hence, the risk neutrality of the insurer is sufficient to obtain the optimality of a disappearing deductible: $C_m^u(\alpha_i X) > 1$.

Besides, due to the presence of the uninsurable reputational risk, indemnities can increase with $X$ even if the effective damage suffered by the firm (her contribution to the Fund) is fixed and equal to $\alpha_i \hat{X}$. This result is also due to the positive correlation between $g$ and $\alpha_i X$.

Actually, here we are dealing with catastrophe risks and an insurer whose portfolio contains the aggregate risk of the Fund bears an additional risk of insolvency following a catastrophe that he has accepted to cover. Besides, it is important to notice that due to the strengthening of environmental legislations in the eighties in the United States and at the beginning of the nineties in Europe, insurers decided to exclude pollution risks from their policies and specific reinsurance groups have had to offer such contracts. Thus, it is not reasonable to assume that the insurer is risk-neutral if we wish to give some practical consistency to our modelization. In the same spirit, empirical facts show that reinsurance groups that accept to cover pollution damages are asking for high insurance premia, which entail high risk premia. It is often argued that the management of large risks entails additional transaction costs, due to risks of insolvency or to the complexity of audits and of claims settlements. This may justify the significant increase in the price of classical insurance. Consequently, it is reasonable to assume that the insurer we are dealing with is risk averse and that administrative costs related to the management

For $g_X = 0$, we would have $\hat{C}^u = 0$ for any $X$ larger than $\hat{X}$ and $\hat{X} = \bar{X}$. 

13
of catastrophe risks are sufficiently high to obtain that, in many cases, the optimal insurance contract displays coinsurance between the insurer and the insured firm beyond a deductible level.

Figure 1 displays the optimal compensation function in that case.

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Figure 1 about here

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A contract with coinsurance beyond a deductible may also be the best risk sharing when the insurer bears convex administrative costs, as shown by Raviv (1979). If the convexity assumption is not the most plausible when dealing with classical risks such as car- or house-insurance risks, it is much more closer to reality when we are focusing on large risks. Consequently, convex costs may also explain the optimality of coinsurance in the management of large risks\textsuperscript{14}.

Another result of this section is given by Proposition 3.

Proposition 3

(i) The optimal level of prevention $e^C_i$ satisfies the following first order condition:

\[
1 = - \int_0^T (g_{e_i} + Q_{e_i})u'(w^C_f(X, \hat{X}))f(X/e^C)dX
\]

\[
+ \alpha_i \int_0^{\hat{X}} (1 + \frac{gX}{\alpha_i} - C'(\alpha X))u'(w^C_f)F_{e_i}(X/e^C)dX
\]

\[
+ \alpha_i \int_{\hat{X}}^T (\frac{gX}{\alpha_i} - \hat{C}'(\alpha_i X))u'(\hat{w}^C_f)F_{e_i}(X/e^C)dX
\] \hspace{1cm} (9)

(ii) When standard insurance is available and when the insurer can obtain information on prevention, the optimal level of prevention decided by the firm increases.

\textsuperscript{14}This assumption is not retained here. With a cost function more general than the one we are using, the parameter $\lambda$ would be replaced by the first derivative of the cost function and the second derivative would appear at the denominator of Equation (7).
This last result is not surprising. Here, the insurer can obtain information about the level of prevention decided by the firm. When a given boat is chartered, its capacity and its safeness are common knowledge. Hence the insurance premium is evaluated with respect to the distribution of the aggregate risk, which depends on the level of prevention.  

If insurance is available, an increase in the level of prevention decreases the level of the premium. In our model, an increase in prevention has also an effect on the marginal indemnities through its impact on the non insurable risk. Indeed, the marginal level of the bad reputation risk $g$ is present in $C'(\alpha_i X)$. Finally, as in standard models with complete information on prevention, the level of prevention increases when the revenue of the insured is hedged. In the next section, we will show that a mixed strategy classical insurance/financial hedging still improves this level of prevention when informations on the risk-reducing policy of the oil firm are also available on capital markets.

Finally, when only standard insurance is available, insurers may ask for high risk premia for accepting to manage a catastrophe risk and the optimal contract displays some coinsurance: as the damage increases, oil firms are less covered and have to bear more and more residual risk. In the next section, the issue is to find complementary mechanisms that are able to diversify risks over a wider range of individuals and to transfer risk to agents such as financial investors. In such a manner, it will be possible to reduce the residual risk borne by the firm after (standard) insurance and to increase available funds for victims in case of an accident.

For the stake of simplicity, we assume that $\hat{X} = \overline{X}$ in what follows.

3 Providing a better hedging strategy through capital markets

A more complete hedging strategy would consist in combining several coverage instruments. Doherty and Dionne (1993) and Mahul (2002) provide such an approach by dividing the risk into two components: an idiosyncratic risk, which can be related to
the specific activities of a given firm, and a systematic risk, related to the risk of the industry as a whole. While the individual risk can be insured by a standard insurance policy, the systematic risk is managed through a participating contract. A participating contract is a policy with a variable premium based on the realized systematic loss. In a second stage, the variability of the insurance premium is hedged either through standard insurance or thanks to adequate financial instruments.

Our problematic is different from the ones of Doherty and Dionne (1993) and Mahul (2002) because 1) the oil industry does not bear an insurable idiosyncratic risk since the effect of bad reputation, which plays this role, is non insurable, 2) The individual risk of the oil firm is correlated to the risk of the Fund, while in the previous analyses both are independent. These differences will have an impact on the link we will obtain between standard insurance and financial hedging. Finally, we also focus on the level of prevention decided by the oil firm.

Now, assume that the oil firm is no longer protected by limited liability and that the Fund to which she belongs has to pay for all oil spills, whatever their size. Thus, the level $\hat{X}$ is no longer consistent here. Nevertheless, the firm can still transfer part of her risk to an insurer, but the maximum damage that will be covered by the contract is equal to $X$ with $X \in [0, T]$. The idea is to limit the implication of the insurer in the coverage of large risks in order to mitigate his bankruptcy risk. We denote $I(\cdot)$ the indemnity schedule. When an oil spill occurs and after having contributed to the Fund, the oil firm obtains an indemnity $I(\alpha_i X)$ if her contribution is less than $\alpha_i \bar{X}$ and the fixed amount $\bar{I} = I(\alpha_i \bar{X})$ for any larger contribution. Still assume that the firm can sell a part $\beta$ of her residual risk $\alpha_i X - \bar{I}$ to an external investor. The price of this transfer depends on $\beta$ and also on the level of prevention $e_i$ adopted by the firm: in this model, financial markets can obtain some information about environmental policies adopted by the firms\textsuperscript{16}. The risk premium asked by the external investor is noted $\pi = \pi(\beta, e_i)$ and it satisfies the properties $\pi_\beta > 0$ and $\pi_{e_i} < 0$. Lastly, the insurer’s unit cost of insurance

\textsuperscript{15}By buying and selling puts and calls of appropriate underlying securities.\textsuperscript{16}See Lanoie et al. (1998) for details about how those informations circulate on financial markets.
is \( \delta \) and it depends on \( \beta \): if the insured commits to cover the worst states of nature on financial markets, the insurer takes into account this information when evaluating the insurance premium. The catastrophe consequences are split between the insurer and the financial markets. Consequently, we can assume that the costs of risk management are lower than in the previous case because of a decrease in the risk premium. Formally, we have: \( \delta(\beta) > 0, \delta(0) = \lambda \) and \( \delta_{\beta} < 0 \).

The maximization program of the oil firm becomes

\[
\max_{I, \beta} R^\beta = \int_0^\infty \left[ u(w_i - \alpha_i X - Q^\beta + I(\alpha_i X) - g(X, \tilde{x}_i)) f(X/e) dX ight. \\
+ \int_0^T \left[ u(w_i - \alpha_i X - Q^\beta + \bar{T} + \beta [\alpha_i X - \bar{T}] \\
g(X, \tilde{x}_i) - \pi(\beta, e_i)) f(X/e) dX - e_i \\
\left. \right] \right] dX \\
\text{subject to} \\
\int_0^\infty \left[ v(W + Q^\beta - (1 + \delta(\beta))I(\alpha_i X)) f(X/e) dX \\
+ v(W + Q^\beta - (1 + \delta(\beta)).\bar{T})(1 - F(X/e)) \geq v(W), \right]
\]

with \( Q^\beta = (1 + \delta(\beta))E[I(\alpha_i X)] \) the insurance premium. Now, we have to define the optimal evolution of \( I \) for any damage lower than the bound \( \overline{X} \) and the optimal level of \( \tilde{\beta} \).

**Proposition 4**

(i) The optimal indemnity function displays a positive deductible. Marginal indemnities for losses between the deductible level and the bound \( \alpha_i \overline{X} \) are given by:

\[
I''(\alpha_i X) = \frac{1 + \frac{\overline{x}_i}{\alpha_i}}{R_u + (1 + \delta(\beta)).R_v} 
\]

with \( R_u \) and \( R_v \) the absolute risk aversion ratios of, respectively, the insured and the insurer.

(ii) For given risk attitudes of the agents and positive hedging from the financial market, the optimal coverage is higher than the one obtained when only standard insurance is available: \( I^{\ast\ast} > C^{\ast\ast} \) for any loss partially covered and less than \( \overline{X} \).
(iii) An increase in the financing of large losses by capital markets increases standard insurance of small and medium losses.

Point iii) enhances the fact that firms should use the wide diversification capability of financial markets to manage the potential large consequences driven by catastrophe risks and they should buy standard insurance for small and medium losses. In the specific framework of the oil industry, this would mean that the general Fund\textsuperscript{17}, which manages small oil spills, should negotiate some coverage conditions offered by standard insurers, while the main claims Fund should rather be managed through interventions on capital markets.

**Lemma 2** Partial financial hedging is optimal if and only if

\[
\pi_\beta. \int_0^T u'(w^\beta_f) f(X/e) dX = \int_0^X I_\beta(\alpha_i X). u'(w^1_f) f(X/e) dX + \int_0^T (\alpha_i X - \bar{T}). u'(w^2_f) f(X/e) dX - \int_0^T Q^\beta_i. u'(w^3_f) f(X/e) dX, \tag{12}
\]

with

\[
\begin{align*}
  w^1_f &= w_i - \alpha_i X - Q^\beta + I(\alpha_i X) - g(X, \bar{x}_i) - \pi(\beta, e_i) \\
  w^2_f &= w_i - \alpha_i X - Q^\beta + \bar{T} + \beta [\alpha_i X - \bar{T}] - g(X, \bar{x}_i) - \pi(\beta, e_i) \\
  w^3_f &= w_i - \alpha_i X - Q^\beta + I(\alpha_i X).1_{\{X \leq \bar{x}\}} + [\bar{T} + \beta (\alpha_i X - \bar{T})].1_{\{X > \bar{x}\}} - g - \pi
\end{align*}
\]

Function \(1\{\cdot\}\) is the indicator function, which takes value one if the condition into brackets is satisfied, zero otherwise.

Equation (12) is obtained by differentiating (10) with respect to \(\beta\). It is a good strategy for the oil firm to seek external financing if the expected marginal cost of an increase in \(\beta\) (left-hand-side-term) equals the expected marginal benefit, obtained thanks

\textsuperscript{17}Actually, what we commonly call the 1992 IOPC Fund is composed of two distinct funds. The first one, the general Fund, is dedicated to the payment of the current administrative costs and to the compensation of small oil spills (less than 4 millions SDRs with one Special Drawing Right = US$ 1.40489 on 14 May 2003), while the main claims Fund is dedicated to large oil spills.
to an increase in the coverage of the small and medium losses (first member in the right-
hand-side-term), to the direct increase of the coverage of large losses (second member) and to the decrease of the price of standard insurance (third member).

Lastly, we have to discuss the level of prevention adopted by the firm in the case of a joint hedging contract. A differentiation of (10) with respect to $e^\beta$, the level of prevention in this model, and integrations by part lead to the following first order condition:

$$
1 = - \int_0^T (g_{e_i} + \pi_{e_i} + Q_{e_i}^\beta) \cdot u'(w_f^\beta) f(X/e) dX
+ \alpha_i \int_0^T \left(1 + \frac{gX}{\alpha_i} - I^*(\alpha_i X) \cdot 1\{x \leq X\} - \beta \cdot 1\{x > X\} \right) \cdot u'(w_f^\beta) F_{e_i}(X/e) dX
$$

(13)

**Proposition 5** The level of prevention adopted by the firm is higher than the one obtained in the model with standard insurance when the opportunity of announcing her prevention policy to markets induces easier access to external financing ($\pi_{e_i} < 0$).

4 Discussion

[to be completed] The new firms’ management of risks tries to encompass all types of risks. Firms have to cope with numerous sources of uncertainties, linked to the production processes, to unanticipated market evolutions, non expected internal organization issues and also with uncertainties related to the existence of large risks. Large risks are often catastrophe risks, with random events the frequency of which may be low, but that induce very large economic consequences, irreversible ecological damages and sometimes loss of human lifes. This is the case for the maritime transport of oil. To manage oil spills, the 1992 IOPC Fund calls for ex post contributions by each oil firm the state of which is member of the Fund. However, no insurance mechanism is designed and only the mutuality principle is applied: The individual contribution corresponds to a percentage of the aggregate risk of the Fund. Because of the limited number of members and also of the huge financial consequences induced by some oil spills, the aggregate
risk cannot be fully spread across the oil firms. Hence it is useful to think about other diversification and/or coverage instruments that would help to smooth the payments of firms through time and also to increase the funds available for compensation. In this paper, we have shown that transferring part of the aggregate risk, namely the part related to catastrophic losses, to investors that have access to capital markets makes standard insurance of small and medium oil spills less costly. The mixed strategy, which consists in using the properties of standard insurance for risks that are reasonably insurable and the wide capability of financial markets to diversify risk across many people in the world for catastrophic losses, seems to be a good compromise. Moreover if firms can send to the markets signals on their environmental policies, financing hedging creates additional incentives to invest in risk-reducing activities....
Figure 1. Optimal Compensation Function when only standard insurance is available
APPENDIX

Proof of Lemma 1

Recall that \( w_f = w_i - \alpha_i X - g(X, \bar{x}_i) \), \( \bar{w}_f = w_i - \alpha_i \bar{X} - g(X, \bar{x}_i) \), \( g_{e_i} = g_{e_i}(X, \bar{x}_i) \), \( g_X = g_X(X, \bar{x}_i) \). A differentiation of (1) with respect to \( e_i \) leads to:

\[
R_{e_i} = - \int_{0}^{\bar{X}} g_{e_i}(X, \bar{x}_i) \cdot u'(w_f) f(X/e) dX - \int_{\bar{X}}^{T} g_{e_i}(X, \bar{x}_i) \cdot u'(\bar{w}_f) f(X/e) dX \\
+ \int_{0}^{\bar{X}} u(w_f) f_{e_i}(X/e) dX + \int_{\bar{X}}^{T} u(\bar{w}_f) F_{e_i}(X/e) dX - 1
\]

With \( F_{e_i}(0, e) = F_{e_i}(T/e) = 0 \), integrations by part of the third and fourth term induce that:

\[
R_{e_i} = - \int_{0}^{\bar{X}} g_{e_i} \cdot u'(w_f) f(X/e) dX - \int_{\bar{X}}^{T} g_{e_i} \cdot u'(\bar{w}_f) f(X/e) dX \\
+ \int_{0}^{\bar{X}} (\alpha_i + g_X) \cdot u'(w_f) F_{e_i}(X/e) dX + \int_{\bar{X}}^{T} g_X \cdot u'(\bar{w}_f) F_{e_i}(X/e) dX - 1
\]

If an interior solution exists, then it satisfies \( R_{e_i} = 0 \). Lemma 1 is demonstrated.

Proof of Proposition 1

Point i) is obtained thanks to a total differentiation of Equation (3) with respect to (w.r.t. in the course) \( e_i \) and \( \bar{X} \).

For Point ii), notice that the higher the prevention, the less the bad reputation risk: \( g_{e_i} < 0 \). Besides, an increase in the aggregate loss \( X \) of the Fund deteriorates the reputation of all firms, so that \( g_X > 0 \). By assumption we also have that \( F_{e_i} \) is positive. Finally, the numerator of (4) is strictly positive for a risk-averse, or risk-neutral, oil firm. The denominator is obtained thanks to a differentiation of (3) w.r.t. \( e_i \). With
\[ g = g(X, \tilde{x}_i) \text{ and } w_f(X, \hat{X}) = w_i - g(X, \tilde{x}_i) - \alpha_i X_1 \{ \tilde{x} \leq \hat{X} \} - \alpha_i \hat{X} \cdot 1_{\{X > \hat{X}\}} \] we have:

\[ R_{e_i e_i} = - \int_0^T g_{e_i e_i} \cdot u'(w_f(X, \hat{X})) f(X/e) dX + \int_0^T g_{e_i}^2 \cdot u''(w_f(X, \hat{X})) f(X/e) dX \]

\[ + \int_0^T g_{X e_i} \cdot u'(w_f(X, \hat{X})) f(X/e) dX \]

\[ - \int_0^T g_{e_i} \cdot u'(w_f(X, \hat{X})) f_e(X/e) dX \]

\[ \hat{X} \]

\[ - \int_0^T (\alpha_i + g_X) \cdot g_{e_i} \cdot u''(w_f) F_{e_i}(X/e) dX - \int_0^T g_{X e_i} \cdot u''(w_f) F_{e_i}(X/e) dX \]

\[ + \int_0^T (\alpha_i + g_X) \cdot u'(w_f) F_{e_i e_i}(X/e) dX + \int_0^T g_{X} \cdot u'(w_f) F_{e_i e_i}(X/e) dX \]

From the definition (2) of \( g(X, \tilde{x}_i) \), we have that \( g_{e_i X} = g_{X e_i} = 0 \). Finally, an integration by part of the third line leads to:

\[ R_{e_i e_i} = - \int_0^T g_{e_i e_i} \cdot u'(w_f(X, \hat{X})) f(X/e) dX + \int_0^T g_{e_i}^2 \cdot u''(w_f(X, \hat{X})) f(X/e) dX \]

\[ - 2 \int_0^T (\alpha_i + g_X) \cdot g_{e_i} \cdot u''(w_f) F_{e_i}(X/e) dX - 2 \int_0^T g_{X e_i} \cdot u''(w_f) F_{e_i}(X/e) dX \]

\[ + \int_0^T (\alpha_i + g_X) \cdot u'(w_f) F_{e_i e_i}(X/e) dX + \int_0^T g_{X} \cdot u'(w_f) F_{e_i e_i}(X/e) dX \]

If the second order conditions are satisfied, then \( R_{e_i e_i} \) is negative. By assumption, we have \( F_{e_i e_i} \leq 0 \), \( g_X > 0 \) and \( u'' < 0 \). Besides, \( g_{e_i e_i} \) is equal to \( p_{e_i e_i}(g(X, x_i) - g(X, 0)) \) (see Equation (2)). Function \( g(X, \cdot) \) is increasing in \( x_i \) and \( p_{e_i e_i} \) is positive or equal to zero, so that \( g_{e_i e_i} \) is positive. Finally \( R_{e_i e_i} \) is negative and \( de_i/d\hat{X} \) given by (4) is positive. Point ii) of Proposition 1 is demonstrated.

---

\[ ^{18} \text{Function } 1_{\{.\}} \text{ is the indicator function, taking value one if the condition into brackets is satisfied, zero otherwise.} \]
Proof of Proposition 2

The optimality conditions related to optimal control that must be satisfied are

\[
\begin{align*}
(i) \quad H_z &= -\mu'(X) \\
(ii) \quad H_\mu &= z(X) \\
(iii) \quad z(0) &= 0 \\
(iv) \quad z(T) &= v(W)
\end{align*}
\]

and \(H_C = 0, \forall X\) such that \(0 < C(\alpha_i X) < X\). From (6) we have \(H_z = 0\) so that \(\mu\) is constant. Conditions (ii), (iii) and (iv) are also satisfied. Besides \(f(X/e)\) is, by definition, always positive. Hence it is possible to work with the simplified Hamiltonian \(H^* = H/f(X/e)\). We have for any \(X\) such that \(0 < C(\alpha_i X) < X\):

\[
H^*_C = 0
\]

\[
\iff u'(w_f^C(X, \bar{X})) - \mu(1 + \lambda) v'(W_f^C) = 0
\]

with \(w_f^C(X, \bar{X}) = w_i - \alpha_i X 1_{\{X \leq \bar{X}\}} - \alpha_i \bar{X} 1_{\{X > \bar{X}\}} + C(\alpha_i X) - Q - g(X, \bar{x}_i)\) and \(W_f^C = W + Q - (1 + \lambda)C(\alpha_i X)\).

First, we have to show that the optimal contract displays a positive deductible. Let us define as \(J(X)\) the function given by (14) and evaluated at \(C(\alpha_i X) = 0\) and \(K(X)\) the same function but evaluated at \(C(\alpha_i X) = \alpha_i X\). By differentiating them w.r.t. \(X\) it is easy to show that \(J(X)\) is increasing in \(X\) and \(K(X)\) is decreasing. Moreover, both functions are equal at point \(X = 0\). Denote them \(L\): \(L = u'(w_i - Q) - \mu(1 + \lambda) v'(W + Q)\).

Two cases must be considered: either \(L\) is negative or \(L\) is positive (the trivial case for which \(L = 0\) is not analyzed).

\(\blacklozenge\ L > 0\)

Since \(J\) is increasing in \(X\), \(L\) is the smallest value it can take. Thus \(J\) is always positive and \(C(\alpha_i X) = 0\) is never optimal\(^{19}\). Besides, \(K\) is decreasing in \(X\). Then it exists a positive level of damage \(\bar{X}\) such that \(K\) is positive on \([0, \bar{X}]\) and \(C(\alpha_i X) = \alpha_i X\)

\(^{19}\)We have \(H^*_{CC} = w''(w_f^C) + \mu(1 + \lambda)^2 v''(W_f^C) < 0\). The second order conditions are satisfied and the result holds.
is optimal on this interval. For damages higher than $\hat{X}$, $K$ becomes negative: from this point, coverage must be constant and an upper limit of insurance is optimal.

$\blacklozenge \ L < 0$

In this case, $K$ is always negative and full coverage is never optimal. Besides, it exists a level of damage $D$ such that $J$ is negative on $[0, D]$ and presents partial coverage for any damage higher than $D$. A positive deductible is optimal.

Following Raviv (1979), we can show that, at fixed insurance premium, an upper limit is always stochastically dominated by pure coinsurance when insurance is costly. The intuition is that the risk averse insured prefers a transfer of indemnities of small damages to higher ones when insurance is costly. In the same spirit, a deductible contract dominates a pure coinsurance contract in the sense of second order stochastic dominance (Gollier and Schlesinger (1996)). Hence, the optimal contract displays a strictly positive deductible as long as the marginal cost of insurance $\lambda$ is positive.

Second, we have to define the optimal marginal indemnities beyond the deductible level. This is done first on $[D, \hat{X}]$ and second on $[\hat{X}, T]$. By differentiating Equality (14) w.r.t. $X$ on $[D, \hat{X}]$ and using it to define $\mu$ we must have, for any loss partially covered on $[D, \hat{X}]$:

$$(-\alpha_i + \alpha_i.C^{st}(\alpha_i X) - g_X).u''(w_f^{C_i}) + (1 + \lambda)^2.\alpha_i.C^{st}(\alpha_i X).\mu.v''(W_f^{C_i}) = 0$$

$$\iff C^{st}(\alpha_i X) = \frac{(1 + \frac{g_X}{\alpha_i}).u''(w_f^{C_i})}{u''(w_f) + (1 + \lambda).\frac{v'(W_f^{C_i}).u''(w_f^{C_i})}{v'(W_f^{C_i})}}$$

$$\iff C^{st}(\alpha_i X) = \frac{(1 + \frac{g_X}{\alpha_i}).R_u}{R_u + (1 + \lambda).R_v}$$

Equation (7) in Point i) is demonstrated. If the insurer is risk neutral we have $R_v$ equal to zero and $C^{st} = 1 + \frac{\alpha_i}{\alpha_i}$. Since all terms are positive, the slope of the compensation function for any damage partially covered is larger than one. The deductible disappears progressively as the damage increases.

Equation (8) in Point i) is obtained thanks to a identical reasoning, but with $X$ in $[\hat{X}, T]$ and $w_f^{C_i} = w_i - \alpha_i \hat{X} + C(\alpha_i X) - Q - g(X, \hat{x}_i)$. By assumption, we have that
$g_{XX}$ is negative, so that marginal indemnities decrease with $X$ for losses higher than $\hat{X}$. Consequently, from a level of damage $\overline{X}$ larger than $\hat{X}$, marginal indemnities are close to zero and the compensation function displays an upper limit.

If the insurer is risk averse and asks for a large risk premium, which means that $\lambda$ is large, the value of $C'*(\alpha_i X)$ may be less than one so that coinsurance for any partially indemnified loss on $[D, \hat{X}]$ is optimal. Point (ii) of Proposition 2 is demonstrated.

**Proof of Proposition 3**

Point i) is obtained thanks to a differentiation of (5) w.r.t. $e_i$. We denote $e_i^C$ the solution of the first order condition

$$R_{e_i}^C = 0$$

$$\Leftrightarrow 1 = -\int_0^T (g_{e_i} + Q_{e_i}) . u'(w_f^C(X, \hat{X})) f(X/e^C)dX + \int_{\hat{X}}^T u(w_f^C) f_{e_i}(X/e^C)dX + \int_{\hat{X}}^T u(w_f^C) f_{e_i}(X/e^C)dX$$

Integrations by part of the two last terms give:

$$\int_{\hat{X}}^T u(w_f^C) f_{e_i}(X/e^C)dX + \int_0^T u(w_f^C) f_{e_i}(X/e^C)dX$$

$$= \alpha_i \int_0^T \left(1 + \frac{gX}{\alpha_i} - C'(\alpha_i X))u'(w_f^C)F_{e_i}(X/e^C)dX \right.$$  

$$+ \alpha_i \int_{\hat{X}}^T \left(\frac{gX}{\alpha_i} - \tilde{C}'(\alpha_i X))u'(w_f^C)F_{e_i}(X/e^C)dX \right.$$.  

24
By replacing the right-hand-side term of this equation in (15), we obtain, at optimum:

\[
1 = - \int_0^T (g_{e_i} + Q_{e_i}).u'(w_f(X, \hat{X}))f(X/e^C) dX \\
+ \alpha_i \int_0^T (1 + \frac{gX}{\alpha_i} - C'(\alpha_i X))u'(w_f(X/e^C))F_{e_i}(X/e^C) dX \\
+ \alpha_i \int_0^T \left( \frac{gX}{\alpha_i} - \tilde{C}'(\alpha_i X))u'(w_f(X/e^C))F_{e_i}(X/e^C) dX \right) 
\]

(16)

Point i) is demonstrated. Point ii) is obtained thanks to a differentiation of (16) w.r.t. \( e_i \) and \( C \):

\[
\frac{de_i}{dC} = \frac{1}{-R_{e_i e_i}^e}. \left[ - \int_0^T (g_{e_i} + Q_{e_i}).(1 - Q_C).u''(w_f(X, \hat{X}))f(X/e^C) dX \\
- \int_0^T Q_{e_i, C}.u'(w_f(X, \hat{X}))f(X/e^C) dX \\
+ \alpha_i \int_0^T (1 + \frac{gX}{\alpha_i} - C'(\alpha_i X))(1 - Q_C).u''(w_f(X/e^C))F_{e_i}(X/e^C) dX \\
+ \alpha_i \int_0^T \left( \frac{gX}{\alpha_i} - \tilde{C}'(\alpha_i X)))(1 - Q_C).u''(w_f(X/e^C))F_{e_i}(X/e^C) dX \right) \right] 
\]

(17)

Marginal compensations \( C'(\alpha_i X) \) are always lower than or equal to \( 1 + \frac{gX}{\alpha_i} \) at optimum (see Equation (7)), while \( \tilde{C}'(\alpha_i X) \) is always lower than or equal to \( \tilde{C}'(\alpha_i X) \) (see Equation (8)). The premium \( Q \) is equal to \( \int_0^T (1 + \lambda)C(X, \hat{X})f(X/e^C) dX \), with \( C(X, \hat{X}) = C \) on \([0, \hat{X}]\) and \( C(X, \hat{X}) = \tilde{C} \) on \([\hat{X}, T]\); Consequently, \( e_{e_i} = \int_0^T (1 + \lambda)C(X, \hat{X})f_{e_i}(X/e^C) dX = -\alpha_i \int_0^T (1 + \lambda)C(X, \hat{X})F_{e_i}(X/e^C) dX \), which is positive, \( Q_C = \)
(1 + \lambda) and \( Q_{e_i C} \) equals zero. Equation (19) becomes:

\[
\frac{d_e}{dC} = \frac{\lambda}{P_{e_i e_i}} \cdot \left[ \int_0^T (g_{e_i} + Q_{e_i}).u''(w_f^C(X, \tilde{X}))f(X/e^C)dX \right]
\]

\[-\alpha_i \int_0^T (1 + \frac{g_X}{\alpha_i} - C'(\alpha_i X)) . u''(w_f^C)F_{e_i}(X/e^C)dX \]

\[-\alpha_i \int_0^T (\frac{g_X}{\alpha_i} - \tilde{C}'(\alpha_i X)).u''(\tilde{w}_f^C)F_{e_i}(X/e^C)dX \]

The second order conditions of this problem are satisfied (the computation is similar to the one presented in the proof of Proposition 1), so that \( R_{e_i e_i}^C \) is negative. Finally, \( \frac{d_e}{dC} \) is positive and Point ii) of Proposition 3 is demonstrated.

**Proof of Proposition 4**

The control variable is \( I(\alpha_i X) \) and the state variable is \( z(X) = \int_0^X v(W + Q^3 - (1 + \delta(\beta))I(\alpha_i t))f(t/e)dt \). The simplified Hamiltonian of Program (10) is

\[
H^{\beta_3} = u(w_f^1).1_{\{X \leq X^\gamma\}} + u(w_f^2).1_{\{X > X^\gamma\}} - e_i + \gamma(X)v(W_f^\beta),
\]

with \( \gamma(X) \) the Lagrange function, \( w_f^1 = w_i - \alpha_i X - Q^\beta + I(\alpha_i X) - g(X, \tilde{X}_i) - \pi(\beta, e_i) \),

\( w_f^2 = w_i - \alpha_i X - Q^\beta + \frac{I}{\alpha_i} \beta [\alpha_i X - \frac{I}{\alpha_i}] - g(X, \tilde{X}_i) - \pi(\beta, e_i) \) and \( W_f^\beta = W + Q^3 - (1 + \delta(\beta))I(\alpha_i X) \). Function \( 1_{\{X > X^\gamma\}} \) is the indicator function, which takes value 1 when the condition into brackets is satisfied, zero otherwise. Still here, the Lagrange function is a constant. We have for any \( X \) in \( ]0, X^\gamma[ \) such that \( 0 < I(\alpha_i X) < X \):

\[
H_f^{\beta_3} = 0
\]

\[
\iff u'(w_f^1) - \gamma(1 + \delta(\beta))v'(W_f^\beta) = 0
\]

(21)

First, we have to show that the optimal contract displays a positive deductible denoted \( D^\beta \). The proof is similar to that proposed for Proposition 2.

Second, by differentiating Equality (21) w.r.t. \( X \) and using it to define \( \gamma \) we must have, for any \( X \) in \( ]D^\beta, X^\gamma[ \) such that \( 0 < I(\alpha_i X) < X \),

\[
(-\alpha_i + \alpha_i. I^*(\alpha_i X) - g_X).u''(w_f^1) + (1 + \delta(\beta))^2.\alpha_i. I^*(\alpha_i X).\gamma.v''(W_f^\beta) = 0,
\]

26
\[ I^*(\alpha_iX) = \frac{(1 + \frac{2\alpha_i}{\alpha_i^2})u''(w_i^2)}{u''(w_i^2) + (1 + \delta(\beta)) \cdot \frac{v''(W_i^2)}{v'(W_i^2)}} \]

\[ \Leftrightarrow I^*(\alpha_iX) = \frac{(1 + \frac{2\alpha_i}{\alpha_i^2}) \cdot R_u}{R_u + (1 + \delta(\beta)) \cdot R_v} \] (22)

Point i) is demonstrated. For point ii), we know that \( \delta \) is decreasing in \( \beta \) and that \( \delta(0) = \lambda \). The marginal indemnities \( I^* \) and \( C^* \) (given by (7)) differ from the term \( \delta \) present at the denominator of \( I^* \). Hence \( I^* \) is always higher than \( C^* \) when \( \beta \) is positive.

Point iii) is immediate. From (22), marginal indemnities increase as \( \beta \) increases.

**Proof of Proposition 5**

The structures of Condition (9) and (13) differ only by the term \(-\pi_{e} \int_{0}^{T} u'(w_{i}^{\beta}) f(X/e) dX\), which is positive. By taking this positive term into account when looking at the expected marginal benefit of prevention and by applying the same reasoning as in the proof of Point ii) of Proposition 3, Proposition 5 is demonstrated.

**References**


