Capital-skill complementarity, labour mobility 
and the welfare effects of trade integration

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Abstract

This paper addresses the role of mobility costs in shaping the effects of trade integration on wage inequality and welfare. We present a three-factor, two-sector model in which the production technology exhibits capital-skill complementarity and the cost of moving across sectors differs between unskilled and skilled workers. We consider a proportional tax on skilled workers' wage that is used to finance a re-training program to reduce the mobility costs of unskilled workers. We show that if training expenditures are sufficiently effective, a positive tax rate can both reduce wage inequalities and reinforce the welfare-enhancing effects of trade integration. In addition we show that, even when the public training programme entails some welfare losses, it can make trade integration Pareto improving.

KEYWORDS: Capital-Skill Complementarity, Intersectoral Labour Movements, Wage Inequality, Trade Integration

JEL Classification: E24, J31, R23

1 Introduction

This paper addresses the interplay between trade integration, labour mobility and wage inequality. We show that the reallocation of workers across sectors, induced by trade integration, might increase across group inequality when workers’ heterogeneity is taken into account. To this end we model a two-sector small competitive economy in which each sector produces a single good combining capital, skilled labour and unskilled labour. Capital is internationally

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mobile; skilled and unskilled workers can move across sectors. We introduce workers’ heterogeneity by allowing the mobility cost to differ between skill categories, and by adopting a production technology which exhibits capital-skill complementarity. The interaction between mobility costs and capital-skill complementarity turns out to be important in order to address the labour market effects of trade integration and, to the best of our knowledge, it has never been explored before in our modelling framework. Remarkably, the mechanism explored in this paper does not require any difference in skill intensity between sectors. We also model a public re-training program, intended at reducing the mobility cost of unskilled workers and financed by a proportional tax levied on skilled workers, and explore the welfare effect of changes in the tax rate under alternative assumptions on the effectiveness of the program.

We characterize both the autarchy and the free trade equilibrium. We study the effects of trade integration in terms of aggregate welfare – measured by an utilitarian social welfare function – and both across and within group wage inequality. We show that if the mobility cost of unskilled workers is high enough to prevent mobility, trade integration has a positive effect on aggregate welfare, but it increases both across and within group inequality. We also show that a lower mobility cost for unskilled workers implies both higher welfare and a lower wage inequality. It is worth emphasizing that the welfare effect of trade integration is well established in the literature, as well as the increase in inequality. What is new in our model is that frictions in the reallocation of unskilled workers during the trade integration process might indeed cause an increase in both across and within group inequality.

The public re-training program entails two conceptually distinct distributional effects. First, by lowering the moving cost for unskilled workers, it allows their reallocation, thus reducing inequality. Second, by taxing the wage of skilled workers, it reduces net of taxes wage differentials between types of workers. Therefore, an increase in the tax rate unambiguously reduces inequality. In general, the effect of the re-training program on aggregate welfare turns out to depend on the efficiency of the training mechanism. In particular, we can show that if training expenditures are sufficiently effective in reducing individuals’ mobility cost, then there exists a positive tax rate such that the provision of public training maximize social welfare. Moreover, even when the public training programme entails some welfare losses, it makes trade integration Pareto improving.

The paper is organized as follows: in Section 2 we briefly review the literature on the effects of trade liberalisation on inter-industry (and intra-industry) labour reallocation. In addition, we discuss the evidence concerning the different level of mobility between sectors and firms of educated and less educated workers. Section 3 provides the basic economic set-up and Section
4 analyzes the effects of trade liberalisation on aggregate welfare and both within and across group wage inequality. It also studies the effects of the introduction of a public training scheme. Section (5) concludes.

2 Related literature

The literature on wage inequality and its relationship with international trade is, by now, extremely vast. The interest in the field has been largely motivated by the dramatic increase in both across and within-skill group wage inequality which has occurred in several developed countries (DCs) and, remarkably, in the US during recent decades (see Katz and Autor, 2000).

The benchmark theoretical linkage between trade integration and across group wage inequality is the well known Stolper-Samuelson theorem: since DCs are assumed to have a comparative advantage in skilled-intensive productions, it follows that the process of specialization induced by trade integration implies an increase in the relative demand for skilled workers, thereby rising the skill premium. A specular pattern of specialization would also entail a reduction of wage inequality in less developed countries (LDCs). However, existing evidence suggests that several LDCs experience rising wage inequality after trade liberalisation episodes (see inter alia Harrison and Hanson, 1999; Arbache et al., 2004). Recent contributions try to solve this puzzle, proposing different linkages between trade integration and wage inequality – see for example Acemoglu (2003), Xu (2003), Gancia and Epifani (2004) and references cited therein.

A second, important prediction of classical trade models is that, when a country opens to international trade, production factor will reallocate across sectors and firms. Indeed, it is this factor reallocation that allows the country to exploit its comparative advantages and to reap the benefits of trade integration. Nevertheless, if the labour market is characterised by imperfect mobility, the benefits of trade integration cannot be fully reached and, at the same time, within group inequality (between similar workers employed in different sectors) would increase. Empirical evidence suggests that the imperfect mobility of workers is a relevant issue, implying that labour reallocation after trade integration is in general low - see Wacziarg and Wallack (2004) and Papageorgious et al. (1991). Lately, a few contributions address the issue linking the effects of trade integration to job security regulations, which is often maintained to be the major sources of frictions in the labour market – see Kambourov (2003) and Andersen and Skanksen (2003).

Remarkably, the joint effect of trade integration and intersectoral labour mobility on both
dimensions of inequality – within and across group inequality – can hardly be addressed in existing theoretical models and, to the best of our knowledge, it has never been addressed before. The existing models are not suitable to explore this effect since they either assume that workers are sector specific, as in Acemoglu (2003), or they adopt a specification of the production technology which implicitly implies that the average skill premium is constant (i.e. a Cobb-Douglas production function).

Here is where our paper tries to contribute to the existing literature. We develop a theoretical model which allows us to study the interplay between trade integration, (imperfect) labor mobility and wage inequality, both across and within skill categories. Two features of the model are crucial in our analysis and allows us to link trade, internal labour reallocation and inequality: moving costs can differ between categories of workers and the production technology exhibits capital-skill complementarity. Both features involve some notion of heterogeneity between skill categories and deserve a brief discussion.

First, we assume that the cost of moving across sectors is higher for unskilled workers than for skilled workers. The idea behind this assumption is that skilled workers have more general abilities, which can easily been transferred between occupations and sectors; on the contrary, less skilled workers are characterized by less trasferrable abilities. Existing evidence seems to support the hypothesis of higher internal mobility (across industry, sectors and geographical areas) of more educated workers: it has been shown that high educated workers possess more ability in the job search and lower transaction costs – see, for instance, Greenwood (1975), Bednarzik (1993) and Helwing (2001) –; they can more easily learn and implement new tasks and technology – see Nelson and Phelps (1966) and Bartel and Lichtenberg (1987) –; and they exhibit an higher propensity to voluntarily change their job – Magnani (2000) and Tomkins and Twomey (2000). Furthermore, there is some evidence that more educated workers spend less time without a job when they are displaced (Bednarzik, 1993; Helwing, 2001), suggesting that higher education is positively correlated with the ability to learn and perform new tasks. This hypothesis seems particularly able to capture the different degrees of workers’ mobility in those sectors affected in last decades by technological changes of a general purpose nature\(^1\).

As pointed out by Aghion, Howitt and Violante (2002), a more general technology allows for a larger degree of transferability of skills across the different sectors of the economy, implying that more skilled workers are more mobile across sectors than unskilled workers. Here we stress that

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\(^1\)Bresnahan and Trajtenberg (1995) coined the term ‘general purpose technologies’ (GPT) to describe certain drastic innovations (e.g. computers) that have the potential for pervasive use and application in a wide range of sectors in the economy.
the assumption doesn’t need to hold for the whole spectrum of qualifications, and indeed it is possible to provide examples of less qualified workers performing more generic tasks which might well change industry or sector more easily than educated workers. With more than two skill groups we could intersect formal qualifications (unskilled vs. skilled) and tasks (sector specific vs. generic) and address more carefully the relative mobility of the resulting groups. In our simplified framework, with only two types of workers, we conform to a standard notion in the literature of education and overlap formal qualification and generic, more easily transferable, abilities.

Second, the constant return to scale production technology exhibits capital skill complementarity: i.e. capital better substitutes unskilled labour than skilled labour. This assumption has received strong empirical support and it has been shown to be crucial in order to establish a link between internal mobility and across group wage inequality – see Devillanova (2004) for a detailed discussion on this point and for references.

As a final remark, notice that in our model the production function is the same in both sectors, apart from a multiplicative parameter capturing the relative productivity of sectors. This assumption has two main advantages. First, it allows us to focus exclusively on the effect of trade integration on wage inequality occurring through the channel identified in this paper – namely, the interaction between the induced labour reallocation and capital-skill complementarity – and to disregard the traditional link which bases on the different skilled-intensity between sectors discussed above. Of course, the two channels are not in contradiction. Second, and related to the previous point, the effect highlighted in this paper is at work in any economy experiencing a trade liberalisation process, independently of its level of development. Indeed, our results can offer a complementary explanation to the puzzle of increasing wage inequality in both DCs and LDCs which follows trade integration.

3 The model

We consider a small economy. The economy is populated by a fixed number of skilled and unskilled workers, who supply their labor services in a competitive labor market. There are two sectors in the economy, each producing a single good, which is sold in a competitive good market. Production requires both types of labor and capital, which is supplied in a perfectly integrated international capital market at the fixed world interest rate. None of the agents in the economy can save: this simplifying assumption is innoquous given our focus

\footnote{Ones might think of a third type of agents, not modelled in the paper, holding capital.}
on wage inequality and the hypothesis of fixed interest rate. Skilled and unskilled workers can move across sectors, but they cannot migrate abroad. We introduce workers’ heterogeneity adopting a production technology which exhibits capital-skill complementarity and by allowing the mobility cost to differ between skill categories. We next detail the production technology and the workers/consumer choice.

3.1 Production

There are two sectors, \( x \) and \( y \), in the economy. In each sector a representative firm produces a single good \( Q = \{X, Y\} \) using the following neoclassical production function:

\[
Q = f(L_q, H_q, K_q) = z_q \left[ b (K_q)^\alpha + (1 - b) (L_q)^\alpha \right] [H_q]^{1-\alpha}
\]

where \( Q \) is the quantity of good produced, \( q = \{x, y\} \) is an index for the sector, \( K_q, L_q \) and \( H_q \) are, respectively, the quantities of physical capital, unskilled labor and skilled labor used to produce \( Q \), \( \alpha, b \in [0, 1] \) are constants and \( z_q \) is a multiplicative parameter capturing the different productivities in the two sectors. In most of the paper we are going to assume \( z_x = z_y = 1 \) - i.e. identical productivity in the two sectors - in order to focus on the reallocation process induced by trade integration.

The production function (1) captures in a very convenient way the presence of capital skill complementarities\(^3\). Indeed, the elasticity of substitution between capital and unskilled labor is \( \frac{1}{1-\alpha} \), which, for \( \alpha > 0 \), is greater than one, the elasticity of substitution between capital and skilled labour. An easy to verify implication of capital skill complementarity is that \( \frac{\partial \pi_q}{\partial K_q} > 0 \), where \( \pi_q = \frac{f_{H,q}(L_q, H_q, K_q)}{f_{L,q}(L_q, H_q, K_q)} \) is the skill-premium - the ratio of skilled to unskilled workers’ productivity - in sector \( q \) and where \( f_{H,q} = \frac{\partial f(L_q, H_q, K_q)}{\partial H_q} \) and \( f_{L,q} = \frac{\partial f(L_q, H_q, K_q)}{\partial L_q} \) are, respectively, the marginal productivity of skilled and unskilled workers in sector \( q \). In words, the more capital is employed in a sector, the higher in that sector is the marginal productivity of skilled workers relative to the marginal productivity of unskilled workers.

Two other features of this framework are worth to be emphasized. First, in (1) skilled and unskilled labor are complementary production factors, with elasticity of substitution equal to one. Therefore, the marginal productivity of unskilled workers is going to be positively affected by the amount of skilled workers in the sector, and vice versa. Second, the production function is the same in both sectors, apart from the multiplicative parameter \( z_q \), and the skilled/unskilled intensity in the two sectors is going to be endogenously determined in equilibrium.

\(^3\)See also Lindquist (2004).
3.2 Workers

The economy is populated by a continuum of skilled workers of measure $\bar{H}$ and by a continuum of unskilled workers of measure $\bar{L}$. For the sake of simplicity and without loss of generality, we normalize $\bar{H} = 1$ and $\bar{L} = 2$. Agents inelastically supply their time endowment, which is normalized to one, in a competitive labor market.

All agents have identical preferences, represented by the following constant elasticity of substitution (CES) utility function:

$$U(C_x, C_y) = U\left(\frac{\gamma C_x^{\theta-1} + (1 - \gamma) C_y^{\theta-1}}{\theta - 1}\right)$$

(2)

where $C_x$ and $C_y$ denote individual’s consumption of the two goods, $\gamma \in [0, 1]$ and $\theta \in [0, \infty]$ determines the elasticity of substitution between goods. Agents maximize (2) by choosing how much to consume and where to work. Both types of workers can move across sectors bearing some costs of reallocation; they cannot internationally migrate.

The indirect utility function is:

$$V(p_x, p_y, m) = \left[\gamma p_x^{1-\theta} + (1 - \gamma) p_y^{1-\theta}\right]^{\frac{1}{\theta - 1}} m_{j,q}$$

(3)

where $p_x$ and $p_y$ are respectively the price of good $X$ and good $Y$, $j = H, L$ is an index of qualification and $m_{j,q}$ is the consumer’s disposable income, which varies across sectors and qualifications. Equation (3) makes it evident that, for given prices, indirect utility is increasing in $m_{j,q}$. Each worker will choose to work in the sector that allows for obtaining the highest disposable income. If an agent changes sector, he/she pays a mobility cost, allowed to differ between skilled and unskilled\(^4\). An agent will choose to move from sector $x$ to sector $y$ if

$$w_{j,y} - c_j \geq w_{j,x}$$

(4)

where $w_{j,q}$ is the wage of type $j$ labor in sector $q$ and $c_j$ is the individual’s moving cost\(^5\). Reallocation of workers occurs until (4) holds with equality. Notice that, if moving were costless, wage equalization between the two sectors would be reached for each type of worker.

We assume $c_L > c_H = 0$, capturing the idea that in contemporary economies skilled workers possess more general skills that can be easily reallocated in different tasks, as discussed in Section 2. $c_H = 0$ is a simplifying normalization, which implies:

\(^4\)Disposable income is going to depend on the public training system too, which will be introduced in the next section.

\(^5\)For each category of workers, mobility costs are symmetrical between sectors, i.e. the cost of moving from sector $x$ to sector $y$ is equal to the cost of moving from sector $y$ to sector $x$ workers.
The generic expression for $c_L$ is:

$$c_L = f + \xi M$$  \hfill (6)

where $M$, is the number of unskilled movers and $f$ and $\xi$ are positive parameters. The component $f$ represents the fixed costs of mobility due to the imperfect adaptability of skills, whereas $\xi M$ captures the costs of congestion associated to labor mobility. In particular (6) implies an aggregate mobility cost which is quadratic in $M$. In our framework, a marginal cost increasing in $M$ is needed in order to get an interior solution for the free trade regime.

### 3.3 Equilibrium

Equating the marginal productivity of capital in each sector to the world interest rate, solving for the quantity of capital and substituting into (1), we get:

$$Q = f(H_q; L_q; z_q) = z_q^\frac{1}{1-\alpha} p_q^\frac{\alpha}{1-\alpha} b \left( \frac{\alpha b}{r} \right)^{\frac{\alpha}{1-\alpha}} H_q + z_q (1-b) L_q^\alpha H_q^{1-\alpha}$$  \hfill (7)

Notice that, \textit{coeteris paribus}, an increase in $z_q$ and/or $p_q$, attracts new investment in the sector, thus increasing physical production and affecting, via capital-skill complementarity, the relative marginal productivity of skilled and unskilled workers in the sector.

Competitive wages of skilled and unskilled workers are\(^6\):

$$w_{H,q} = p_q \frac{\partial f(H_q; L_q; z_q)}{\partial H_q} = z_q^\frac{1}{1-\alpha} p_q^\frac{1}{1-\alpha} d + z_q g l_q^\alpha$$  \hfill (8)

$$w_{L,q} = p_q \frac{\partial f(H_q; L_q; z_q)}{\partial L_q} = p_q z_q^\varphi l_q^{\alpha-1}$$  \hfill (9)

where $d = b \left( \frac{\alpha b}{r} \right)^{\frac{\alpha}{1-\alpha}}$, $g = (1 - \alpha) (1 - b)$ and $\varphi = \alpha (1 - b)$ are positive constants and $l_q = \frac{L_q}{H_q}$ indicates the skills ratio in the sector, determined by the mobility choices of workers: the higher this ratio, the more unskilled labor is employed in a sector relatively to skilled labor.

Full employment implies:

$$H_x + H_y = \bar{H} = 1$$  \hfill (10)

$$L_x + L_y = \bar{L} = 2$$  \hfill (11)

\(^6\)Our assumption on the population structure insures that $\frac{\partial f(H_q; L_q; z_q)}{\partial H_q} > \frac{\partial f(H_q; L_q; z_q)}{\partial L_q}$ and $\frac{\partial g(H_x; L_x)}{\partial H_x} > \frac{\partial g(H_x; L_x)}{\partial L_x}$, i.e. that the marginal productivity of skilled labour is always higher than the unskilled one.
Normalizing $p_x = 1$, and indicating with $p = \frac{p_y}{p_x}$ the relative price of commodity Y, the equilibrium of the economy determines the set of wages ($w_{H,y}$, $w_{H,x}$, $w_{L,y}$ and $w_{L,x}$), the commodity price $p$ and a distribution of workers between sectors ($H_x$, $H_y$, $L_x$ and $L_y$) such that workers/consumers optimally take their consumption and location decisions, firms maximize profits and markets clear.

Notice that, from (10) and (11), $H_y$ and $L_y$ univoquely determine the distribution of workers between sectors. Thus we are left with a system of seven unknown endogenous variable ($w_{H,y}$, $w_{H,x}$, $w_{L,y}$, $w_{L,x}$, $p$, $H_y$ and $L_y$). Given $p$, the four equations for wages (8) and (9), $q = x, y$, and the two conditions for migration (3), $j = H, L$, determine wages and the workers’ distribution. In some specification of the numerical solution below, we are going to assume that, missing a training program, $c_L$ is so high that unskilled workers would never find it optimal to move. In this case $L_Y$ is fixed\(^7\) and (4) holds with strict inequality.

In order to solve for the equilibrium, we have to distinguish between the autarchy and free trade domestic relative price $p$.

**Autarchy.** In absence of international trade, $p$ is determined by market clearing conditions in the commodity markets - i.e. aggregate consumption of good $q$, $\tilde{C}_q$, equals production in sector $q$. By the properties of the CES utility function we get:

$$p^A = \frac{p_y}{p_x} = \frac{1 - \gamma}{\gamma} \left( \frac{\tilde{C}_y}{\tilde{C}_x} \right)^{-\frac{1}{\beta}} = \left( 1 - \frac{\gamma}{\gamma} \right) \left[ \frac{z^{1 - \alpha} p_{A}^{-\alpha} b \left( \frac{ab}{p} \right)^{\alpha} H_y + z (1 - b) L_y H_y^{1 - \alpha}}{b \left( \frac{ab}{p} \right)^{\alpha} H_x + (1 - b) L_x H_x^{1 - \alpha}} \right]^{-\frac{1}{\beta}}$$

where the subscript $A$ indicates autarchy.

**Free trade.** Under the assumption of small economy, in free trade the relative price $p^{FT}$ is determined in the international market and it is now *exogenous*. In order to simplify the exposition of the results, we are going to assume that $p^{FT} > p^A$, implying that Home country has a comparative advantage in sector $y$.

### 3.4 Public training program

We also model a public re-training program, intended at reducing the mobility cost of unskilled workers, financed by a proportional tax levied on skilled workers. Specifically, we assume that the training expenditure are targeted to unskilled workers in the sector where their wage is

\(^7\)This implies that, in the absence of public education expenditure, unskilled workers are specific factors, as defined by Mussa (1974).
lower and they reduce the unskilled workers fixed component of the mobility cost. The relevant cost for unskilled workers is now:

\[ \bar{c}_L = f - \chi e + \xi M \]  

(13)

where \( e \) indicates per capita training expenditures and \( \chi \) is a parameter capturing the effectiveness of the program. The idea behind (13) is that if workers’ reallocation cost is affected by the adaptability of their skills - which, in our simplified framework, maps into the distinction between skilled and unskilled workers - it can be reduced by programs aimed at providing more adaptable skills. Just as an example, one can think of \( f \) as the cost of a course to learn a computer package. If \( \chi = 1 \), our benchmark case, one euro expenditures in training causes one euro reduction in the individual’s reallocation costs, just as it where a voucher covering part of the fee of the course. \( \chi > 1 \) allows for the presence of economies of scale and/or externalities in the provision of a public training program.

Few recent contributions have stressed the role of employment protection legislation (EPL) and other institutional settings in preventing labor mobility. Mobility costs introduce a further important source of friction in the labour market, other than the EPL. This turns out to be crucial in order to evaluate the instruments available to the policy maker to enhance labour reallocation. However, if EPL are believed to be the only/main source of low mobility, a reduction of reallocation costs could be reached by simply reducing job security regulations, at no cost. In our framework, this possibility can be modeled by simply assuming a perfectly effective training program, i.e. \( \chi = \infty \). If, on the contrary, the adaptability of workers to new tasks is a major concern, changes in EPL would not have the expected effect; at the same time, policies aimed at increasing mobility become costly and their optimality cannot be given for granted.

The introduction of the training program requires only few marginal adjustments to the previous set-up. The relevant wage for skilled workers is now the after tax wage:

\[ \bar{w}_{H,q} = (1 - t)w_{H,q} \]  

(14)

where \( t \) is the tax rate. Moreover, we consider a balance budget constraint for the training program:

\[ E = t \left( H_y w_{H,y} + H_x w_{H,x} \right) \]  

(15)

where \( E \) are total expenditures in training. Once \( t \) is fixed, total expenditures are uniquely determined by (15). We assume that total expenditures are divided equally among all the unskilled workers in the sector, hence per worker expenditure is \( e = \frac{E}{L_x} \), where we are assuming
that, without any loss of generality, if any reallocation of workers occurs, it is going to take place from sector \( x \) to sector \( y \) and therefore public training is offered to unskilled workers in sector \( x \).

Once we substitute (6), and \( w_{H,q} \) with, respectively, (13) and (14), the equilibrium of the economy is described as in Section 3.3, except that now we have one free variable \( t \). We characterize the equilibrium of the economy as a function of the tax rate.

4 Results

We here study the effect of trade integration on both within and across group wage inequality, and on aggregate welfare. More specifically, we focus on four main outcomes: within group unskilled wage inequality\(^8\); across group wage inequality in each sector, which under the assumption of competitive labor market is given by the skill premium \( \pi \); aggregate across group wage inequality (the average wage premium \( \pi \)); aggregate welfare, measured by an utilitarian social welfare function \( W \). Notice that the choice of an utilitarian welfare function is clearly reductive, since it does not attach any disutility to inequality, but it is conservative from the point of view of our results.

To focus on essentials, we consider the case in which the two sectors have identical productivities - i.e. \( z_y = z_x = 1 \) - and the utility function is a Cobb-Douglas symmetric in the two goods. In this case, it is straightforward that in the autarchy equilibrium \( p = 1 \), workers are distributed symmetrically between sectors, within group inequalities are null and the wage premium is the same in the two sectors. Starting from a symmetric autarchy equilibrium, we can more easily concentrate on factor reallocation solely driven by increasing trade integration.

4.1 Free Trade

Suppose now that the Home country opens to international trade. Under the assumption of small economy, with free trade the relative price is determined in the international market and is now exogenous. We assume that trade integration causes an increase in the relative price \( p \), implying that if any reallocation of workers occurs, it is going to take place from sector \( x \) to sector \( y \).

In this case, the aggregate utilitarian social welfare function \( W \) is:

\[
W = V_H + L_x V_{Lx} + V_{Lyc} + (1 - L_x) V_{Lyc}
\]

\(^8\)Having assumed zero reallocation costs for skilled workers, their wage is always equalized between sectors.
where $V_H$ is the indirect utility function of skilled workers in both sectors (of measure 1), $V_{Lx}$ is the indirect utility function of unskilled workers in sector $x$ (of measure $L_x$), $V^{nc}_{Ly}$ is the indirect utility function of unskilled workers in sector $y$ (of measure 1) who do not bear any reallocation cost and, finally, $V^c_{Ly}$ is the indirect utility function of those workers (of measure $1 - L_x$) who in equilibrium are in sector $y$ and bear some reallocation costs.

4.1.1 Immobile unskilled workers

We first consider a situation in which, starting from $p^A = 1$, trade integration entails a marginal increase in $p$ which is not sufficient to make the wage differential larger than the moving cost - i.e. $|w_{Ly} - w_{Lx}| < c_L$ and $L_y = L_x = 1$. In other words, the trade-induced sectoral wage differential is higher than the reallocation costs for unskilled workers in sector $x$. Under this assumption, the following proposition summarizes the main consequences of opening up the economy to international trade:

**Proposition 1** Free trade raises aggregate welfare and both aggregate and within group inequality. Moreover, it has an ambiguous effect on wage premium in sector $y$ whereas it certainly increases the wage premium in sector $x$.

**Proof.** See Appendix 1. ■

Proposition 1 points out that an increase of the relative price $p$ due to trade integration affects all the variables of interest. In particular, it raises total output and welfare because of some reallocation of mobile factors - skilled workers and capital - toward the most profitable sector. However trade integration also has distributional consequences. First of all, by inducing a reallocation of skilled workers from sector $x$ to sector $y$, and because of the complementarity between the two types of labor, it increases within group wage inequality of (immobile) unskilled workers. Second, the same reallocation of skilled workers also affects the across group wage inequality in each sector, by altering the skill composition. Third, the arrival of new capital in the economy - attracted by the trade-induced sector $y$ increased profitability - causes an increase in the average across group wage inequality, because of capital skill complementarity. The overall effect is an increase of inequality.

4.1.2 Mobile unskilled workers

In the more general case, when both skilled and unskilled workers are allowed to move across sectors, since the conditions characterizing the equilibrium of the economy are highly non
linear, we are unable to provide a closed form solution and we have to compute numerically the model. The values of the parameters are: $b = 0.4, \tau = 0.1; \alpha = 0.5; \xi = 0.05; \theta = 1; \gamma = 0.5; f = 0.208$. We check the qualitative robustness of the results to all the admissible range of the parameters of the model and we only discuss this issue when results are sensitive to the parameter specification.

When Home country opens to free trade - i.e. $p = p^{FT} > p^A = 1$ - skilled workers immediately react to the wage differential between sectors and reallocate toward sector $y$ - see figure 1. At the same time, international mobile capital accrues to sector $y$. The overall effect is an increase of the skilled wage, as figure 2 makes it clear. Given the level of the fixed cost $f$, unskilled workers start to reallocate for $p > 1.2$ - the level of the free trade price for which equation 4 has a positive sign. For $p = 1.46$ reallocation is complete and only sector $y$ produces (see figure 3). The effect on within group inequality is shown in figure 4, where we plot $w_{L,y}/w_{L,x}$ as a function of $p$. It worth noting that within group inequality is strictly increasing in $p$. For $p < 1.2$, this is a general result, due to the complementarity between the two types of labor and the fact that unskilled workers are fixed. For $p > 1.2$ the assumption of non zero reallocation cost plays an important role: indeed, at $p = 1.2$ most skilled workers have already moved to sector $y$; for higher levels of $p$, as figure 5 and 6 show, the skill intensity $H_L$ in both sectors is smaller, and unskilled wages is lower (see figure 7 and 8).

As across-group inequality is concerned, figure 9 shows that the relationship with free trade price is non monotonic. Let consider first the two limit cases. When $p = 1$, the economy is perfectly symmetric, the sectoral skill intensities (see figures 5 and 6) and the sectoral skill-premia (see figures 10 and 11) are identical. Consider instead the level of $p$ such that the economy is completely specialized: the skill intensity in sector $y$ is the same as in autarchy, but, as figure 9 highlights, across-group wage inequality is now higher. This is due to capital-skill complementarity: namely, the increase in $p$ attracts new capital in the economy, rising the skill premium; when the economy specializes in production of $Y$, this unambiguously rises across-group inequality in that sector. The behaviour of aggregate across-group wage inequality in between these two extreme points is more tricky to be analysed. Let we first focus on sector $x$. By figure 6, skill intensity in the sector $x$ decreases, unambiguously increasing the wage premium in the sector. In sector $y$ two forces are at work: the (non monotonic) behaviour of the skill intensity in the sector, which should first reduce and then increase the skill premium; the arrival of new capital, which, by capital-skill complementarity, rises the wage premium. The net effect of these two forces on wage inequality in $y$ is plotted in figure 10. The average wage premium captures both the behaviour of (the level of) wage inequality in $x$ and $y$ and changes.
in the relative weights of the two sectors. In particular, for \( p \) between 1 and 1.2, the dramatic increase in wage inequality in \( x \) more than compensate the reduction of wage inequality in \( y \). For higher values of \( p \), inequality increases in sector \( y \) too. However, factor reallocation progressively lowers the weight of sector \( x \) in determining aggregate wage inequality. The latter reaches a maximum at \( p = 1.35 \) and is lower for higher values of the free trade price. Remarkably, average across-group wage inequality is always higher in free trade with respect to autarchy.

Notice, finally, that aggregate welfare increases with \( p \) (figure 12). Indeed an higher \( p \) implies that more (international) capital enters sector \( y \). At the same time, the higher the free trade price, the higher the incentive to specialize in order to exploitation the comparative advantage of the country.

Summing up, the analysis of this section shows that the trade liberalisation causes an increase in aggregate welfare, but also an increase in across and within group wage inequality\(^9\). We next study the effect of the reallocation cost on those variables.

### 4.1.3 The effect of fixed cost

We now study the equilibrium solution of the model when, for given \( p \) we let the mobility costs of unskilled workers to vary. We consider a free trade relative price \( p = 1.2 \) and we study how the level of the fixed cost \( f \) affects the variables of interest of our model.

For \( f > 0.208 \) unskilled workers do not move because within group wage differential is lower than the fixed cost \( f \) (the constant horizontal interval figure 13). When \( w_{Ly} - w_{Lx} > f \) some unskilled workers in \( x \) move toward sector \( y \). Skilled workers and capital follow the reallocation of unskilled workers toward sector \( y \).

Figure 14 shows the effect of \( f \) on the within group inequality. The graph stops when \( f \) is low enough to allow all workers to be reallocated in sector \( y \). As figure 14 illustrates, the lower \( f \), the lower the within group inequality in equilibrium. It is worth stressing that this result is due to the complementarity between skilled and unskilled workers.

The second important result is that a lower fixed cost \( f \) implies a lower aggregate wage inequality (figure 15). The intuition of this result is that, for given \( p \), with lower \( f \) unskilled workers can move from a highly unequal sector to a less unequal sector (see figure 16). The different levels of sectoral inequality is a consequence of the different degrees of mobility between skilled and unskilled workers, implying different skill intensities in the two sectors (see figure 17). Notice that, specularly to the case analysed in the previous section, the emergence of lower

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\(^9\) Of course, within group wage inequality is not defined when the economy is fully specialised.
aggregate wage inequality when fixed cost are lower is accompanied by higher wage premium in both sectors.

The third result is that when $f$ is lower, aggregate welfare is higher (see figure 18). The reason is, trivially, that the moving cost represents a waste of resources in the aggregate and limit the profitability of the reallocation of the production factors. This means that the lower these costs are, the more a country can exploit its comparative advantages.

4.1.4 Taxation and the retraining program

The previous sections highlights that: i) trade integration increases welfare and both across and within group wage inequality; ii) a lower unskilled workers’ mobility cost reduces both across and within group wage inequality. Here we explore the possibility of using (part of) the increase in welfare induced by trade integration in order to reduce inequality, by cutting unskilled workers mobility cost. In particular, we introduce a re-training program for unskilled workers in sector $x$, financed by a flat tax on skilled wages. As before, we fix $p = 1.2$ and consider a situation in which, with no training program, unskilled workers would never move ($f = 0.208$). It follows that, for $t = 0$, unskilled workers are equally allocated in the two sectors (i.e. $L_x = L_y = 1$). Obviously, since they are perfectly mobile, skilled workers are unevenly distributed between sectors. For the moment we will consider a low efficiency of the re-training program, i.e. $\xi = 1$.

Not surprisingly, the effects on wage inequality of an increase in $t$ mimic the ones illustrated in the previous section. Indeed, from (13) and (15) it is evident that, given $L_x = 1$, a higher tax rate implies a lower reallocation cost for unskilled workers. In particular, we find that both within and across group wage inequality is decreasing in $t$ (see figure 19 and 20). The main differences with respect to previous section, are the following. First, across-group wage inequality is reduced both via the general equilibrium effects analysed before and via taxation of skilled wages, which reduces the net-of-tax wage differentials between skilled and unskilled workers. Second, differently to the the case of an exogenous reduction of $f$, aggregate welfare decreases as $t$ increases, since the reduction of friction to mobility is now costly - see figure 21. When the re-training program shows low efficiency - $\xi = 1$ - the latter effect more than compensates the positive effect of factor reallocation on efficiency. We come back to this point later. We are in the presence of a trade-off: the re-training program reduces inequality but at the same time decreases aggregate welfare.

Is it possible to exploit the welfare gain induced by trade integration in order to make all agents better off? In order to answer this question, table 1 compares the autarchy equilibrium
with the free trade one. The first two columns show that, when $t = 0$, trade integration has a positive effect on aggregate welfare, but it hurts unskilled workers in sector $x$. For $t = 6.4\%$ all workers reallocate to sector $y$ and the economy is fully specialised. However, the mobility cost borne by the unskilled workers is still too high and the ones who have to move across sectors would have been better off in autarchy than in free trade. If, however, we further increase $t$, unskilled workers indirect utility can be increased. In particular, given our parameters, a level of taxation higher than $t = 14.2\%$ allows to obtain an equilibrium which is Pareto superior to the autarchy one. Notice that, from the point in which the economy is completely specialized changes in $t$ have no effect on the aggregate welfare, because now the retraining program acts simply as an (indirect) redistributive mechanism. Notice, finally, that the condition $E = tw_H \leq f$ is always satisfied. In sum, even when the public re-training program entails some welfare losses, it can make trade integration Pareto improving.

Finally, figure 22 illustrates the case in which $\xi > 1$. In this case we consider a re-training program able to produce some increasing returns: one euro used to finance the re-training mechanism allows for obtaining more than one euro exploiting the comparative advantage of the economy.\footnote{One can also think to programs targeted to reduce the mobility cost of those unskilled workers who actually move.} In this case, it is possible for the tax/training program to produce a welfare enhancing effect. For instance, with $\xi = 5$, total reallocation is reached with $t = 0.013$ and welfare increases with respect to the free trade case with no training program.

5 Conclusions

The discussion over welfare and distributional effects of trade integration is by now a long standing one. In this paper we suggest a new channel through which trade integration can affect these variables, which relies on the heterogeneity between skill categories. In particular, we allow the mobility cost to differ between skill categories, and we adopt a production technology which exhibits capital-skill complementarity. We also study the role of a costly re-training program, intended at reducing individuals’ intersectoral mobility, in shaping the effects of trade integration.

Two are the main contributions of our study. First, the model proposed here is able to account for the effect of trade integration on both across and within skill categories wage inequality when workers are (imperfectly) inter-sectorally mobile. This is, to the best of our knowledge, new to the literature. Notice, incidentally, that the redistributitional effect of inte-
igration may cause a lack of political consensus for trade liberalisation, preventing the country to benefit from the gains accruing from international specialisation. Second, we show that even under the conservative assumption of a re-training program entailing some welfare losses, its implementation can make trade integration Pareto improving.

Further research should be directed in relaxing some of the simplifying assumptions here adopted and in testing empirically the implications of our model.

References


6 Appendix

Proof of Proposition 1

**Within group inequality** Given the full mobility of skilled workers and the full immobility of unskilled workers, in equilibrium it must be that: \( w_{H_y} = w_{H_x} \) and \( L_y = L_x = 1 \). From (8), in equilibrium the following holds: \( p^{\frac{1}{1-\alpha}}d + pg(H_Y)^{-\alpha} = d + g(H_x)^{-\alpha} \). Since \( p > 1 \), this condition is satisfied if and only if \( H_y > H_x \), which also implies that skill-intensity in sector \( y \) is higher than in sector \( x \). It follows that \( w_{L_y} = p\varphi H_y^{1-\alpha} > w_{Lx} = \varphi H_x^{1-\alpha} \), meaning that the
within-group inequality in the unskilled category is higher with respect to the autarchy case (where \( w_{Ly} = w_{Lx} \)).

**Wage premium** We start considering sector \( y \). The wage premium is defined as

\[
\frac{w_{Hy}}{w_{Ly}} = \frac{(pz)^{\frac{1}{1-\alpha}} d + p z g \left( \frac{L_y}{P_y} \right)^\alpha}{pz \varphi \left( \frac{L_y}{P_y} \right)^\alpha - 1} = \frac{p^{\frac{1}{1-\alpha}} z^{\frac{1}{1-\alpha}} d + z g \left( \frac{L_y}{P_y} \right)^\alpha}{z \varphi \left( \frac{L_y}{P_y} \right)^\alpha - 1}
\]

(16)

As it is evident, the increase in \( p \) has an ambiguous effect on the wage premium in sector \( y \). Indeed both the skilled and the unskilled wages increase, but the change of their ratio may be positive or negative depending on the values of the parameters and on the magnitude of the increase of \( p \).

Wage inequality in sector \( x \) is defined as:

\[
\frac{w_{Hx}}{w_{Lx}} = \frac{d + g \left( \frac{L_y}{H_y} \right)}{\varphi \left( \frac{H_y}{L_x} \right)^{1-\alpha}}
\]

Thus if \( p \) increases, the wage premium increases as well. Indeed, if \( p \) increases, \( H_y \) increases and \( H_x \) decreases, making the numerator increasing and the denominator decreasing. As a result the ratio increases.

**Aggregate wage inequality** Let now consider the effect of trade integration on the aggregate wage inequality, which is defined as:

\[
\bar{w}_p = \frac{\frac{w_{Hy} H_y + w_{Hx} H_x}{H_y + H_x}}{\frac{w_{Ly} L_y + w_{Lx} L_x}{L_y + L_x}} = \frac{2 \left[ d + g \left( 1 - H_y \right)^{-\alpha} \right]}{pq H_y^{1-\alpha} + q \left( 1 - H_y \right)^{1-\alpha}}
\]

In order to evaluate the effect of the opening to free trade on \( \bar{w}_p \) it is sufficient to study the sign of the first derivative of \( \bar{w}_p \) with respect to \( p \) calculated in \( p = 1 \). The first derivative is:

\[
\frac{\partial \bar{w}_p}{\partial p} = \frac{2 \alpha g \left( 1 - H_Y \right)^{-\alpha-1} H_Y'}{pq H_Y^{1-\alpha} + q \left( 1 - H_Y \right)^{1-\alpha}} + \frac{-2 \left[ d + g \left( 1 - H_Y \right)^{-\alpha} \right] \left[ q H_Y^{1-\alpha} + pq (1 - \alpha) H_Y^{1-\alpha} H_Y' + (1 - \alpha) q (1 - H_Y)^{-\alpha} (-1) H_Y' \right]}{pq H_Y^{1-\alpha} + q \left( 1 - H_Y \right)^{1-\alpha}}
\]

(17)

Now we study the sign of this derivative in \( p = 1 \), where \( H_y = H_x = \frac{1}{2} = H \).
Since the denominator of the previous is always positive, the sign of the derivative is fully determined by the sign of the numerator:

\[
\text{sign} \left\{ \frac{\partial \bar{w}_p}{\partial p} \right\} = \text{sign} \left\{ \left[ 2\alpha g H^{-\alpha-1}_Y \right] (q H^{1-\alpha}_Y + q H^{1-\alpha}_Y) + 
-2 \left[ d + g H^{-\alpha}_Y \right] \left[ q H^{1-\alpha}_y + q (1 - \alpha) H^{-\alpha}_Y H'_Y + (1 - \alpha) q H^{-\alpha}_Y (-1) H_Y \right] \right\}
= \text{sign} \left\{ \left[ 2\alpha g H^{-\alpha-1}_Y \right] 2q H^{1-\alpha}_Y - 2 \left[ d + g H^{-\alpha}_y \right] q H^{1-\alpha}_Y \right\}
\]

Simplifying we obtain:

\[
\text{sign} \left\{ \frac{\partial \bar{w}_p}{\partial p} \right\} = \text{sign} \left\{ 4\alpha g q H^{-2\alpha-2} - 2dq H^{1-\alpha} - 2gq H \right\}
\]

Calculating the first derivative of \( H \) with respect to \( p \) in \( \bar{H} \), we have:

\[
\text{sign} \left\{ \frac{\partial \bar{w}_p}{\partial p} \right\} = \text{sign} \left\{ 4\alpha g q H^{-2\alpha-2} - 2dq H^{1-\alpha} - 2gq H \right\}
\]

since \( 0 < \alpha < 1 \).

This proves that an increase in the domestic relative price \( p \) makes the aggregate income inequality to increase.
Figure 1. Skilled workers: effect of free-trade price

![Figure 1. Skilled workers: effect of free-trade price](image1.png)

Figure 2. Skilled wage: effect of FT price

![Figure 2. Skilled wage: effect of FT price](image2.png)
Figure 3. Unskilled workers: effect of FT price

Figure 4. Within group inequality: effect of FT price
Figure 5. Skill intensity in sector Y: effect of FT price

Figure 6. Skill intensity sector X: effect of FT price
Figure 7. Unskilled wage sector Y: effect of FT price

Figure 8. Unskilled wage sector X: effect of FT price
Figure 9. Aggregate wage inequality: effect of FT price

Figure 10. Inequality sector Y: effect of FT price
Figure 11. Inequality sector X: effect of FT price

Figure 12. Aggregate welfare: the effect of FT price
Figure 13. Unskilled workers: effect of mobility cost

Figure 14. Within group inequality: effect of mobility cost
Figure 15. Aggregate wage inequality: the effect of mobility cost

![Graph showing aggregate wage inequality](image)

Figure 16. Sectoral wage inequality: the effect of mobility cost

- Wage inequality in sector Y
- Wage inequality in sector X
Figure 17. Skill intensity sector Y and X: the effect of fixed cost

Figure 18. Aggregate welfare: the effect of fixed cost
Figure 19. Within group inequality: the effect of taxation $\kappa = 1$

Figure 20. Aggregate wage inequality: the effect of taxation $\kappa = 1$
Figure 21. Aggregate welfare: the effect of taxation ($\chi=1$)

Table 1. Pareto Gains from Trade

<table>
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<th>Autarchy</th>
<th>Free trade, $t=0$</th>
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<td>0.150</td>
<td>0.117</td>
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<tr>
<td>Utility unskilled X</td>
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<td>0.055</td>
<td>0.048</td>
<td>0.106</td>
<td>0.116</td>
</tr>
</tbody>
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Figure 22. Aggregate welfare: the effect of taxation ($\chi=5$)