“Deep Pockets”, Research and Development Persistence and Economic Growth

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Abstract

This paper studies endogenous growth driven by an expanding variety of product where lenders limit credit up to collateralizable value of existing patents. Due to R&D investment risk, there is a composition effect between innovative firms currently constrained and innovative firms anticipating future contraints (hence accumulating current profit and decreasing current debt). Results are: (i) patent behaviour is lumpy and show some persistence; (ii) the steady state aggregate debt/patent ratio is below the leverage ceiling due to the composition of current versus future financial constraints; (iii) this debt/patent ratio determines a leverage driven steady state growth of the economy.

Keywords: Endogenous Growth, Research and Development, Credit rationing.
1. Introduction

There is now considerable empirical evidence that variables related to financing constraints such as leverage and/or cash flow availability are correlated with R&D investment in several countries (see Bechetti and Sierra’s [2001] and Hall’s [2002] recent surveys). Blundell, Griffith and Van Reenen [1999] provide a “deep pocket” argument stating that: “A more traditional interpretation of the innovation-market power correlation is that failures in financial markets force firms to rely on their own supernormal profits to finance the search for innovation. The availability of internal sources of funding (‘deep pockets’) are useful for all forms of investment, but may be particularly important for R&D”. Causality between cash flow and R&D investment goes both ways, which suggests that pre-innovation rents as well as post innovation rents are related to R&D investment (Hall, Mairesse, Branstetter and Crepon [1999]). Patents, innovations and cash-flow are correlated and patents and innovations show strong history dependence (Geroski, Van Reenen, Walters [2001]). However, most firms are not able to innovate persistently so that R&D investment is often lumpy (Geroski, Van Reenen, Walters [1999]).

These features emphasize the particular importance of adverse selection and moral hazard problems faced by providers of external finance for intangible and R&D investment which is fostered by at least four factors. First, asymmetric information arises endogenously because innovators do not want to disclose full information on their R&D projects, for fear that that lenders might give this information to competitors in the patent race (Bhattacharya and Ritter [1985]). Second, R&D investment is riskier than physical investment. Third, knowledge of the specific research area is required for an efficient ex post control, which involves a costly investment for bankers. Finally, banks often take tangible assets with an efficient second hand market as ex ante guarantees for loans. But R&D investment is mostly intangible, except the existing returns provided by previous patents. With incomplete debt contracts, collateral may limit credit (Kiyotaki and Moore [1997]).

Since innovation is viewed as a major factor of growth where monopoly rents provide incentives to entrepreneurs, several researchers investigated how the financing of R&D affect economic growth (see Pagano [1993] and Levine [1997] for surveys). King and Levine [1993], Bose and Cothren [1996] and Blackburn and Hung [1998] emphasize that the cost of R&D includes various agency costs due to adverse selection or moral hazard. Financial intermediation provide risk sharing which leads to an improved allo-

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cation of tangible and intangible investment resources, allowing for an increase in the aggregate productivity of capital (De la Fuente and Marin [1996]). Risk may either come from technological reasons such as the division of labour (Saint-Paul [1993]) or from liquidity risk (Bencivenga and Smith [1991]). Financial intermediaries also allow information on risks of investments to be learnt (Greenwood and Jovanovic [1990]). Aghion, Dewatripont and Rey [1999] emphasize that debt is a discipline device, which forces entrepreneurs to maximise the profits of the innovative firm (it enforces them not to delay adoption of new technology) and is a substitute to competition to provide incentives.

This paper studies endogenous growth driven by an expanding variety of product (Romer [1990], Grossmann and Helpman [1991]) where lenders limit credit up to collateralizable value of existing patents and where investment opportunities are stochastic, leading to a composition between currently constrained firms and other firms. It aims at describing some of the observed empirical characteristics of R&D investment already presented in this introduction. Along the same lines as Blundell, Griffith and Van Reenen [1999] empirical R&D cost function and Kiyotaki and Moore [1997] borrowers behaviour and incomplete debt contract, this paper focuses then on the three following issues. First, it derives persistent and lumpy patent and debt behaviour at the microeconomic level. Second, it determines the economic conditions for the existence of steady state aggregate leverage as there is a composition effect between innovative firms currently constrained and innovative firms anticipating future constraints (hence accumulating current profit and decreasing current debt) due to R&D investment risk. Third, it determines the economic conditions for the existence of a leverage driven steady state growth rate.

The model departs from King and Levine [1993], De la Fuente and Marin [1996] and Blackburn and Hung [1998] R&D, finance and growth models in that it takes into account two means of finance: debt and retained earnings, and not only debt. It departs from Kiyotaki and Moore [1997] small open economy credit cycle model in relaxing the fixed size constraint of the whole economy capital stock, in considering a closed economy with endogenous interest rate, and in that assets (patents) price do not fluctuate. It departs from Aghion, Dewatripont and Rey [1999] in that the debt contract is not able to provide all the incentives to drive entrepreneurs to first best behaviour. It departs from Bachetta and Caminal [2000] credit cycle model which also emphasize composition effects between financially constrained and unconstrained firms: in this model, innovative firms which are not currently constrained are expecting to be constrained in the future. It departs from Romer [1990], Grossmann and Helpman [1991] and Aghion and Howitt [1992] in the way future monopoly rents coming from expected innovative rents drive growth: in this paper, expected monopoly rents on existing patents also increase the price of collateral, hence the amount of loans and economic growth. It departs from Aghion, Harris, Howitt, Vickers [2001] in the way past profits (pre-innovation rents) coming from previous innovative activities are also driving R&D activity (“escape-competition” in neck and neck industries):
in this paper, current and past profits finance R&D investment due to a debt ceiling constraints; in particular profits are accumulated, "digging deep pockets" over periods where the innovator has no profitable ideas for R&D investment. With respect to Aghion, Bloom, Blundell, Griffith and Howitt [2002] distinction between a "Schumpeterian" situation (where rents and lower competition are good for R&D) and a "Darwinian" situation (where low rents and more competition are good for R&D) which may lead to an inverted U relationship between innovation and competition, this paper adds the "deep pockets" argument on the Schumpeterian left hand side of the inverted U curve.

The paper is organised as follows. The microeconomic behaviours of agents are described in section 2. Section 3 provides the conditions for steady state aggregate growth. Section 4 concludes the paper with a discussion of the results and related research.

2. The model

2.1. Households

A continuum of wage-earners, distributed on \([0, L]\), maximizes a constant intertemporal elasticity of substitution utility function discounted over an infinite horizon:

\[
U_t = \sum_{\tau=0}^{\infty} u(c_{t+\tau}) (1 + \rho)^{-\tau} \quad \text{with} \quad u(c_t) = (c_t^{1-\sigma} - 1)/(1-\sigma) \quad \text{for} \quad \sigma > 0 \quad \text{and} \quad \sigma \neq 1 \quad \text{or with} \quad u(c_t) = \ln(c_t) \quad \text{for} \quad \sigma = 1. 
\]

Consumption at time \(t\) is \(c_t\), the rate of time preference is \(\rho\), the discount rate is denoted \(\beta = 1/(1+\rho)\), and the inverse of the elasticity of substitution is \(\sigma\). Households supply inelastically one unit of labor in the final goods industry paid at a real wage rate \(w_t\) and they have no disutility of labor. They lend to entrepreneurs and earn a rate of return \(r\) (the interest factor is denoted \(R = 1 + r\)) on their individual wealth \(b_{t-1}^h\) so that their wealth dynamics is given by \(b_t^h = (1+r)b_{t-1}^h + w_t - c_t\). The initial wealth \(b_0^h\) is given and identical for all households. Then, optimal consumption growth \(g_c\) is given by \(1 + g_c = c_{t+1}/c_t = C_{t+1}/C_t = (\beta R)^{\frac{1}{\sigma}}\), where \(C_t = c_t L\) denotes aggregate consumption. Consumption growth rate increases with the return on savings and decreases with the rate of time preference and the elasticity of substitution. Optimal consumption is \(C_t = C_0(1+g_c)^t\) and \(U_0\) is bounded if \((1+r)^{1-\sigma} < 1 + \rho\). For \(\sigma \geq 1\) this condition is always fulfilled. For \(0 < \sigma < 1\), the interest rate has to remain in the following range: \(R \in ]1+\rho, (1+\rho)^{1/(1-\sigma)}[=]1 + r_{\min}^c, 1 + r_{\max}^c[\).

2.2. Production of the final good

As in other "increasing product variety" models (Romer [1990], Grossman and Helpmann [1991]), the economy has three types of activities: a final good sector, whose price is taken as numeraire, an intermediate goods sector, whose output is used in the production of the final good and an R&D sector which discovers blue-prints allowing the creation of new intermediate goods. Producers of the final good operate in perfect
competition. The final good $Y_t$ is produced from intermediate inputs, which fully deprecate over one period and have to be bought again each period. Intermediate goods are defined on a set $\{X(i), i \in [0, N_t]\}$. The quantity $X(i)$ is the amount of intermediate good $i$ used in the production process of the final good. The value $N_t$ represents the most recently invented intermediate good, so that the interval $[0, N_t]$ is the variety of intermediate goods available in the economy. Technical progress is described as the invention of new intermediate goods which adds to the range of intermediate goods already invented, and implies an increase of $N_t$ over time. Then, the constant return to scale production function of the final good is given by:

$$Y_t = AL^{1-\alpha} \int_0^{N_t} X(i)^{\alpha} di \text{ with } 0 < \alpha < 1$$

(2.1)

The representative producer of final goods buys intermediate goods at the given price $p_i$. Producers can buy patents from innovators. The subscript $t$ will not be precised when the period considered is not ambiguous. The representative producer of the final good demands a quantity of each intermediate input $i$, denoted $X(i)$ following a profit maximising behaviour:

$$(X(i), L) \in ArgMax \left(Y - wL - \int_0^{N_t} p_i X(i) \, di \right)$$

(2.2)

which gives the neo-classical demand function for intermediate inputs and for labour:

$$\alpha AL^{1-\alpha}X(i)^{\alpha-1} = p_i \text{ for } i \in [0, N_t]$$

(2.3)

$$\left(1 - \alpha\right) \frac{Y}{L} = w$$

(2.4)

The equation of the marginal product of each intermediate product and its price therefore implies the following demand function:

$$X(i) = \left(\frac{\alpha A}{p_i}\right)^{1/\left(1-\alpha\right)} \text{ for } i \in [0, N_t]$$

(2.5)

### 2.3. Production of intermediate goods

At each date $t$, an innovator receives a rent from a producer of intermediate goods who uses his blueprint (an innovative firm has previously discovered $N_t$ blueprints). Intermediate goods producers sell them to producers of final goods with a price including a mark-up on marginal costs. Producing one unit of intermediate good costs one unit of final output $Y_t$ to be produced. Then the monopoly price is:

$$p_i = \frac{1}{\alpha} > 1$$

(2.6)
The price is identical for all intermediate goods and the monopoly profit per intermediate good sold is:

$$\pi = \left( \frac{1 - \alpha}{\alpha} \right) A^{1/(1-\alpha)} \alpha^{2/(1-\alpha)} L$$  \hspace{1cm} (2.7)

2.4. R&D Sector

At every period, a continuum of risk neutral entrepreneurs indexed by $j$ (when necessary) distributed over the interval $[0, 1]$ are engaged in the R&D activity. They can borrow or lend at the interest rate $r$ so that they maximise the present value $V$ of non negative consumption given by their firms dividends $d_{t+1} \geq 0$ discounted by the interest factor:

$$V_0 = \sum_{t=0}^{+\infty} d_{t+1} R^{-t}$$  \hspace{1cm} (2.8)

They hold an initial endowment of a number of blueprints $n_0$ equal to an existing stock of intermediate goods. They face the equation of motion of the stock of blueprints:

$$n_{t+1} = i_t^n + \left( 1 - \tilde{\delta}_t \right) n_t$$  \hspace{1cm} (2.9)

The innovative firm faces the threat of obsolescence and/or imitation of a random proportion $\tilde{\delta}_t$ of their stock of their existing patents ($0 \leq \tilde{\delta}_t < 1$), which is independently and identically distributed across inventors (because there is a large number of inventors, there is no aggregate uncertainty). The number of new blueprints $i_t^n$ obtained in a period depends on a costly and risky search activity, which includes the entrepreneur specific human capital input $h_t$ and of luck $\tilde{\epsilon}_t$. With probability $1 - \theta$, the inventor has no positive net present value ideas for inventing new intermediate goods during this period and does not invest in R&D ($\tilde{\epsilon}_t = 0$) with no R&D output ($i_t^n = 0$). With probability $\theta$, the inventor find profitable ideas during this period and is able to invest in R&D ($\tilde{\epsilon}_t = 1$). This captures the empirical observation that R&D investment is often lumpy. This leads to the following linearized cost function of R&D investment, $\tilde{\epsilon}_t \cdot 1_{h_t=1} \cdot q \cdot \left( n_{t+1} - \left( 1 - \tilde{\delta}_t \right) n_t \right)$, where $q$ is a technological cost parameter and $1_{h_t=1}$ is a dichotomous variable equal to zero when the entrepreneur withdraw her specific human capital and else equal to one. Like households, entrepreneurs are able to supply inelastically one unit of human capital ($h_t = 1$) in their own firm (instead of the final goods industry) and they have no disutility of labor. Once the R&D activity has started, only the entrepreneur possesses the skills necessary to invent new designs from this input.

An entrepreneur has always the ability to threaten its creditors by withdrawing her human capital input, consume the credit, repudiate its debt contract and find other creditors for next periods (Hart and Moore [1994], Kiyotaki and Moore [1997]). Under
the assumption that observing $\tilde{\varepsilon}_t$ and $1_{b_t=1}$ is too expensive for creditors, new lenders are not able state ex-post if no investment at last period was due to lack of investment opportunity or to the withdrawing specific human capital. Creditors (households) protect themselves by collateralizing the stock of existing blueprints over which the firm has a monopoly rent and take care never to let the size of the debt repayment to exceed the liquidation value of the stock of patent next period after depreciation. The innovative firm receives a rent on each period on each of the blueprints it has discovered before. The rents received at date $t + 1$ are summed over his stock of blueprints at date $t$ and amount to $\pi n_t$. The market value of the blueprint is equal to the discounted flow of profits gained on the production of an intermediate good.

Lenders take into account that an expected proportion $\delta = E_t \left( \tilde{\delta}_t \right)$ of the innovations become obsolete at each period, so that the discount rate is increased by a factor $\delta$ (they can diversify the obsolescence risk by lending to many entrepreneurs). The credit constraint is then expressed as:

$$ Rb_t \leq (1 - \mu) \left( \sum_{t=1}^{t=+\infty} R^{-t} (1 - \delta)^{t-1} \pi \right) n_t = (1 - \mu) \frac{\pi}{r + \delta} n_t $$

or as a debt/patent ratio $x_t$ bounded by a endogenous ceiling $x^c$ (for homogeneity, the unit cost of R&D investment $q$ multiplies patents):

$$ x_t = \frac{b_t}{q n_t} \leq x^c \left( \mu, \pi, \delta, q, R \right) = \left( \frac{1 - \mu}{1 + r} \right) \left( \frac{\pi}{q (r + \delta)} \right) \tag{2.10} $$

Lenders may loose a proportion $(1 - \mu)$, with $0 \leq \mu < 1$, of collateral because of legal or bankruptcy costs related to the change of property rights over the patents. In this Kiyotaki and Moore’s [1997] setting, debt $b_t$ is a prefered means of finance rather than new share issues. Entrepreneurs simultaneously enjoy a non tradable by-product of the stock of innovations they invented which is measured as a non-tradable final output $Y_t = d_m n_t$, and can be only consumed by them (it can also be understood as a private benefit of running a business). This non tradable by-product only prevent the situation in which firms’ owners continually postpone strictly positive consumption leading to a zero utility, that is entrepreneurs indifference between producing or not:

$$ d_t \geq d_m n_{t-1} > 0 \tag{2.11} $$

The innovative firm’s flow of funds constraint states that dividends should be equal to the profits at date $t$ earned from previously discovered blueprints, to which are added new debt net of interest repayment and substracted the cost investment in R&D:

$$ \pi n_{t-1} + b_t = \tilde{\varepsilon}_t q \left( n_t - \left( 1 - \tilde{\delta} \right) n_{t-1} \right) + Rb_{t-1} + d_t - d_m n_{t-1} \tag{2.12} $$
In order to determine the investment behaviour and the debt policy of innovative firms facing the equation of motion of blueprints, the credit constraint and the flow of funds constraints, several assumptions regarding the behaviour of innovative firms and the technology and finance parameters are required.

Assumption A1: \( \pi > q (r + \delta) \) (imperfect capital markets: the marginal profit for R&D investment is higher than the marginal cost of debt).

Assumption A2: \( \mu > 1 - \left( \frac{q(r+\delta)}{\pi} \right) (1 + r) \). Bankruptcy costs are are sufficiently high so that the debt/patent ceiling is strictly below unity, which is consistent with A1.

Under assumptions A1 and A2 and the strict positivity of the real interest rate \( (R > 0) \) and of the probability of finding a profitable investment opportunity \((\theta > 0)\), follows this proposition:

**Proposition 1.** Innovating firms can be in one of these two regimes at each period:

(i) Regime 1 (Maximal debt): Innovators having an opportunity to invest at date \( t \) will choose a binding debt ceiling and consume no more than its current (private) output of nontradable good \( (d_t = d_{m_{n_{t-1}}}) \). Patents are determined by the flow of funds and the debt ceiling ratio leads to an equation of motion for patents for firms investing in R&D on period \( t \) which describes how leverage \((b_t/n_t)\) amplifies the persistence of innovation:

\[
\begin{align*}
    b_t &= \left(1 - \frac{\mu}{1 + r}\right) \left(\frac{\pi}{r + \delta}\right) n_t \quad \text{(2.13)} \\
    n_t &= \left(1 + \frac{\pi}{q} - \frac{\delta}{q}\right) n_{t-1} - R^{b_{t-1}}_t \quad \text{(2.14)}
\end{align*}
\]

(ii) Regime 2 (Decreasing debt): Innovators which have no opportunity for profitable investment now will also consume \( d_t = d_{m_{n_{t-1}}} \) as they expect to find new profitable ideas and to face a financial constraint in the future. Patents are determined by the low of motion. The flow of funds equations determines the law of motion of debt: retained earnings are used to to reimburse part of their debt and accumulated for future R&D investment.

\[
\begin{align*}
    b_t &= Rb_{t-1} - \pi n_{t-1} \quad \text{(2.15)} \\
    n_t &= \left(1 - \frac{\pi}{q}\right) n_{t-1} \quad \text{(2.16)}
\end{align*}
\]

**Proof.** See appendix 1.

Therefore, consumption is always kept at the minimum level corresponding to the private output of the entrepreneur, which can be neither reinvested nor used for reimbursment purposes. The consequence is that investing firms will always be credit-constrained. One could relax this constraint by supposing that only a fraction of
investing firms choose to invest the full possible amount when indifferent between investing and saving. This would have consequences on the aggregate amount of investment, on the growth rate and on the equilibrium rate of interest.

3. R&D Persistence and Economic Growth

3.1. Aggregate Patent Dynamics

Given the optimal investment behaviour and credit policy of firms described by proposition 1, we derive now the equations of motion for the entrepreneurs aggregate patent and debt. Debt and patents equations are linear in patent and debt in both cases. One can appeal to the law of large number for the hazard of finding profitable ideas and the hazard of obsolescence and aggregate across entrepreneurs to derive the equations of motions of patents and debt without having to keep track of the distribution of the individual entrepreneurs patents and debt (Aggregate patents and debt are denoted by capital letters $N_t$ and $B_t$). Since the population of entrepreneurs is unity, we first have the equation of motion of the number of patents:

$$N_t = (1 - \theta) (1 - \delta) N_{t-1} + \theta \left( \frac{1 + \frac{\pi}{q} - \delta}{1 - \left( \frac{1}{1+r} \right)} N_{t-1} - \frac{R B_{t-1}}{q} \right)$$ (3.1)

The flow of funds equality leads to the equation of motion of aggregate debt:

$$B_t = q N_t - q (1 - \delta) N_{t-1} + R B_{t-1} - \pi N_{t-1}$$ (3.2)

We investigate R&D sector steady states where debt and patent grow at the same rate, which amounts to a constant aggregate debt/patent ratio ($x_t = x_{t-1}$). The equation of motion of patents can be written as a patent growth factor $G_N$:

$$G_N \left( \theta, \pi, x^c, x_{t-1}, \delta, q, R \right) = \frac{N_t}{N_{t-1}} = (1 - \theta) (1 - \delta) + \theta \frac{1 + \frac{\pi}{q} - \delta - R x_{t-1}}{1 - x^c}$$

This patent growth factor decreases the costs of R&D investment parameters (debt service $R x_{t-1}$, technological cost, expected rate of obsolescence) and increases with the monopoly profits rewarding innovation $\pi$, these effects being amplified by maximal leverage $(1/(1 - x^c))$ which also decreases with the cost of R&D investment and increases with monopoly profits. Then, the equation of motion of aggregate debt can be written as a debt growth factor:

$$G_B = \frac{B_t}{B_{t-1}} = R + q \frac{N_t}{B_t} - q (1 - \delta) + \pi \frac{N_{t-1}}{B_{t-1}}$$ (3.3)

$$G_B \left( \pi, x_t, x_{t-1}, \delta, q, R \right) = \frac{R - \left( 1 + \frac{\pi}{q} - \delta \right) a N_{t-1}}{1 - a N_{t-1}} = \frac{R - \left( 1 + \frac{\pi}{q} - \delta \right) \frac{1}{x_t}}{1 - \frac{1}{x_t}}$$ (3.4)
The equation of motion of aggregate debt can indeed be written as a patent growth factor $G_{BN}$:

$$G_{NB} \left( \pi_x, x_t, x_{t-1}, \delta, q, R \right) = \frac{x_t}{x_{t-1}}G_B = \frac{N_t}{N_{t-1}} = \frac{1 + \frac{\pi}{q} - \delta - R x_{t-1}}{1 - x_t} \tag{3.5}$$

This patent growth factor is similar to the one of investing firms except that the leverage during the period is not necessarily maximal. This growth factor of innovations is determined by the growth factor of internal funds, with wealth net of debt at the denominator and retained earnings at the numerator.

At the steady state, $x_t = x_{t-1} = x$ is given by the equality between (a) the patent growth factor derived from the equation of motion of debt $G_{NB}$ and (b) the patent growth factor derived from the equation of motion of patent $G_N$ which depends linearly on the steady state debt/patent ratio $x$.

$$\left(1 - \theta\right) \left(1 - \delta\right) + \theta \left(\frac{1 + \frac{\pi}{q} - \delta - R x}{1 - x_c}\right) = \frac{1 + \frac{\pi}{q} - \delta - R x_{t-1}}{1 - x_t} \tag{3.6}$$

The patent growth rate on the left hand side is a decreasing function of debt/patents ratio $x$ as long as $\frac{\pi}{q} - \delta > r$ (in $G_{NB}$, the leverage effect $x_t$ offsets the size of debt repayment $x_{t-1}$). After polynomial division with respect to $x$ which clearly shows that the patent growth $G_{NB}$ increases linearly with leverage (debt/(value of the firms’ patent less debt) and in an hyperbolic fashion with the debt/patent ratio $x$. The steady state debt/patent ratio is the intercept a rising hyperbola with a decreasing line in the plane $(G, x)$ for $x \in [0, x_c]$ with $0 \leq x_c < 1$. We are then able to state assumption 2 and proposition 2.

Assumption 3: $\theta \geq \theta_{\text{min}}^x = \frac{(1-x^c)\frac{\pi}{q}}{x^c(1-\delta)}$. The proportion of investing innovators at a given date is above a minimal threshold, which has the property to be always below unity. This assumption guarantees a strictly positive aggregate debt providing returns on households savings:

**Proposition 1.** R&D sector steady state patent and debt growth. Under assumption A1 ($\pi > q(r + \delta)$), A2 ($x^c < 1$) and assumption A3 ($\theta \geq \theta_{\text{min}}$):

(i) For a proportion of investing innovators $\theta \geq \theta_{\text{min}}$, a unique steady state patent and debt growth exist, with a constant aggregate debt/patent ratio $x^* \leq x^c$. This steady state debt/patent ratio rises to the individual debt/patent ceiling if all innovators found profitable ideas and invested ($\theta = 1$) and increases with increases of the endogenous debt/patent ceiling $x^c$. The steady state aggregate debt/patent ratio increases with the proportion of investing firms $\theta$ and decreases unambiguously with bankruptcy costs $\mu$. For other R&D costs and benefits, the results are ambiguous and depend on the composition of firms $\theta$. For high values of $\theta$, the aggregate debt/patent ratio increases with and monopoly rents rewarding innovation $\pi$ and decreases with the unit R&D investment cost $q$ (for
\( \theta > \theta_x \), expected obsolescence rate \( \delta \) (for \( \theta > \theta_x \)) and the marginal cost of debt \( r \) (for \( \theta > \theta_R \)). For alternative conditions on the share of currently constrained firms, the signs are reverted.

(ii) The steady state growth of patents, for a given interest rate, is given by \( G_N \left( x^* \right) = G_{NB} \left( x^* \right) \). As it increases with the aggregate debt/patent ratio, the effects depicted in (i) are qualitatively similar on the growth rate of patents. The steady state patent growth rate increases with the proportion of investing firms \( \theta \) and decreases unambiguously with bankruptcy costs \( \mu \). For other R&D costs and benefits, the results are ambiguous and depend on the composition of firms \( \theta \). For high values of \( \theta \), the steady state patent growth rate increases with and monopoly rents rewarding innovation \( \pi \) and decreases with the unit R&D investment cost \( q \) (for \( \theta > \theta_x^N \)), expected obsolescence rate \( \delta \) (for \( \theta > \theta_R^N \)) and the marginal cost of debt \( r \) (for \( \theta > \theta_R^N \)). For alternative conditions on the share of currently constrained firms, the signs are reverted.

\textbf{Proof.} See appendix 2.

The inversion of dependance of the aggregate debt/patent ratio on profits, interest rate and depreciation is explained by the composition effect of firms. Firms in the second regime in proportion \( 1 - \theta \) anticipate financial constraints. In this regime, higher profits and lower depreciation and lower interest rate has an opposite decreasing effect on leverage with respect to the endogeneous debt ceiling regime. As a consequence, when \( \theta \) decreases, the effects of the second regime tends to dominate the effect of the current financially constrained regime. This model fits with a weak correlation at the aggregate level between debt/equity ratio and cash flows.

3.2. Steady State Growth Rate of the Economy

Steady state equilibria are given by equality of the growth rate of consumption \( G_C \left( R \right) \), of new goods \( G_N \left( R, x, \theta \right) \) and of debt \( G_B \left( R, x \right) \). This amounts to the equality between \( G_C \left( R \right) \) and \( G_N \left( R, x^*, \theta \right) = G_B \left( R, x^* \right) \) which allows to compute the equilibrium real interest rate \( R^* \) and a strictly positive growth rate with bounded utility under some conditions:

\[
\left( \beta R \right)^{1/\sigma} = (1 - \theta) (1 - \delta) + \theta \left( \frac{1 + \frac{\pi}{q} - \delta - R x^* \left( R, \theta \right)}{1 - x^c \left( R \right)} \right)
\]

Eliminating the interest rate, leads to an expression of the growth rate the steady state growth rate \( G^* \). The financed constrained positive growth is only feasible if \( 1 < G^* < G_{\max} \) and for \( 1 + \rho < R^* < R_{\max} \equiv (1 + \rho)(G_{\max})^\sigma = (1 + \rho)^{\frac{1}{1-\sigma}} = G_{\max} \)
when $0 < \sigma < 1$. This amounts to the following conditions on the share of investing innovative firms such as $\theta^G_{\min} < \theta < \theta^G_{\max}$, with $\theta^G_{\min}$ and $\theta^G_{\max}$ defined implicitly by:

\[
G_N (r = \rho, \theta = \theta_{\min}) = G_C (r = \rho) = 1 \quad (3.7)
\]

\[
G_N (r = r_{\max}, \theta = \theta_{\max}) = G_C (r_{\max}) = G_{\max} \quad (3.8)
\]

that is:

\[
\theta^G_{\min} = \frac{\delta}{1+\frac{\rho - R^* (R, \theta)}{q}} - (1 - \delta)
\]

\[
\theta^G_{\max} = \frac{G_{\max} - (1 - \delta)}{1+\frac{\rho - R^* (R, \theta)}{q}} - (1 - \delta)
\]

when $0 < \sigma < 1$.

Then we define the following condition:

Assumption 4 (A4): $\max \left( \theta_{\pi}^{G}, \theta_{\min}^{G}, \theta_{N}^{G} \right) < \theta < \min \left( \theta_{\max}^{G}, \theta_{\pi}^{G}, 1 \right)$: these conditions on the composition of firms indicate that the steady state debt/patent is strictly positive (A3), that the growth rate of the economy is strictly positive (which is equivalent an equilibrium interest rate higher than the households rate of time preference $\rho < r^*$), that the growth rate of patent decreases with the interest rate (a sufficient condition for unicity), that utility is bounded in the steady state when $0 < \sigma < 1$ and that the equilibrium interest rate is strictly below the profit rate ($r^* < \frac{\pi}{q} - \delta$).

**Proposition 2.** Under sufficient conditions on the bankruptcy cost parameter $\mu$ and on the proportion of currently investing firms $\theta$, that is under conditions A1 (imperfect capital market), A2 (bankruptcy costs such that the debt ceiling is below unity), and conditions on the proportion of investing firms A3 (high enough so that steady state debt/patent ratio is strictly positive), and A4, there exist an equilibrium interest rate $R^*$ and a steady state growth rate $G^*$ for the economy with bounded utility in imperfect capital markets.

This growth rate decreases with the constant elasticity of substitution $\sigma$ and the rate of time preference $\rho$. It increases with the proportion of investing firms $\theta$ and decreases unambiguously with bankruptcy costs $\mu$. For other R&D costs and benefits, the results are ambiguous and depend on the composition of firms $\theta$. For high values of $\theta$, the steady state patent growth rate increases with monopoly rents rewarding innovation $\pi$ and decreases with the unit R&D investment cost $q$ (for $\theta > \theta_N^{\pi}$), expected obsolescence rate $\delta$ (for $\theta > \theta_N^{\delta}$). For alternative conditions on the share of currently constrained firms, the signs are reverted.
4. Conclusion

Developing an endogenous growth model with lenders limiting credit up to collateralizable value of existing patents and with a composition between innovative firms currently constrained and innovative firms anticipating future contraints (hence accumulating current profit and decreasing current debt), this paper provides the following results.

First, the resulting patent behaviour takes into account lumpy and persistent behaviour, which is amplified by leverage and the monopoly rents; a specification which matches a recent empirical model of innovation and patent estimated by Blundell, Griffith and Van Reenen [1999].

Second, there exist a steady state endogenous aggregate leverage (or debt/patent ratio) below the leverage ceiling, taking into account firms currently constrained and firms decreasing leverage because they anticipate future constraints.

Third, there exist a steady state aggregate growth rate which increases with the steady state aggregate leverage and with the share of currently constrained firms, besides the usual positive effects of monopoly rents rewarding innovation (Romer [1990], Grossman and Helpman [1992] and Aghion and Howitt [1992]) and negative effects of variables included in the cost of R&D investment.

Direction of future research could consider empirical implications based on this leverage driven growth.

References


5. Appendix 1

The Lagrangian of the entrepreneur program is:

\[(n_t, b_t) \in \text{Arg} \max E_0 \sum_{t=1}^{+\infty} R^{-t} \left[ d_t + \lambda_t^b \left(1 - \mu \right) \frac{\pi}{r + b} n_t - R b_t \right] + \lambda_t^d (d_t - d_m n_{t-1}) \]

(5.1)
where $\lambda^B_t$ is the Lagrange multiplier related to the debt ceiling constraint, $\lambda^d_t$ is the Lagrange multiplier related to the non-zero consumption constraint, and with consumption $d_t$ given by the flow of funds constraint:

$$d_t = \pi n_{t-1} + b_t - \tilde{\varepsilon}_t q \left( n_t - \left(1 - \hat{\delta} \right) n_{t-1} \right) - R b_{t-1} + d_m n_{t-1}$$

(5.2)

We also know that the random variable $\tilde{\varepsilon}_t$ is such that:

$$\tilde{\varepsilon}_t = 0 \Rightarrow n_t = \left(1 - \hat{\delta}_t \right) n_{t-1}$$

(5.3)

The Euler equation on debt $b_t$ is:

$$1 + \lambda^d_t - R \lambda^B_t + R^{-1} E_t \left( 1 + \lambda^d_{t+1} \right) (-R) = 0 \Rightarrow \lambda^d_t = R \lambda^B_t + E_t \left( \lambda^d_{t+1} \right)$$

(5.4)

$$\lambda^d_t = R \sum_{\tau=0}^{+\infty} \lambda^b_{t+\tau} + \lim_{t \to +\infty} E_t \left( \lambda^d_{t+\tau+1} \right)$$

Consumption is kept at its minimal level if $\lambda^d_t > 0$ which is the case if $\lambda^B_t > 0$ (the credit constraint is currently binding, as long as the (real) interest rate is strictly positive $R > 0$) or if the expectation that consumption will be kept at its minimal level next period is strictly positive $E_t \left( \lambda^d_{t+1} \right) > 0$ which recursively is that case if the credit constraint next period will be binding or if the expectation that consumption will be kept at its minimal period in two periods is strictly positive. Hence, a necessary condition for consumption to be over its minimal level is that the entrepreneur is never financially constrained at any future period. His expectation to have a profitable investment opportunity next period is $\theta > 0$, so that, when repeating indefinitely the game, this occurrence will happen at least once for sure. At this date, the entrepreneur will face the credit constraint under the condition that the marginal gain is higher than the marginal cost of R&D investment $\pi > q (r + \delta)$. If one assumes perfect capital markets (i.e. if one rejects the principal agent problem related to the entrepreneur specific human capital input, related to the assumption that $\tilde{\varepsilon}_t \cdot 1_{h>0}$ is costly to observe by creditors), credit market clears, $\pi = q (r + \delta)$, one finds a standard "increasing product variety" growth model (Romer [1990], Grossman and Helpmann [1991]).

Under this assumption, the entrepreneur faces two regimes: a first one where he is currently constrained, a second one where he is not financially constrained now but he knows he will be constrained at some date in the future. The optimal path for these two regimes ($n_t, b_t$) are fully described by the law of motion of patents, the flow of funds and in the first regime by the debt ceiling constraint (the Euler equation on patent is helpful to determine the Lagrange multiplier on debt $\lambda^B_t$). In regime one, the entrepreneur faces an investment opportunity and takes debt up to the credit ceiling ($b_t = b^c$). Then the flow of funds allows to determine the number of patents
\( n_t \) which increases with retained earnings. In regime 2, patents are determined by the law of motion when no investment \( n_t = \left(1 - \tilde{\delta}_{t-1}\right) n_{t-1} \) and the flow of funds allows to determine debt \( b_t \) which decreases with retained earnings.

6. Appendix 2

A) Existence and unicity of the debt/patent steady state:

The steady state \( x \) is given by the implicit quadratic equation (we denote \( \varpi = \frac{\tilde{\delta}}{\eta} \) and \( x^c = y \)):

\[
H(x) = R + ((1 + \varpi - \delta) - R) \frac{1}{1 - x} - (1 - \theta) (1 - \delta) - \theta \left(\frac{1 + \varpi - \delta - Rx}{1 - y}\right) = 0
\]

This function is strictly increasing with \( x \):

\[
\frac{\partial H(x)}{\partial x} = \frac{1 + \varpi - \delta - R}{(1 - x)^2} + \theta \frac{R}{1 - y} > 0
\]

A solution exists for \( 0 < x \leq y < 1 \) under conditions \( H(0) < 0 \) and \( H(y) > 0 \) as the function \( H(x) \) is continuous on this interval. \( H(0) < 0 \) leads to condition A3 on \( \theta \):

\[
H(0) = 1 + \varpi - \delta - (1 - \theta) (1 - \delta) - \theta \frac{1 + \varpi - \delta}{1 - y} < 0
\]

\[
\Rightarrow \quad \theta > \theta_{\text{min}}^x = \frac{\varpi (1 - y)}{y (1 - \delta) + \varpi} > 0 \quad (A3)
\]

Remark: \( \theta_{\text{min}}^x = \frac{\varpi - y \varpi}{y - y \varpi + \varpi} < 1 \).

\( H(y) > 0 \) is always fulfilled as long as \( \theta \leq 1 \):

\[
H(y, R, \varpi, \delta, \theta, y) = R + \frac{1 + \varpi - \delta - R}{1 - y} - (1 - \theta) (1 - \delta) - \theta \frac{1 + \varpi - \delta - yR}{1 - y} \geq 0
\]

Remark: in the case \( \theta = 1 \), the solution is \( x = y \).

B) Partial derivatives of the steady state debt/patent ratio given by the implicit function theorem:

\[
\frac{\partial x}{\partial Z} = \frac{-\frac{\partial H}{\partial \theta}}{\left(\frac{\partial H}{\partial x} > 0\right)}
\]

B1) Non ambiguous sign: \( \frac{\partial H}{\partial \theta} < 0 \).

\[
\frac{\partial H(x, R, \varpi, \delta, \theta, y)}{\partial \theta} = 1 - \delta - \frac{1 + \varpi - \delta - Rx}{1 - y} < 0
\]
B2) Non ambiguous dependance on the debt ceiling and on bankruptcy costs ($\frac{\partial H}{\partial \mu} < 0$):

$$\frac{\partial H(x, R, \omega, \delta, \theta, y)}{\partial y} = -\theta \frac{1 + \omega - \delta - Rx}{(1 - y)^2} < 0$$

B3) Ambiguous signs:

$$\frac{\partial x}{\partial R} = \frac{-\frac{1}{1-x} + \frac{\theta}{1-y} \left[ 1 - \frac{1 + \omega - \delta - Rx}{1 - y} \left( \frac{y}{x} \right) \left( \frac{1}{R} + \frac{1}{(R - 1 + \delta)} \right) \right]}{\frac{1 + \omega - \delta - R}{(1-x)^2} + \frac{\theta}{1-y} R}$$

$$\frac{\partial x}{\partial \omega} = -\frac{\frac{1}{1-x} - \frac{\theta}{1-y} \left( 1 + \frac{1 + \omega - \delta - Rx}{1 - y} \frac{y}{x} \right)}{\frac{1 + \omega - \delta - R}{(1-x)^2} + \frac{\theta}{1-y} R}$$

$$\frac{\partial x}{\partial \delta} = -\frac{\frac{1}{1-x} + 1 - \theta + \frac{\theta}{1-y} \left[ 1 + \frac{1 + \omega - \delta - Rx}{1 - y} \frac{y}{R - 1 + \delta} \right]}{\frac{1 + \omega - \delta - R}{(1-x)^2} + \frac{\theta}{1-y} R}$$

We define:

$$Z_R = \frac{1 + \omega - \delta - Rx}{1 - y} \left( \frac{y}{x} \right) \left( \frac{1}{R} + \frac{1}{(R - 1 + \delta)} \right)$$

$$Z_\omega = \frac{1 + \omega - \delta - Rx}{1 - y} \frac{y}{\omega}$$

$$Z_\delta = \frac{1 + \omega - \delta - Rx}{1 - y} \frac{y}{R - 1 + \delta}$$

then:

$$\frac{\partial x}{\partial R} < 0 \iff \theta (1 - x (\theta)) > \frac{1 - y}{1 - Z_R} \iff \theta \geq \theta_R^x$$

$$\frac{\partial x}{\partial \omega} > 0 \iff \theta (1 - x (\theta)) > \frac{1 - y}{1 + Z_\omega} \iff \theta \geq \theta_\omega^x$$

$$\frac{\partial x}{\partial \delta} < 0 \iff \frac{(1 - x (\theta))}{x (\theta)} > \frac{1 - y}{y + Z_\delta} \iff \theta \geq \theta_\delta^x$$

C) Partial derivatives for the growth of patents (ambiguous signs): $G_N$

$$\frac{\partial G_N}{\partial R} = \frac{1}{1 - x} \left( -x + \frac{1 + \omega - \delta - R}{1 - x} \frac{\partial x}{\partial R} \right)$$

$$\frac{\partial G_N}{\partial \omega} = \frac{1}{1 - x} \left( 1 + \frac{1 + \omega - \delta - R}{1 - x} \frac{\partial x}{\partial \omega} \right)$$

$$\frac{\partial G_N}{\partial \delta} = \frac{1}{1 - x} \left( -1 + \frac{1 + \omega - \delta - R}{1 - x} \frac{\partial x}{\partial \delta} \right)$$
Then:

\[
\frac{\partial G_N}{\partial R} < 0 \iff \frac{\partial x}{\partial R} < \frac{x(\theta)(1 - x(\theta))}{1 + \omega - \delta - R} \iff \theta \geq \theta_R^N
\]

\[
\frac{\partial G_N}{\partial \omega} > 0 \iff \frac{\partial x}{\partial \omega} > \frac{-(1 - x(\theta))}{1 + \omega - \delta - R} \iff \theta \geq \theta_{\omega}^N
\]

\[
\frac{\partial G_N}{\partial \delta} < 0 \iff \frac{\partial x}{\partial \delta} < \frac{1 - x(\theta)}{1 + \omega - \delta - R} \iff \theta \geq \theta_{\delta}^N
\]