AN EMPIRICAL INVESTIGATION
OF THE EFFECTS OF FUTURES
ON MONEY DEMAND IN THE
U.S.

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ABSTRACT

This paper addresses the classical theme of money demand using a different estimation technique from traditional papers. Futures represent the widest and biggest innovation of financial markets; modern monetary economics should include financial innovation in the money demand function since it contributes to provide stability. The country chosen for empirical estimation is the US since it provides complete data and meaningful evolution of markets. The estimation method chosen (Phillips and Hansen) is new to the monetary literature, and preliminary results confirm the role of financial innovation in explaining market instability and portfolio substitution in money demand.

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Key words: money demand, futures, substitution, fractional cointegration, spectral analysis.
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An Empirical Investigation of the Effects of Futures on Money Demand in the US

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1 Introduction

This paper addresses the classical issue of money demand using a different estimation technique from traditional papers. Futures represent the widest and biggest innovation of financial markets; modern monetary economics should include financial innovation in the money demand function since it contributes to provide stability.

We are interested in the relationship among money holdings, income, Federal Funds rate, and Futures, which is not instantaneous. Suppose to start from a stable equilibrium situation and that something “disturbs”. These noises do not suddenly transmit to the others. Then, what we are looking for is a long-run relationship among these variables. Before cointegrating theory, the usual way to work was using first differenced variables or correcting for autocorrelation. Both the first and the second methods do not take into account the long-run nature of the equilibrium. In particular, first differencing variables drop out a lot of information, because they do not contain information about their long-run attitude anymore. That is clearly an inefficient way of proceeding and produces inefficient estimates. Our estimation procedure looks for a way in which money demand representation can gain in stability from financial innovation. Reasons why financial innovation should be included are deeply discussed.

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2 Money Demand Theory Overview

Money demand is an economic theme, which has fascinated economists over the centuries and no unique result has been ever reached. Money demand, and money allocation in portfolio depend on the definition on money and wealth and on the possible combinations, depending on technology available and risk attitude.

Using very simple notation, we can synthesise the evolution of money demand specifications and start with the well know quantitative theory of money (MV=PQ), moving to the Fisher interpretation of it (MV(r)=PQ) and then look at the Keynesian liquidity preference (Md=(r,Y)) where money holdings are not only function of income (or consumption) but depend also on the alternative investment possible, following the speculative motive to hold money, together with precautionary and transactional. Tobin introduced the concept of average money holdings M=(2bT/r)^1/2 where b is the brokerage charge to convert bonds into money, r is the interest rate and T is the number of transactions. This is known also as the square-root law. Empirical studies on money demand have followed economic theory and tried to find out a stable representation of money demand\(^1\). Financial innovation and evolving regulation might change the process of allocation of wealth in the form of money or substitutes, and empirical and theoretical studies follow this process and try to find stable and meaningful functions over time and countries.

Money demand and its relationship with growth and inflation are central themes in modern monetary and financial policy, but a stable money demand function is the basic tool to identify the correct relationship with these final goals. Central banks in the last decade have started following monetary rules weighting goals (growth, inflation, unemployment) and relative drawbacks and effects with respect to fiscal policy.

\(^1\) Walsh, 1998 and Pawley, 1993.
Following Barro and Santomero (1972) and Coenen and Vega (1999) we observe that a stable representation of the money demand should include alternative assets’ return to explain portfolio shifts and allocations in the short run. Barro and Santomero introduced, much before Regulation Q considered them, interest bearing deposits as assets explaining the behaviour of money demand of households in the US. Coenen and Vega more recently explain how the European money demand can be considered stable after introducing some financial assets weighted with their returns and risk; these financial asset are representative of the European financial activity. The Keynesian money demand $M_d=(r,Y)$ is enriched with innovation $(r^*)$ so that it can be represented implicitly as $M_d=(r,Y,r^*)$. Financial innovation is a big opportunity for investors, who, in the market, decide which innovation is going to survive and evolve, and monetary authority, which looks at the demand for money and its stability as an explanatory variable together with consumption and investments, should consider these portfolio shifts and their (adverse) effects. Our aim is not to sustain that central banks should act on derivatives markets to stabilise the money demand since we believe in a radical change in money government and management, which has already taken place; policy goals should be ordered market conditions, good monitoring and supervision systems, and transparency, and no quantitative rules or direct interventions on markets. Since final policy variables are linked with intermediate targets (money, consumptions, asset prices) in a dynamic and two directions way, the final effect of quantitative operations could be different from the expected. An example is the policy developed by the Central Bank of New Zealand after the Asian crisis, which was theoretically correct but in practise contributed to recession and the spread of the crisis in the area. Quantitative goals have, in our view, to be replaced with qualitative rules, and accountability is an example. The only speaking of Alan Greenspan, Chairman of the Federal Reserve Bank, about rising interest rates in 2004 moves stock markets and credit as if he already did. This is the environment we face and try to deal with.
3 Future and Other Derivatives Markets Dimension: Why Are They So Loved by Investors?

Derivatives have grown in dimension, liquidity and variety of contracts during the last 30 years; almost every stock exchange worldwide has a derivatives division of trading, while OTC attract customers for their flexibility and being off-balance. These simple qualities help to explain why they have grown so fast and in the dimension we will describe. Derivatives are financial assets and reasons moving investments decision toward them can be grouped in liquidity and diffusion over worldwide markets; efficient price formation (low volatility and high liquidity); tax rules; costs saving strategies. In details:

a) The BIS Quarterly Review reports data about amounts outstanding, turnover, gross market value, and counterpart distribution and quality on worldwide derivatives markets (OTC and exchange traded). According to the last available survey with data referring to December 2003, aggregate value of turnover of exchange traded financial contracts reached 874 trillion U.S. dollars, with a 26% increase over the last year. The growth speed in previous years was similar, with 17% in 2001 and 55% in 2002. About 90% of contracts are interest rates based, and 55% of them are traded in the U.S.. Europe and Asia follow with a gap. Over 2003 currency derivatives and Asian based contracts were growing at a high speed thanks to the weak dollar and the Asia recovery process taking place. Looking at OTC derivatives, at June 2003 (last available data) notional amounts outstanding reached the value of 170 trillion US dollar, and gross market value 8 trillion US dollar. OTC and exchange traded data are different in their measure and significance, but the dimension of OTC, regardless of their being un-standardised and unregulated, means that investors find what they are looking for in these markets. OTC markets in December 2003 trade 197.177 billion dollars notional amount and 6.987 billion U.S. dollars gross market value, and 60% of all contracts are interest rates based. Foreign exchange OTC notional amount contracts amount to 24.484 billions of U.S. dollars and more than 50% of them are outright forward and foreign exchange swaps. Gross market values
amount of foreign exchange OTC reached 1.301 billions of U.S. dollars, and the share of forward and swaps is almost the same as measured with notional amounts.

To have an idea of the amount of resources devoted to derivatives markets it is sufficient to say that the public debt of Italy in 2003 amounts to 1.372,33 billions of euro, and the Italian GDP is 1.302,33 billions; then the ratio between notional amount of OTC and the Italian “scary” public debt (using a 1:1 exchange rate between the euro and the U.S. dollar) is 1,44. Resources traded on the NYSE, one of the most important stock exchanges for market capitalisation, amount to 17,3 trillion\(^2\) dollars in the same period, which is around 8,7% of OTC notional amount worldwide traded.

b) Volatility is a measure of markets instability and we can look at two types of volatility: actual and implied. The first is the annualised standard deviation of changes in asset prices. Implied volatility is based on option prices, which incorporates a premium reflecting the time-varying risk aversion (Jeanneau and Micu, 2003). Correlation between volatility and liquidity can be analysed to extract some risk attitude behaviour of traders, and to understand how information flows over markets. Using intra-day or day-to-day data, a positive correlation between volatility and liquidity (measured either with turnover or open interest) is found; but using monthly data, no significant correlation (or even negative in case of S&P500 option) has been found (Jeanneau and Micu, 2003).\(^3\) When a positive relationship between volatility and liquidity is found, it supports the hedging behaviour, where traders move risk to those better able to bear it, and supports the hypothesis that some private information is present on markets (and speculators exploit it). Day-to-day (or intra-day) data are influenced by announcements, private and public information, while monthly data are influenced only by main macroeconomic news, which repeat every months; only surprising announcements can emerge at monthly data level.

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\(^2\) Data refers to market capitalisation of year 2003 (NYSE, 2004). Data about Italian debt and GDP come from the Bank of Italy online database.

\(^3\) Derivatives used in the analysis are S&P 500 Stock Index Futures and Options, and 10-years US Treasury Notes Futures and Options. Results using actual and implied volatility are not very different.
Volatility in derivatives exchange traded markets has not shown to be higher than in underlying assets markets, and liquidity shows similar (and even better) values. OTC markets have higher variance of prices and less standardisation of positions and contracts, but have not yet caused any dramatic failure in stable periods; in case of turbulence (against a currency, or country, or shares) they tend to exacerbate the effects on other markets, because of the leverage effect. Econometric evidence shows that options’ introduction tends to lower the underlying assets’ volatility, while futures on stock indexes do not show this property uniformly (Bank of Italy, 1995, page 79). In general, the introduction of derivatives tends to lower the bid-ask spread of the market, thus increasing market efficiency.

Matching price property\(^4\) and price discovery effect\(^5\) complete the characteristics of derivatives, and contribute to their market efficiency.

North American financial and non-financial derivatives markets are the most liquid and developed worldwide; the other important financial market is the UK, which is outside the euro area and has the most important Europe based exchange-traded market.

Europe (excluding the UK) and Asia follow but with a gap due to different reasons. Europe has not a unique exchange traded market although the euro should have incentive that, and costs of trading are sometimes lower in the USA. Asia had suffered of lower monitoring and control in the past (mainly before the 1997-98 crisis) and investors prefer to choose a safe and sound market (OTC and exchange traded).

The BIS reports that at the end of 2002 the greatest amount of OTC contracts was traded with dealers and other financial institutions; a very marginal role is played by non-financial customers (like Government and firms). However, in the 2004 Survey about derivatives activities, the BIS is expected to give a deeper picture of counterparts involved in derivatives, especially OTC; at the moment, it is not possible to completely locate traders

\(^4\) Matching price is the mathematical property of derivatives prices of being positively correlated with underlying assets markets movements, so that prices match and no arbitrage is possible.

\(^5\) Price discovery effect is the property of derivatives prices to converge with underlying assets markets prices, so that they are discovered by market participants.
and non financial institutions within countries, markets and currencies. Moreover, a better
definition of trader is going to be given by the BIS in order to distinguish between inter-
dealer and customer transactions, and improved definitions of in-house or related party
deals will be used to complete statistics.

c) A very interesting issue in economic analysis, which also helps explaining the
choice of financial instruments to invest in, is tax rules; as explained by Anson (1999)
according to the North American system, derivatives used by firms to hedge are considered
as costs in the balance sheet, and then losses and costs can be used to diminish pre tax
profits; firms using derivatives to speculate\(^6\) cannot do that. Another important factor is that
OTC derivatives are off-balance sheet items, and are not supposed to influence directly the
rating of the firm\(^7\). The tax saving and elusion are important incentives to invest and can
influence market liquidity, costs reduction and hedging strategies of firms.

d) Governments can manage the cost of debt using interest rates swaps, as explained
by Piga (2001), and the impact on welfare can be important. In Piga’s survey, Sweden is far
the biggest user of interest rate swap (IRS) in public debt management; in June 2000 50%
of Swedish debt was covered by IRSs. Costs can be lowered using foreign exchange swaps
(FES) as well. Examples are Denmark and Canada, which have high openness degree and
flexible exchange rates toward the EMU and the United States, respectively, and hedge
against exchange rate variations.

Welfare effects can be much more complicate to analyse and any country has its
reasons and procedures to use derivatives to manage debt level and costs; the issues of risk
loving behaviour and medium-long term effects of management can modify the final effect
of the strategy, but a general lack of communication and explanation of these operations
makes Piga’s analysis quite personal and not completely objective.

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\(^5\) Price discovery effect is the ability of derivatives price (taken today) of being equal to future (expiring date)
discounted underlying asset price (see J. Hull, 2002, chap. 1-8). No arbitrage is possible.

\(^6\) Hedging is when firms lower or move risks of their core business, while speculation is related with trading
of non-related-core-business assets/liabilities.
Countries having floating exchange rates can use derivatives to stabilise the rate and to manage reserves at a lower cost, but cannot succeed against a speculative attack, as the Asian example of 1997-98 teaches us (e.g. Thailand bath).

Central banks can use derivatives to manage the interest rate level and variation in absence of the government bond markets, like Switzerland (Hooyman, 1993) or to intervene in the open and inter-bank market (Tinsley, 1998).

To sum up, resources devoted to OTC and exchange traded derivatives are far bigger than public debt of some European countries, and exceeds resources traded on some stock exchange; derivatives have grown thank to their a) providing liquidity and being traded on worldwide markets; b) being efficient assets (have normal level of volatility, and high liquidity); c) exploiting tax rules (saving and elusion); d) lowering costs.

4 Effects for Money Control and Analysis of Financial Innovation

Money aggregates are indicators used by modern central banks to look at possible inflationary pressures. For example, the Deutsche Bundesbank controlled the intermediate target of M3 to reach the goal of zero inflation. Money aggregate however suffers of a theoretical problem: substitution effect. Money aggregate like M3 does not weight its components with user or opportunity costs, leading to give the same importance, from a portfolio approach viewpoint, to coins or Government bonds. As observed by Barnett (1997), this is not correct if we want to measure money demand and its variation, or the impact of money manoeuvres on real cash balance. Liquidity, opportunity costs, and substitution are important features in money management.

Central banks are independent from the national Government and Treasury because of their ability to control targets, but financial innovation modifies this ability in the short run. Some monetary authorities have chosen particular (weighted) indexes to look at, like

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7 This last lack is maybe going to change since some international investment banks have announced that they want to consider OTC derivatives operation’s exposure of firms in their loans decisions.
the Monetary Condition Index (MCI), given the impossibility to use money aggregates as instruments\(^8\). The only instrument for monetary policy is the interest rate, given flexible exchange rates regimes worldwide.

Other central banks have chosen divisia index to help the analysis and government of targets; the Fed looks at divisia M1 and M2 to fine tune interest rates manoeuvres, but these indexes are quite difficult to explain to the public, leading to a modification of communication, which is the base of credibility and coherence of central banker.

The Bank of Italy (1995a) states that, according to the money view, perfect substitutability between credit and bonds is the base for money manoeuvre to be effective (using interest rates). Derivatives increase markets’ liquidity and substitutability, and increase the speed of the transmission mechanism of monetary impulses; the distributive effect has to be considered. In fact, if individual risk can be shifted, at a macro level it cannot be cancelled, and it is of central importance to understand if those bearing more risks are really able to manage them (i.e. ordered market conditions). According to the credit view, imperfect substitutability between credit and bonds is supposed and the introduction of derivatives, highly substitutable with bonds and credit, can alter dramatically monetary policy actions and effects. The Bank concludes that a credit-based monetary policy is not going to be effective facing financial innovation, while interest rate manoeuvres aiming at ordered market conditions, taking into consideration the evolution of financial markets, and quasi-money instruments, are probably the best development of modern central banking.

The use of derivatives for monetary purposes (futures, forwards and options) is also a cost-lowering way of managing interest and exchange rates (Cf. Bank of Italy, cit., pag. 84-85), and is not prohibited by current national and international regulation.

For a given money supply level, the use of derivatives by Central Banks implies a lower liquidity-deposit ratio, and this increases the potential money creation through the

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\(^8\) MCIs are neither instruments nor targets; they represent the weighted goals of monetary policy, in terms of interest and exchange rates.
credit channel. In general, the Central Bank can use derivatives, like exchange-traded futures, to fine tune monetary policy in the short run, together with traditional open market instruments. Other derivatives, especially OTC could lead to a potential conflict with the goal of stability and transparency of the authority. In any case derivatives are not a suitable instrument to influence the long run segment of the yield curve (Rossetti, 2000). Rossetti\(^9\) observes that the impact of derivatives on money demand should be to diminish cash reserves, i.e. diminish transactional demand for money, to increase substitutability with financial assets, and to speed up the transmission mechanism of movements of interest rates on different maturities. Given higher substitutability (bond-credit, credit-cash) monetary policy that acts trough the banking system could become less effective.

Pawley (1992) observes that financial innovation needs to be modelled into the demand for money as to identify a stable function. Broad demand for money is influenced by the change in the own rate of money; this elasticity is of central importance for money control and has not been sufficiently considered. Elasticity of the rate and substitutability represent therefore the core problem of modern monetary economics in the globalised economy.

Savona (2002) links the liquidity paradox stated by Keynes (1936), “everyone feels to be liquid but the market itself is not”, with the disruptive impact of derivatives; central banks in the last 15 years have fought against excessive speculation\(^10\) using their function of lenders of last resort, and responding on market movements to avoid credit crunch (or even collapse). This is definitely a new way of managing money order, at home or internationally, and uses up-to-date instruments, like futures and options. Money quantity is not directly affected, but order and stability. Again, quantitative goals loose importance and qualitative analyses should dominate in research departments.

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\(^9\) In R. Violi (2000) where the issue of derivatives and monetary policy is deeply analysed by prominent economists and practitioners.
5 A Justification to the Inclusion of Future into the Money Demand Function

Money demand function needs to be updated with financial innovation as to reach a stable representation of the relationship

\[ 11 \]; money aggregates’ definition reflects this evolutionary process of financial and money markets, but each central banks chooses the definition of the aggregate to look at which best suits policy and reaction function. Portfolio substitution and demand for liquid assets are the main issues of this first (rough) estimation of money demand function. This really intuitive analysis is functional to approach the model of assets’ demand. If the demand for money is influenced by financial innovation, namely derivatives, a substitution model is needed.

Money demand should be represented by a wide money aggregate, like M3 (as defined by the Federal Reserve and the European Central Bank) as to make it possible to look at portfolio substitution effects and liquidity allocation in the area. Money aggregates are calculated on the main hypothesis of perfect substitutability between assets compounding the aggregate, and this is probably the main drawback of the approach. Perfect substitution means that no risk premium is considered (e.g. between coins and 1-month Treasury Bills) and this is not possible in finance theory, which looks at assets demand related to risk. M3 is the sum of coins, bank accounts, and short-term assets (like Government’s bonds), and it represents the liquidity of the economy.

The dynamics of short-run broad money demand adjusts to financial innovation, while the theory tells us that in the long-run money should be a stable function of income and interest rate

\[ 12 \].

Money demand should be modelled through the use of weighted monetary indexes, i.e. Divisia Index, introduced in the literature by Barnett (1992 and 1997) and developed and measured by the Federal Reserve Bank of St. Louis (they call it Monetary Services

\[ 10 \] Some examples are the Federal Reserve and the LTCM in 1998, the Bank of Thailand and the Thai bath attack in 1997-98, the Bank of Italy and the lira in 1992, but many others could be named.

\[ 11 \] See M. Pawley, 1993.

Index, MSI), because they address directly the problem of un-perfect substitutability contrary to traditional money aggregates, which are simple sums of assets.

The money demand function in the implicit form can be written as \((m/p) = f(r, y, \text{future})\), where \((m-p)\) is real cash balance (money demand), and is a function of interest rate \((r)\), income \((y)\), and the financial innovation (future) representative of market and portfolios in terms of liquidity, and open interest.

The critic moved to the introduction of the future into the money demand function is that it is positively correlated with the underlying asset (the Dow Jones Industrial Average index) so that we could have confusion in the function, i.e. the coefficient we see is related with the underlying and not with the future, i.e. the futures is not an independent asset. The property of futures’ prices being correlated with the underlying is an efficiency characteristic and is called price discovery effect\(^{13}\); main differences, however, are the dimensions of the two markets, the liquidity degree, risk profile and costs-profits. We know that the derivatives’ market is hundreds of times bigger than the underlying. Liquidity of futures markets is the same or higher that the underling markets’; in some cases the underlying asset does not exist as such (like in the DJIA case) and then futures have higher liquidity by definition. Looking at costs and potential profit, the leverage effect of futures allow for higher (potential) profit and lower costs of investments. Risk profile of futures is the same as the underlying, given matching price and price discovery effect, but because of their high liquidity and general efficiency, they are preferred for hedging activity. Moreover, futures and traditional stock exchange show different economic functions (derivatives exhibit leverage, hedging and substitutability)\(^{14}\), while hedging and leverage cannot be exploited in the same way in the underlying market. We can conclude that they represent different assets and satisfy different functions in the money demand and asset allocation of investors.

\(^{13}\) See J. Hull 2002 for math details.

\(^{14}\) For a wider discussion of economic functions of derivatives see P. Savona, 2003, and C. Vrolijk, 1997, for monetary effects.
Given that we want to enrich the specification of money demand function and include an innovation, which represents market evolution, we should include a liquid future, representative of the US economy, not highly volatile, used for hedging or speculation, and showing normal costs (in terms of bid-ask spread) on the market. The future will be included in the estimation over the period it contributes to lower financial volatility (1999 to 2000) so that we can detect its stabilising ability.

6 A Brief Introduction to Long Memory Processes

Generally, stochastic processes are divided into stationary and non-stationary. In this work we want to allow for the possibility, suggested by recent theories about stochastic processes, of nuances in stationarity and non-stationarity definition, opening the analysis to the intermediate cases represented by long memory processes. Consequently, we adopt a wide definition of cointegration, extending time series analysis to the frequency domain.

The process $x_t$ is stationary, if it has zero mean, finite variance and autocovariances that do not depend on time, but only on the respective lags. Stationary processes can be generalized by ARMA (AutoRegressive Moving Average) models, such that:

$$x_t = \frac{\theta(L)}{\phi(L)} \epsilon_t,$$

with $\theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \ldots + \theta_q L^q$ and $\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p$  

where $\theta(L)$ and $\phi(L)$ are polynomials in the lag operator $L$, $\epsilon_t$ is white noise distributed ($E \epsilon_t = 0, \ E \epsilon^2 = \sigma^2, \text{ and } E \epsilon_{t-\tau} \epsilon_t = 0$ for $\tau \neq 0$), and $\phi(L)$ is invertible.

The process $x_t$ can also be explicitly written as a weighted sum of current and past innovations:

$$x_t = \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j}$$

(1)

where the coefficients $\psi$ represent how much of something happened in the past is currently remembered.
Stationary processes quickly “forget”; for them it holds that:
\[ |\psi_j| \leq constant \cdot |\psi|, \text{ with } |\psi| < 1 \]

A general representation of a non-stationary process is the ARIMA’s (AutoRegressive Integrated Moving Average):
\[ x_t = (1 - L)^d \frac{\theta(L)}{\varphi(L)} \epsilon_t \]  

(2)

Here, the variance of \( x_t \) grows over time. It is worth noting that this process becomes an ARMA, once differentiated.

Suppose to have now:
\[ x_t = (1 - L)^d \frac{\theta(L)}{\varphi(L)} \epsilon_t \]  

(3)

where \( d \in \mathbb{R} \) and the fractional differencing operator is defined by:
\[ (1 - L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d) L^k}{\Gamma(-d) \Gamma(k+1)} \]  

(4)

Such a representation reduces to ARMA for \( d = 0 \) and to ARIMA for \( d = 1 \). For \( d \) assuming non integer values, there is the so-called ARFIMA (AutoRegressive Fractionally Integrated Moving Average) representation; ARFIMA processes are stationary for \(-\frac{1}{2} < d < \frac{1}{2}\) and mean-reverting for \(-1 < d < 1\). Then, for \( \frac{1}{2} < |d| < 1 \), they are non-stationary but mean-reverting. In particular, for \( d < 1 \), the process is stationary, but it very slowly “forgets” and, for that reason, it is called long memory process.

In the case of ARFIMA models, the coefficients of the expansion can be proved to be:
\[ \psi_j = constant \cdot j^{d-1} \]

In conventional analysis, ARFIMA models are not considered; in effect, unit-root tests are designed to distinguish only the case of \( d \) taking the value of 1 or 0. In effect, the
Dickey-Fuller and Phillips-Perron unit root tests only distinguish between stationarity and non-stationarity. They do not allow a proper assessment of what the importance of the non-stationary component of a time series can be. As they only consider two mutually exclusive alternatives, when the series of interest simultaneously has the feature of a random walk as well as those of stationary series, it is a proved and accepted result that these tests tend to accept the null hypothesis of non-stationarity.

Including intermediate cases can give more robustness to the analysis.

7 Fractional Cointegration and Time Series Analysis in the Frequency Domain

Why are we interested in long memory processes? Suppose we want to estimate the relationship:

\[ y_t = \beta \cdot x_t + u_t \]  \hspace{1cm} (5)

where \( y_t \) and \( x_t \) are integrated of order \( d \) long memory processes

\[ \begin{pmatrix} y_t \\ x_t \end{pmatrix} \sim I(d) \]

\( y_t \) and \( x_t \) are fractionally cointegrated\(^{16}\), if it is possible to find a linear combination of them that lowers the memory of the process, such that:

\[ y_t - \hat{\beta} \cdot x_t = u_t \sim I(d'), \text{ with } d' < d, \]

where \( \hat{\beta} \) is a certain estimate of the parameter \( \beta \).

This is the definition of cointegration, i.e. of long run equilibrium among integrated processes, that we are going to apply.

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\(^{15}\) It is worth noting that each stationary process is mean reverting, but a mean-reverting process is not necessarily stationary.

\(^{16}\) See, for example, P. M. Robinson and D. Marinucci, 1998.
In order to check whether this concept of cointegration applies to the variables involved in our analysis on money demand, we need to introduce time series analysis in the frequency domain.

In the frequency domain or Fourier space, a stochastic process is regarded as the sum of deterministic cycles with uncorrelated stochastic weights.

The idea of spectral analysis of time series is to transform stochastic processes in order to abstract from their covariance function as much information as possible. In this view, the spectral density of a stationary process in the time domain is the frequency domain counterpart of a covariance function, that is its Fourier transform

\[ f(\lambda) = \frac{1}{2\pi} \sum_{\tau} \gamma(\tau) e^{-i\lambda \tau} \]  

(6)

where \( \lambda \) is the frequency, \( i = \sqrt{-1} \), \( e^{i\lambda \tau} = \cos \lambda \tau - i \sin \lambda \tau \), and

\[ \gamma(\tau) = \int_{-\pi}^{\pi} f(\lambda) e^{-i\lambda \tau} d\lambda \]  

(7)

It is worth noting that: \( f(\lambda) \) has a period of \( 2\pi \), that is \( f(\lambda) = f(\lambda + 2\pi j) \), where \( j \) is an integer; under certain conditions\(^{17} \), \( f(\lambda) \) assumes real values; \( f(\lambda) = f(-\lambda) \), such that the analysis of the spectral density can be restricted to the range \( \lambda \in [0, \pi] \).

\( f(\lambda) \) is a function of \( \lambda \) and expresses the value of the spectral density for each \( \lambda \in [0, \pi] \): the bigger is the spectral density at that frequency the bigger is the variance, then the process tends to repeat with that frequency.

For a white noise process \( \varepsilon_t \), with \( \gamma_\varepsilon(0) = \sigma_\varepsilon^2 \) and \( \gamma_\varepsilon(\tau) = 0 \) for \( \tau \neq 0 \), \( f_\varepsilon(\lambda) = \sigma_\varepsilon^2 / 2\pi \), then its spectrum is flat and independent of \( \lambda \), because each frequency has an equal weight in determining the variance of the white noise.

For the ARMA process in (3), the spectral density is easily showed to be:

\(^{17} \text{See P.J. Brockwell and P.A. Davis, 1991.} \)
\[ f_x(\lambda) = \frac{|\Phi(e^{-i\lambda})|^2}{\varphi(e^{-i\lambda})^2} f_e(\lambda) = \frac{|\Phi(e^{-i\lambda})|^2}{\varphi(e^{-i\lambda})^2} \frac{\sigma_x^2}{2\pi} \]  

(8)

Its spectrum is still bounded but not flat anymore.

A non-stationary process has an unbounded spectrum as frequencies go to zero.

Finally, the spectral density of a long memory process is bounded, but concentrated on the lowest frequencies, because it is dominated by long memory components.

[spectral densities of simulated processes about here]

8 The Money Demand Model and Results

Our aim is to check if a long-run equilibrium among money, income and prices exists and to understand if in a period of high instability futures play a crucial role in re-addressing these variables towards their equilibrium path.

Thus, the model to be estimated is:

\[ M_t / P_t = \alpha + \beta \cdot Y_t + \gamma r_t + \delta \cdot Futures_t + \delta' \cdot [t \in (a,b)] \cdot Futures_t + u_t \]  

(9)

where we use the monetary services index (MSIM3)\(^{18}\) for \( M_t \), the consumers price index\(^{19}\) (P) for \( P_t \), the industrial production index\(^{20}\) (INDUROD) for \( Y_t \), and the Federal Fund rate\(^{21}\) (FEDFUND) for \( r_t \), the settlement price of future contract continuous rolled over on the Dow Jones Industrial Average Index\(^{22}\) (FUTURE) for \( Futures_t \), and \( u_t \) is an error term. \([t \in (a,b)]\) (DUMFUT) is an indicator function that introduces a structural break in the

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\(^{18}\) Source: Federal Reserve Bank of Saint Louis.

\(^{19}\) Source: Datastream.

\(^{20}\) Source: Datastream.

\(^{21}\) Source: Federal Reserve Bank of Saint Louis

\(^{22}\) Source: Datastream.
model for the period of high instability denoted by the time interval \((a, b)\), which started on 1/1999 up to 12/2000 when it takes on value 1 (0 otherwise)\(^{23}\). It has the meaning of a slope-correcting dummy, a significant value which underlines the powerful role of futures in determining the long-run equilibrium among the variables of interest. We added a dummy variable (DUMTT) to take into account the Twin Towers effect on financial and money markets in the US.

All data are monthly and start in October 1997 up to August 2003, because the future contract started on that date.

According to our theory, we expect that \(\beta\) is positive and substantially different from 0, that \(\delta\) is not substantially different from 0, and that \(\delta'\) assumes a positive and significant value, because we want to show that futures play a role in re-establishing the equilibrium in high instability periods.

In Figg. 1-4, the spectral density of each variable is reported. Each spectrum shows to have a high bounded peak at the lowest frequencies, denoting its long memory nature.

In choosing the coefficient estimation technique, we had to challenge two problems: variables simultaneity and the short sample period available. In effect, the relationship among these variables is not instantaneous; for example, disturbances in Federal Fund rate do not instantaneously transmit to money, or income.

Different ways to identify and test for a long-run relationship among variables are suggested by cointegration techniques. Lots of single equations and multiple equations models are available, each of them with its own advantages and drawbacks:

1. the Engle-Granger’s two-step procedure is based on a mechanism that do not take into account the problems of variables’ endogeneity and it also imposes untested common factor restrictions, that dramatically lower the power of tests, in the event of any of them being false;

\(^{23}\) Spectral densities refer to detrended series, since spectral analysis does not distinguish between deterministic and stochastic trend series.
2. VAR system, like in Johansen’s full-information maximum likelihood cointegration allows to address the issue of simultaneity. However, it, being based on a maximum-likelihood estimation technique, extends specification problem linked to any single equation to the whole system;

3. Instrumental Variables techniques allows to address the issue of simultaneity of variables;

4. OLS estimators are super-consistent with nonstationary variables (converge to the real parameters values at a faster rate, \( n \) instead of \( \sqrt{n} \)), but its asymptotic distribution is severely biased, such that it makes it impossible any statistical inference;

5. Phillips-Perron Fully Modified OLS (FM-OLS) method that uses the super-consistency property of OLS estimators and accurately calculates the asymptotic distribution of the estimator of a cointegration relationship, allowing for statistical inference\(^{24}\) but with nonstationary time series requires particular attention to the choice of instruments\(^{25}\).

Each of these methods looks for a cointegration relationship among non-stationary variables. What is common to all of them is just the definition of stationarity and nonstationary process.

Conventional analysis only distinguishes between stationary and non-stationary processes.

Following Phillips and Hansen’s Monte Carlo results (1990) showing a better small sample properties of FM-OLS estimators with respect to the others, our choice fall on it. FM-OLS, synthetically, consists of correcting the asymptotic distribution of the OLS parameter estimator for the bias terms due to the dependence among variables\(^{26}\).

\(^{24}\) See for example D. Hamilton, 1994.

\(^{25}\) See, for example, P. Phillips and B. Hansen, 1990.

\(^{26}\) Ibidem.
Table 1 Fully Modified Phillips-Hansen Estimates
Parzen weights, truncation lag = 16, Trended Case
Dependent variable is MSIM3/P
70 observations used for estimation from 1997M11 to 2003M8

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-Ratio[Prob]</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>-.21622</td>
<td>.022709</td>
<td>-9.5214[.000]</td>
</tr>
<tr>
<td>INDUPROD</td>
<td>.0063410</td>
<td>.2051E-3</td>
<td>30.9104[.000]</td>
</tr>
<tr>
<td>FEDFUND</td>
<td>-.018770</td>
<td>.3804E-3</td>
<td>-49.3393[.000]</td>
</tr>
<tr>
<td>FUTURE</td>
<td>.6164E-6</td>
<td>.7691E-6</td>
<td>.80146[.426]</td>
</tr>
<tr>
<td>DUFUT</td>
<td>.2604E-5</td>
<td>.8068E-6</td>
<td>3.2280[.002]</td>
</tr>
<tr>
<td>DUMTT</td>
<td>-.014820</td>
<td>.0027857</td>
<td>-5.3199[.000]</td>
</tr>
</tbody>
</table>

Truncation lags order chosen by running the regression for all possible lags and by choosing the order that gives the whiter regression error.

The FM-OLS estimate confirms all our expectations about parameters signs and statistical significance.

Actually, the spectral density of the regression residuals (see Fig. 5) shows a peak at the lowest frequencies, but sharper than the ones showed by the original time series in Figg. 1-4. Hence, it shows to have less memory than the generating processes.

The last result, according to the definition of cointegration adopted, shows that a long-run relationship among the considered variables exists, and that it fulfils in the signs and the statistical significance of regression coefficients the hypotheses maintained. The magnitude of the Federal Fund is not directly comparable with that of future price, since the first is a rate and the second a price; signs are as expected and the substitution effect in real money balance is detected. Dummy variables are both significant, so that the Twin Towers effect influenced the money demand process over the period and corrects the intercept. The future dummy on the contrary, underlines the stabilising effect of futures over a period of
high volatility of financial markets (1/1999-12/2000); without futures the money market subject to high volatility moves from its long-run equilibrium path and fails to come back. This is why the FUTURE coefficient is not statistically different from zero outside the instability period.

9 Conclusions

Monetary theory aims at defining a stable representation of money demand function; this is actually not achievable, since financial markets introduce innovations on a daily basis and run faster than regulation, and theoretic analysis; our aim then is to look at an empirical specification of money demand which includes one of the most traded, liquid financial asset, both at domestic and international levels. Country chosen for empirical investigation is the one where financial innovation is highly developed and available to all investors, since we are looking at a substitute of traditional assets into money demand function.

The Phillips and Hansen estimator fully exploits super-consistency of OLS estimator over small samples and periods, and differently from OLS estimator, whose asymptotic distribution is not reliable for non-stationary variables, let estimate a regression and infer statistically. This is the main reason basing on which we choose this estimator and not other cointegration techniques, given the limits of available data.

Results show that fractional cointegration is useful for underlying the role of future prices, i.e. financial innovation, in explaining instability in money demand; futures help money demand function to come back to a stable long-run equilibrium path after instability periods. Traditional monetary literature has paid attention to modified money markets and institution, but a stable money demand function needs to be identified in order to provide meaningful information about inflation pressures and financial order. The Keynesian money demand function is the basic starting function, but the substitution effect in portfolio is considered too, by measuring money with monetary services index. The long-run
equilibrium solution, i.e. money as a function of income and price, is confirmed by these results, but the inclusion of futures lets increase the descriptive power of the money demand function.
10 Bibliography


Fig. 1

Standardized Spectral Density Function of MSIM3/P Parzen window

- Parzen
- +2 S.E.
- -2 S.E.

Frequency
Fig. 2

Standardized Spectral Density Function of INDUPROD Parzen window

- Parzen
- +2 S.E.
- -2 S.E.
Fig. 3

Standardized Spectral Density Function of FEDFUND Parzen window

Parzen

+2 S.E.

-2 S.E.

Frequency
Fig. 4

Standardized Spectral Density Function of FUTURE Parzen window

Parzen

+2 S.E.

-2 S.E.
Fig. 5

Standardized Spectral Density Function of FM-OLS regression residuals Parzen

Parzen

+2 S.E.

-2 S.E.