Investigating Post-WWII U.S. Macroeconomic Dynamics with Multiple Filters

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Abstract  
This paper estimates a new-Keynesian DSGE model of the U.S. business cycle by employing a variety of business cycle proxies, either one-by-one or jointly. Objects such as posterior densities, impulse-response functions, and forecast error variance decompositions are shown to be remarkably sensitive to filtering. This uncertainty notwithstanding, shocks to trend inflation are given robust support as the main inflation driver in the post-WWII era. 

Keywords: Filtering, business cycle proxies, new-Keynesian business cycle model, trend inflation, estimated dynamics.  

JEL classification: C32, E32, E37.

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1 Introduction

When willing to take a macroeconomic model of the business cycle to the data, an econometrician has to choose on how to filter the raw data to work with the frequencies of interest. While some researchers employ statistical filters - e.g. Hodrick-Prescott - to accomplish this task, others make assumptions on processes such as technology and preferences in order to detrend the data in a theoretically-consistent manner. Both approaches have pros and cons. Statistical filters are robust to model misspecification, but are somewhat ad hoc - why should one prefer Hodrick-Prescott to linear-detrending? - and may induce biases in the estimated cyclical component (Cogley and Nason (1995)). On the other hand, theoretically-consistent detrending is surely appealing, and logically in line with the employment of micro-founded models, but also obviously prone to biases induced by trend misspecification - what if technology is not a log-difference stationary process? Unfortunately, different filtering choices may lead to dramatically heterogeneous representations of the business cycle (Canova (1998)). Moreover, the misspecification of the trend component in rational expectations models may drastically alter policy functions and equilibrium laws of motions, so calling for an 'adjustment' by the structural parameters to compensate for such distortions when the model is confronted with the data (Cogley (2001)). Then, one may very well wonder how sensitive the results obtained with estimated econometric models are to different filtering.

This paper asks the question ‘How relevant is filtering to the investigation of the post–WWII U.S. macroeconomic dynamics?’ To answer this question, I estimate a new-Keynesian model of the business cycle (NKBC henceforth) with a variety of different proxies of the business cycle to scrutinize the impact of filtering on objects typically investigated by applied macroeconomist. In particular, I aim at assessing the impact of filtering on i) the posterior densities of the parameters of the structural new-Keynesian model I focus on, ii) the impulse response functions to monetary policy shocks, and iii) the contribution of the estimated shocks to macroeconomic volatilities.

The concern for the first object is easy to justify, in the light of the effort made by econometricians to assess the value of key-parameters such as e.g. the slope of the Phillips curve (key to measure the sacrifice ratio), the degree of 'habit formation', the intertemporal elasticity of substitution (that affects the impact of monetary policy

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1 The literature on filtering to construct proxies of the business cycle is extremely vast. For recent discussions on this issue, see Canova (2007) (third chapter), Cogley (2008), and Proietti (forthcoming).

2 Throughout this paper, I will use the terms 'detrending' and 'filtering' interchangeably. In fact, as pointed out by Canova (2007, Chapter 3), 'detrending' refers to the process of making economic series (covariance) stationary, while 'filtering' has a much broader applicability, and refers in general to 'manipulations' operated to the frequencies of the spectrum. In the context of filtering, macroeconomists often focus on frequencies corresponding to - say - 8 to 32 quarters to extrapolate the business cycle component out of macroeconomic time-series.
moves on the demand side of the economy), and the systematic reaction to inflation and output fluctuations by monetary policy authorities. The slope of the Phillips curve and the intertemporal elasticity of substitution are clearly key-determinants to assess the impact of a monetary policy move on inflation and the business cycle. Habit formation also affects the transmission mechanism, and it is an element entering the design of optimal monetary policy (Amato and Laubach (2004)). The interest in the estimation of the monetary policy parameters as well as shocks’ volatility is grounded by the discussion on the drivers of the Great Moderation, i.e. the generalized reduction in the unconditional volatilities of several U.S. macroeconomic series observed since the beginning of the 1980s. Evidence in favor of a shift towards a more aggressive policy stance occurred with the advent of Paul Volcker as Chairman of the Fed has been put forward, among others, by Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), Boivin and Giannoni (2006b). By contrast, some authors stress the role of milder structural shocks as the main driver of the great moderation (Sims and Zha (2006), Smets and Wouters (2007), Justiniano and Primiceri (2008b), Canova, Gambetti, and Pappa (2008)).

Impulse response functions to monetary policy shocks are typically estimated to grasp the quantitative impact that policy surprises may exert on the economy. In undertaking this part of the study, I will distinguish between unexpected monetary policy shifts - i.e. 'standard' monetary policy shocks - and unexpected changes in the inflation target - still a monetary policy shock, but whose origin is conceptually very different. Another important dimension I investigate in this paper is the participation of some structural shocks to the dynamics of inflation and output, an exercise useful to identify the drivers of the post-WWII U.S. economy.

To assess the role played by different filtering strategies, I will first estimate the NKBC model by using different filters one-by-one. Then, I re-do this exercise by employing all the filters jointly as proposed by Canova and Ferroni (2009). Borrowing ideas from the 'estimation in a data-rich environment’ put forward by Boivin and Giannoni (2006a), Canova and Ferroni (2009) elaborate a methodology to estimate business cycle models with different 'contaminated proxies’ of the business cycle, which turns out to be deliver superior results in terms of consistency and efficiency of the estimated model parameters as well as impulse response functions when compared to standard 'single-filter' approaches.

My findings read as follows.

- Different filtering techniques lead to remarkably heterogeneous business cycle proxies in terms of turning points, volatility, and persistence. By contrast, they comove (to some degree) and share low-power when it comes to isolate business cycle frequencies. These findings, obtained with a sample updated to 2008:II,
echo those presented by Canova (1998) and Proietti (forthcoming), and offer solid support to the research question asked in this paper.

- Such heterogeneity induce (in some cases dramatically) disparate posterior densities of the parameters of the small-scale, new-Keynesian model I concentrate upon. In particular, I find a substantial amount of 'proxy-induced uncertainty' surrounding the slope of the Phillips curve, the degree of 'habit formation', the intertemporal elasticity of substitution, the long-run monetary policy response to inflation and output gap oscillations, the persistence and volatility of the structural shocks I consider. These results, conceptually in line with those presented in Canova (2008), Ferroni (2008), and Canova and Ferroni (2009), open the issue of robustness to different filtering-choices as regards the identification of the U.S. macroeconomic dynamics.

- The diversity in the business cycle proxies remarkably affects the estimated impulse response functions to monetary policy shocks. In particular, the responses of the model-consistent 'output gap' to an unexpected move of the federal funds rate and to shocks to the inflation target are clearly proxy-specific, above all when assessed in the Great Moderation period.\(^3\) Interestingly, filter-uncertainty also affects the reaction of inflation and the policy rate to the investigated shocks.

- Filter-induced heterogeneity is also present when looking at the forecast error variance decomposition. However, some commonalities emerge, the most evident being the role of trend inflation shocks for the variance of inflation and the policy rate. This result - above all as regards inflation - line up with recent findings by Kozicki and Tinsley (2005), Cogley and Sbordone (2008), Ireland (2007), Bjørnland, Leitemo, and Maih (2007), Cogley, Primiceri, and Sargent (2009), Castelnuovo, Greco, and Raggi (2008), and Benati (2008b), and corroborate research aiming at understanding the reasons underlying the drifts in the low frequency of inflation observed in the 1970s and 1980s, possibly related to learning by the U.S. monetary policy authorities (Cogley and Sargent (2005b), Primiceri (2006), Sargent, Williams, and Zha (2006), and Carboni and Ellison (2008)).

\(^3\)In this paper I will interpret the empirical proxies of the business cycle as measures of the 'output gap'. Justiniano and Primiceri (2008a) work with a medium-scale DSGE model and show that the degree of adherence of the theoretically relevant 'gap' to statistically detrended output depends on how the 'gap' is defined. In fact, while the gap defined with the counterfactual potential (efficient) output that would prevail under perfect competition closely resembles detrended output, that computed with the counterfactual natural level of output - that would prevail under flexible prices and wages but in presence of inefficiencies due to firms' market power - is extremely more volatile than detrended output. Interestingly, Justiniano and Primiceri (2008a) show that mark up shocks are in fact 'empirically equivalent' to measurement errors, because both capture the very high frequencies of price and wage inflation. Under this alternative interpretation, potential and natural output move one-to-one, and the implied theoretical gaps closely resemble detrended output.
While sharing in part the methodology and well as the modeling assumptions with the authors cited above, my contribution is fundamentally different as regards the object of my investigation, which ultimately aims at understanding how differences in the construction of business cycle proxies may drive conclusions on the U.S. macroeconomic dynamics.

The remainder of the paper is structured as follows. Section 2 proposes the new-Keynesian model I employ for my analysis. Section 3 presents the different measures of the business cycle I work with, and discuss their properties. In Section 4 I discuss some issues on the estimation of the macroeconomic model I focus on, with a particular emphasis on the multiple filters approach. Section 5 presents my findings concerning posterior densities, inflation gaps, impulse response functions, and forecast error variance decompositions. Section 6 defines the contacts with the existing literature. Section 7 concludes.

2 The model with time-varying trend inflation

The model I consider is a new-Keynesian business cycle framework:

\begin{align*}
\pi_t &= \pi_t^* + \beta E_t(\pi_{t+1} - \pi_{t+1}^*) + \kappa x_t + \varepsilon_t^\pi, \\
x_t &= \gamma E_t x_{t+1} + (1 - \gamma)x_{t-1} - \tau(R_t - E_t \pi_{t+1}) + \varepsilon_t^x, \\
R_t &= (1 - \rho_R)(\phi_R(\pi_t - \pi_t^*)) + \phi_x x_t + \phi_R R_{t-1} + \eta_t^R, \\
\pi_t^* &= \rho_s \pi_{t-1}^* + \eta_t^\pi, \\
\varepsilon_t^z &= \rho_z \varepsilon_{t-1}^z + \eta_t^z, z \in \{\pi, x\}; \eta_t^j \sim i.i.d. N(0, \sigma_j^2), j \in \{R, *, \pi, x\}.
\end{align*}

Eq. (1) is an expectational new-Keynesian Phillips curve (NKPC). Such curve dictates the evolution of the inflation rate \(\pi_t\) as a function of the contemporaneous inflation target \(\pi_t^*\), the expected value of the future realization of the inflation gap (the wedge between raw inflation and its target), whose loading is the discount factor \(\beta\), and the output gap \(x_t\), whose influence on the inflation rate is regulated by the slope \(\kappa\). The presence of trend inflation in the NKPC may be rationalized by firms’ full indexation to contemporaneous trend inflation (Woodford (2007)), an assumption empirically corroborated by Ireland (2007).\footnote{In presence of partial indexation, the inflation schedule displays extra terms and interactions between the steady-state inflation level and some structural parameters entering the NKPC. For a recent theoretical analysis, see Ascari and Ropele (2007a). Cogley and Sbordone (2008) and Benati (2008b) tackle this issue from an empirical standpoint.} Goodfriend and King (2008) employ a very similar model to analyze the U.S. inflation drift observed in the 1970s.

The NKPC at hand is purely forward looking. This choice is motivated by the sound evidence pointing towards a zero-weight assigned to past inflation in presence of trend.
inflation (Cogley and Sbordone (2008), Benati (2008b)). Moreover, indexation to past inflation is hardly structural (Benati (2008a), Benati (2008b)). Consequently, I refrain from modeling inflation persistence by means of any indexation scheme.

The IS eq. (2) describes the evolution of the cyclical component of the real GDP, which is a function of expected and past values - weighted by $\gamma$ - as well as by the ex-ante real interest rate, the latter loaded by the intertemporal elasticity of substitution $\tau$. Strictly speaking, $\gamma$ is a transformation involving the degree of habit formation of the representative agent, and $\tau$ a transformation involving the degree of relative risk aversion and that of habit formation. More precisely, it is possible to show that $\gamma \equiv 1/(1 + h)$, where $h$ is the degree of ‘habit formation’, and $\tau \equiv h/\left[\sigma(1 + h)\right]$, with $\sigma$ being the relative risk aversion of the representative consumer. Notice that the unitary upper bound naturally associated to $h$ induces a lower bound for $\gamma$ reading $1/2$, and strongly influences $\tau$. However, in a recent contribution, Fuhrer and Rudebusch (2004) support backward-looking versions of the U.S. IS equation. Moreover, their estimates of $\tau$ are hardly consistent with those emerging after the imposition of the cross-equation restrictions listed before. I then estimate $\gamma$ and $\tau$ as free parameters, so allowing for a very flexible structure of aggregate demand.

Eq. (3) is a Taylor rule that suggests a gradual response by the Fed to oscillations of the gaps in inflation and output. Trend inflation (4) is assumed to follow a perfectly observed autoregressive process (with unconditional mean normalized to zero), an assumption I share with a variety of previous studies (Cogley and Sargent (2005a), Ireland (2007), Woodford (2007), Goodfriend and King (2008), and Cogley, Primiceri, and Sargent (2009)). When setting $\sigma^2 = 0$, i.e. when assuming a constant inflation

\footnote{To be precise, Benati (2008a) proposes a very extensive analysis involving ten OECD countries and the Euro Area aggregate. He shows that under stable regimes with clearly defined nominal anchors (U.K., Canada, Sweden, New Zealand, Switzerland under inflation targeting, the Euro Area under the European Monetary Union), inflation can be modeled with a purely forward looking NKPC. His findings point against the notion of price indexation being a structural parameter. The United States have not officially adopted an inflation targeting monetary policy strategy. However, several contributions have supported the shift towards a more aggressive monetary policy at the end of the 1970s. The association of a lower value for the U.S. price indexation parameter in the Great Moderation subsample to a more aggressive systematic monetary policy by the Federal Reserve Bank is conceptually in line with Benati’s (2008a) position on the indexation parameter being a reduced-form one.}

\footnote{Given that the Fed has never officially announced its inflation target, this is a somewhat problematic assumption. As pointed out by Walsh (2008), misperceptions of the inflation target by the private sector may de facto be interpreted as inflationary shocks by an econometrician who assumes trend inflation to be perfectly known. To tackle this issue, one should model the signal-extraction problem faced by the private sector, or in general allow for a learning process over the inflation target. I plan to tackle this issue in the future. For a theoretical analysis along this line with a calibrated model of the business cycle, see Erceg and Levin (2003). Kozicki and Tinsley (2005) embed an imperfectly perceived inflation target in a VAR framework. Milani (2006) considers rational expectations vs. learning in a small scale model similar to the one employed in this paper and compares the estimates of trend inflation obtained under either scenarios.}
target, this model collapses to standard AS/AS model of the kind recently employed in empirical analysis (Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), Lubik and Surico (forthcoming), Boivin and Giannoni (2006b), Benati and Surico (2008), Benati (2008a)). Notice that the time-varying inflation target process (4) is allowed to exert an impact on the economy along two dimensions: directly, given its presence in the NKPC, and indirectly, through the movements it induces in the policy rate and, consequently, in the business cycle.

Standard assumptions on the stochastic processes (5) close the model.

3 Different business cycle proxies: A comparison

How to approximate the model-consistent business cycle measure $x_t$, which enters eqs. (1)-(3)? To answer this question, one has to extract the cyclical component from the real-GDP raw time series. I concentrate on six different trends, very popular among macroeconomists. First, I consider the measure of potential output provided by the Congressional Budget Office, which employs a production-function approach to compute a measure of sustainable output.\(^7\) I employ such a measure to filter low-frequency movements of the real GDP out of the raw series, and I label this empirical proxy 'CBO'. The second transformation is obtained by applying the popular Hodrick-Prescott ('HP') filter with standard weight 1,600. The third transformation fits a linear trend to the raw series without allowing for any break in the sample ('LIN'). By contrast, the fourth manipulation ('LBR') fits a linear trend and allows for a break in both the constant and the slope parameter in 1980:III as in Canova and Ferroni (2009). An alternative I pursue is the application of the Baxter and King (1994) band-pass filter ('BP') so to extract cycles within the [8,32] quarters periodicity, with 12 quarters left as leads/lags. Finally, I take the growth rate of the raw series ('FD') as proxy of the GDP’s cyclical component, a choice that relies upon the random walk with drift as a model for the real GDP trend.\(^8\) I perform all these transformations by considering the sample 1954:III-2008:II, a sample longer than the one I employ to estimate the NKBC model. This choice’s aim is that of tackling initial-condition issues concerning some of the filters at hand.

Figure 1 - left column displays the business cycle empirical proxies obtained with the six filters described above. One may spot similarities and differences across these proxies. Some comments are in order. First, ‘eyeball econometrics’ suggests a positive correlation across proxies, which is also confirmed by the figures reported in Table 1.

\(^7\)A detailed explanation on the computation of the CBO potential output may be found at http://www.cbo.gov/ftpdocs/30xx/doc3020/PotentialOutput.pdf.

\(^8\)The filters I consider are very widely employed in the macroeconomic literature. However, the list one may think of is very large. Canova (1998), Canova (2007, Chapter 3), and Proietti (forthcoming) offer a thorough description on a wider variety of filters.
However, such correlation varies - in some cases, dramatically - when moving from a pair to another. The highest correlation - 0.94 - regards the pair (HP,BP), while the lowest - 0.10 - involve (LIN,FD). In general, FD is poorly correlated with the rest of the business cycle indicators. This is due to somewhat erratic behavior displayed by this proxy, which also signals shorter cycles with respect to alternatives. The different proxies under investigation display a relevant amount of heterogeneity also in terms of business cycle dating. Taking the NBER recessions (identified by the grey bars in Figure 1) as reference, one may observe that CBO and HP perform reasonably well. By contrast, LIN just misses to capture the 1969:IV-1970:IV, 1973:IV-1975:I, and 1980:I-1980:III recessions, which are considered as simple slowdowns - i.e. realizations of decreasing but positive output gaps, while LBR shows a somewhat better ability in matching such recessions. Still sticking to the dating issue, FD shows the worst performance, with no clear indication of any particular recession, with the exception of the early 1980s one, indeed caught by all the proxies at hand. The magnitude of booms and busts is clearly filter-dependent, with some filters - e.g. LIN - possibly magnifying the deviations with respect to ‘potential’ output and others - e.g. FD - dampening them. Table 1 confirms the high volatility in terms of estimated variance of the cyclical component of output. The highest figure - 11.61 - is associated to the LIN filtered proxy, whose variance is much larger than those of the widely employed CBO and HP - respectively 5.76 and 2.90 - and definitely greater than the one of the real GDP growth rate, with the ratio between the two being close to sixteen! Interestingly, when allowing for a break in the trend coefficients, the variance of linearly detrended business cycle proxy drops of about 40%, so getting much closer to those of HP and CBO. The FD indicator returns the lowest variance - 0.73, and the BP filter induces the second lowest variance - 1.68.

Such heterogeneity is also reflected by the AutoCorrelation Functions depicted in Figure 1 - middle panel. In terms of autocovariance structure, a very different story is told by filters like HP and BP when contrasted to FD, with the latter showing a very quick drop of persistence after a few lags and a mild oscillatory behavior around zero thereafter, while the former display higher persistence and wide oscillations over the twenty-five lags considered. Accounting for the break in the linear trend induces a switch of the sign for most of the autocovariances of LBR with respect to LIN. Table 1-last row, however, suggests that the estimated persistence of the business cycle is very high, with the exception of the FD manipulation. Figure 1 - right panel reveals also a common feature of the log-Spectra of the proxies at hand, i.e. the significant errors concerning the frequencies retained by our proxies. Ideally, business cycle indicators should retain frequencies corresponding to the range 8 to 32 quarters (identified in the normalized frequency domain in the Figure by the vertical black dotted bars). Notably, the log-Spectra reveal that all our proxies attribute an excessive power both to low-
frequencies and to very-high frequencies. In general, problems of leakage (loss of power at the edges of the business cycle frequency band) and compression (increase of power in the middle of the band) seem to be a common element of empirical proxies of the business cycle, a commonality also stressed by (Canova (1998), Canova (2007), Chapter 3), Canova (2008), Proietti (forthcoming), and Canova and Ferroni (2009)).

In short, commonly applied filters tend to comove but are very heterogeneous across some dimensions - dating, magnitude, average length, and persistence of the business cycle. Indeed, they share errors as regards the extraction of the frequencies of interest from raw data. Which are the implications in terms of model estimation and policy evaluation? The next Sections shed some light on these issues.

4 Model estimation with multiple filters

To appreciate to what extent detrending may affect estimation, I first employ all the proxies scrutinized in the previous Section one-by-one. Then, I will perform estimations by considering the different proxies jointly. Canova and Ferroni (2009) point out that this procedure has three main advantages. First, it does not require the researcher to take a strong a-priori stand on how to model the trend and the shocks driving it. Given the uncertainty surrounding the evolution of factors like technology and preferences, the fact of being able to remain agnostic on which filter to use is likely to work towards the reduction of biases due to trend misspecification. Second, this methodology allows to employ cyclical data computed with filters having very different features, e.g. one vs. two-sided filters, univariate vs. multivariate, deterministic vs. stochastic, and so on, so making parameter estimates more robust to filter misspecification. Third, errors in the attribution of the business cycle frequencies are proxy-specific. If such errors display a somewhat common pattern across proxies, the joint employment of different empirical indicators of the business cycle should reduce small sample biases in parameters estimates. If such errors are more idiosyncratic, this estimation procedure should wash them out so delivering more precise estimates. Canova and Ferroni’s (2009) Monte Carlo exercises confirm that the joint employment of multiple filters reduce the biases of the estimated parameters as well as impulse response functions.

To estimate the model (1)-(5), I therefore set up the following measurement equation:

\[ \text{log-Spectra is computed with the 'pwelch' Matlab function. A Bartlett window kernel of size 21 was employed to smooth the periodogram and obtain a consistent estimation of the spectra.} \]
\[
\begin{bmatrix}
FFRATE_t \\
INFLGDP_t \\
\tilde{x}_{1t} \\
\vdots \\
\tilde{x}_{Nt}
\end{bmatrix}
= \begin{bmatrix}
\bar{\pi} \\
\pi \\
\vdots \\
\pi
\end{bmatrix}
\begin{bmatrix}
\tilde{\pi}_{1t} \\
\tilde{\pi}_{2t} \\
\vdots \\
\tilde{\pi}_{Nt}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\begin{bmatrix}
\lambda_1 & \ldots & 0 \\
0 & \ddots & 0 \\
0 & \ldots & \lambda_N
\end{bmatrix}
\begin{bmatrix}
x_t \\
\vdots \\
x_t
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\begin{bmatrix}
u_{1t} \\
\vdots \\
u_{Nt}
\end{bmatrix}
\]

where \(FFRATE_t\) is the quarterly federal funds rate at time \(t\), \(INFLGDP_t\) is the quarterly GDP deflator inflation, \(\tilde{x}_t = [\tilde{x}_{1t}, \ldots, \tilde{x}_{Nt}]'\) is the \((N \times 1)\) vector of empirical proxies of the business cycle computed with \(N\) different approximations of the trend, \(\bar{x} = [\bar{x}_{1t}, \ldots, \bar{x}_{Nt}]'\) is a \((N \times 1)\) vector of proxy-idiosyncratic constants to demean the filters, \(\lambda\) is a \((N \times N)\) diagonal matrix of 'loadings' relating the model consistent cyclical component \(x_t\) to the \(N\) empirical proxies \(\tilde{x}_t\), and \(u_t = [u_{1t}, \ldots, u_{Nt}]'\) \(\sim i.i.d. (0_{N \times 1}, \text{diag}(\sigma^2_{u1}, \ldots, \sigma^2_{uN}))\) is a \((N \times 1)\) vector of serially and mutually uncorrelated filter-specific measurement errors. If a given filtered measure \(\tilde{x}_{nt}\), \(n \in \{1, \ldots, N\}\) represented exactly the model consistent business cycle, one should expect the slope \(\lambda_n\) to be statistically equal to one, and \(\sigma^2_{un}\) to be statistically equal to zero. Then, the slope parameters indicate the weights assigned by the data to the 'signals' delivered by a given filter as regards the cyclical component of real GDP when contrasted with the model-consistent business cycle indicator. The associated measurement errors indicate the uncertainty surrounding such signals. When implementing the multiple filter strategy, I normalize \(\lambda_{CBO} = 1\), and I interpret the remaining \(\lambda_{ys}\) as relative loadings with respect to the first one. By contrast, when estimating the model with a single proxy, the measurement equation (6) is featured by \(N = 1\), \(\tilde{x}_t = [\tilde{x}_{nt}]'\), \(u_t = [u_{nt}]'\) only, and the restriction \(\lambda = [\lambda_n]' = 1\) is imposed. A measurement error to the business cycle equation in (6) is allowed also when a single proxy is employed.

Notice that pre-filtering is applied neither to inflation nor to the federal funds rate. In the model at hand, inflation is filtered by the trend inflation process (4), which allows to construct a model consistent inflation gap measure. As for the federal funds rate, the absence of pre-filtering enables a consistent comparison of the results of this paper to those offered by previous contributions, which typically do not perform any manipulation on the raw nominal rate.

**Bayesian estimation**

I perform econometric estimations by relying upon Bayesian techniques. Bayesian techniques are by now standard in the literature (see An and Schorfheide (2007) and Fernandez-Villaverde (2009) for detailed reviews, and Canova and Sala (forthcoming) for a discussion of the pros and cons of this methodology vs. alternatives). I then need to set priors so to augment the likelihood of the model with some a-priori knowledge.
Following Cogley, Primiceri, and Sargent (2009), I set the autoregressive parameter $\rho_s$ of the inflation target process (4) to 0.995 to capture low-frequency movements in inflation. Consequently, the zero-frequency of the inflation process is almost entirely explained by shocks to trend inflation. This does not necessarily imply, however, that shocks to trend inflation are, by construction, the main driver of the short-run volatility of inflation. By contrast, the contribution of the standard monetary policy shock for the zero-frequency of the inflation rate is negligible. This difference enhances the identification of the two monetary policy shocks.\footnote{Notice that the presence of the inflation target process in the NKPC (1) would enable the model to handle a unit-root in the trend inflation process (4). However, when experimenting with $\rho_s = 1$, I verified i) a dramatic drop of the impact of the trend inflation shock on the zero-frequency of the inflation rate, and ii) a large increase in the portion of the inflation variance explained by the standard monetary policy shock, which got much closer to the explained variance due to the trend inflation shock. Given that modeling trend inflation as a unit root might hinder the identification of the two monetary policy shocks, I decided against it and in favor of $\rho_s = 0.995$.}

The remaining priors are fairly standard, and roughly in line with Benati and Surico (2008), Benati (2008a), Benati (2008b), and Cogley, Primiceri, and Sargent (2009) as regards the parameters in common between their models and the one under investigation. In particular, I aim to be relatively uninformative as far as the persistence parameters are concerned, and I allow the domain of the volatilities of the model to be wide enough to let the data free to indicate the relative impact of the various shocks on the U.S. economic system. As it is customary in the literature, I calibrate the discount factor $\beta$ to 0.99. Table 2 reports the priors for the structural parameters employed in the estimation phase. Finally, I assume the loadings of the empirical proxies to be independently distributed as $\lambda_i \sim N(1,0.5)$. Measurement errors are also assumed to be independently distributed and follow $u_{it} \sim \text{Inverse Gamma}(0.25,2)$.\footnote{The figures reported in brackets refer to the mean and standard deviation of the distributions of interest.}

I use as raw data the U.S. GDP deflator, log-real GDP, and federal funds rate (average of monthly observations), all downloaded from the website of the Federal Reserve System.\footnote{URL: http://research.stlouisfed.org/fred2/} In line with e.g. Cogley, Primiceri, and Sargent (2009), I consider the following two subsamples: 1960:I-1979:II, which corresponds to the ‘Great Inflation’ period before the appointment of Paul Volcker as Fed’s chairman, and 1982:IV-2008:II, which corresponds to the post-‘Volcker experiment’/‘Great Moderation’ sample.\footnote{To perform Bayesian estimation I employed Dynare 4.0, a set of algorithms - developed by Michel Juillard and collaborators - freely available at http://www.cepremap.cnrs.fr/dynare/. The mode of each parameter’s posterior distribution was computed by using the ‘csminwel’ algorithm elaborated by Chris Sims. A check of the posterior mode, performed by plotting the posterior density for values around the computed mode for each estimated parameter in turn, confirmed the goodness of the optimizations. I employed such modes to initialize the random walk Metropolis-Hastings algorithm to simulate the posterior distributions. The inverse of the Hessian of the posterior distribution evaluated at the posterior mode was used to define the variance-covariance matrix of the chain. The initial VCV}
log-real GDP is filtered and all series demeaned prior to estimation.\textsuperscript{14}

5 Empirical results

Tables 3 and 4 collect figures concerning the posterior distributions of some representative models, and Figure 2 plots the densities of all estimated models across the two subsamples. Several comments are in order. First, all the posterior medians appear to be economically sensible. Second, and more importantly for this study, one may spot striking differences across models as far as some of the key parameters are concerned. In particular, the intertemporal elasticity of substitution, the extent to which agents are forward-looking in the IS curve, and the long-run reaction of the Fed to inflation and output gap fluctuations appear to be proxy-specific. In the light of the fact that counterfactuals are typically run by relying on such densities or, often, by conditioning on their means/medians, one may very well wonder how reliable the conclusions drawn on the basis of such exercises should be considered in the light of the just documented proxy-induced uncertainty.

Comfortably, there are also similarities across the estimated models. The long-run reaction to inflation gap oscillations increase when moving to the Great Moderation subsample, even if not necessarily so in a statistical sense. This finding is in line with Cogley, Primiceri, and Sargent (2009), and resembles the one proposed by Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), Boivin and Giannoni (2006b). Notice that, as in Cogley, Primiceri, and Sargent (2009), the difference between the systematic reactions to inflation gap fluctuations in the two subsamples is not large. This might be due to the imposition on equilibrium uniqueness in the estimation phase.\textsuperscript{15} Another possible explanation could be the different object at hand, i.e. inflation gap in this matrix of the forecast errors in the Kalman filter was set to be equal to the unconditional variance of the state variables. I used the steady-state of the model to initialize the state vector in the Kalman filter. I simulated two chains of 200,000 draws each, and discarded the first 75\% as burn-in. To scale the variance-covariance matrix of the random walk chain I used a factor so to achieve an acceptance rate belonging to the [23\%, 40\%] range. To assess the stationarity of the chains, I considered the convergence checks proposed by Brooks and Gelman (1998). I conditioned the estimation of the model to the unique-solution parameter region.

\textsuperscript{14}I also verified the robustness of the results presented in the paper to estimations with undemeaned data and constants in the measurement equations.

\textsuperscript{15}The debate on the evidence in favor of an indeterminate equilibrium in the pre-Volcker subsample is very lively. On the one hand, Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), and Boivin and Giannoni (2006b) lends support to indeterminacy. Castelnuovo and Surico (2008) show that indeterminacy may offer a rationale for the price puzzle typically found when estimating the effects of a monetary policy shocks with VAR models. Surico (2006) discusses the perils coming from merging two subsamples characterized by different equilibria when conducting empirical exercises on NKPCs. By contrast, Sims and Zha (2006), Justiniano and Primiceri (2008b), and Cogley, Primiceri, and Sargent (2009) cast doubts on multiple equilibria as a relevant feature to describe the economic situation in the 1960s and 1970s.
study vs. raw inflation in Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), Boivin and Giannoni (2006b).\footnote{For preliminary investigations on this issue, see Castelnuovo, Greco, and Raggi (2008).}

A robust result I obtain is the generalized reduction of the volatilities of the structural shocks in the second subsample, a finding that captures in first approximation the evidence put forward by Justiniano and Primiceri (2008b) with a framework allowing for time-varying conditional volatilities. Notably, the variance of the inflation target shock is lower in the second subsample. This finding, which I share with Stock and Watson (2007) and Cogley, Primiceri, and Sargent (2009), candidates the reduction in trend inflation volatility as one of the possible drivers of the Great Moderation.\footnote{The drop of the trend inflation volatility shock may at a first glance appear to be negligible. However, one must bear in mind that the autoregressive root \( \rho_s = 0.995 \), then a drop of \( \varepsilon \) in the volatility of the trend inflation shock translates into a reduction of about \( \frac{1}{1-0.995^2} \approx 100\varepsilon \) in the volatility of the trend inflation process.} Notice also that the IES is not stable across subsamples. This is somewhat unfortunate, given that I aim at performing counterfactuals that assume that the parameters of the model are structural in the sense of Lucas, i.e. policy-independent. I assume that the estimated decrease of IES may be due to variations in the degree of habit formation/relative risk aversion unrelated to the change in the systematic U.S. monetary policy. Then, when conducting my simulation exercise, I will treat such parameter as policy-independent.\footnote{I refrain from providing comparisons on the single versus multiple-filter approach based on forecasting exercises. As already commented, given that I work with two-sided filters, a forecasting exercise would violate the information set of the rational agent at time \( t \).}

### 5.1 Comparison with the standard price indexation model

Before moving to remaining empirical findings, it is worth scrutinizing how the model I focus on performs with respect to the standard price-indexation model displaying no trend inflation. To do so, I consider the version recently scrutinized by e.g. Benati and Surico (2008) and Benati (2008a), which is featured by the following NKPC and Taylor rule:

\[
\begin{align*}
\pi_t &= \frac{\beta}{1 + \alpha \beta} E_t \pi_{t+1} + \frac{\alpha}{1 + \alpha \beta} \pi_{t-1} + \kappa x_t + \varepsilon_t, \quad (7) \\
R_t &= (1 - \phi_R) (\phi_x \pi_t + \phi_x x_t) + \phi_R R_{t-1} + \eta_t^R. \quad (8)
\end{align*}
\]

Eq. (7) displays the parameter \( \alpha \), which identifies non-reoptimizing firms’ indexation to past inflation. Benati (2008a) engages in an extensive investigation with data of the main industrialized countries, and rejects the ‘structural’ interpretation of this parameter. In fact, such a parameter turns out to be unstable when moving across different monetary policy regimes. Then, a model modeling indexation might very likely be ill-suited to perform policy analysis, above all counterfactuals.
Eq. (8) is a standard Taylor rule postulating a systematic reaction to inflation oscillations by the Fed. In a constant-inflation target world, inflation and inflation gap are coincident objects. However, as already mentioned, (time-varying) trend inflation has been empirically supported as one of the relevant features of the post-WWII U.S. economy (Cogley and Sargent (2005a), Ireland (2007), Castelnuovo, Greco, and Raggi (2008), and Cogley, Primiceri, and Sargent (2009)). If this is the case, a standard Taylor rule with a constant inflation target is likely to offer a misspecified representation of the U.S. monetary policy conduct.

To engage in a formal comparison between my benchmark (NKBC) model (1)-(5) model and the alternative indexation (IND) framework displaying no trend inflation, I estimate also the model composed by equations (2), (5), (7), (8) with different proxies of the business cycle. In so doing, I follow Benati (2008a) and model the inflation shifter $\varepsilon_t$ as a white noise, so giving the indexation parameter $\alpha$ the highest chance of grasping the U.S. inflation persistence. Tables 3 and 4 collect the (log) Marginal Likelihoods of the NKBC and IND models (last two rows).¹⁹ Some comments are in order. First, the NKBC model with trend inflation is clearly preferred in four of the five comparisons reported in Table 3 - Great Inflation. Indeed, price indexation acts as an imperfect ‘substitute’ of the time-varying inflation target. This result squares up with those put forward by Cogley and Sbordone (2008) and Benati (2008b), which favor a new-Keynesian Phillips curve formulation with trend inflation and no indexation when contrasted to a model endowed with price indexation. The only exception is represented by the FD scenario, which supports instead the IND model. This is possibly due to the low volatility of the FD cyclical component, which may perhaps capture some of the low frequencies in inflation otherwise caught by trend inflation. Notably, the posterior odd clearly favors the NKBC model with multiple filters MF are considered.

A somewhat different picture arises when considering the Great Moderation subsample. In this latter case, three models out of five - HP, LIN, FD - support the price indexation model. This is perhaps not too surprising: the Great Moderation period is characterized by low and fairly stable inflation, and the role of trend inflation for the dynamics of raw inflation is inferior with respect to that played to track it in the 1970s. However, the CBO filter and the MF panel of filters still support the benchmark NKBC model. Overall, when considering both subsamples, the trend inflation model appears to be preferable. Moreover, it is naturally suited to investigate the role that trend inflation shocks have played for the post-WWII U.S. economy. Then, in the reminder of the paper I will exclusively focus on such a model.²⁰

¹⁹Preliminary attempts to estimate the IND model with the priors reported in Table 2 failed due to the difficulty of computing posterior modes of the models at hand. I verified that a smooth convergence was instead possible by manipulating the prior mean of the slope of the NKPC. The estimations of the IND model are then conditional on $\kappa \sim \text{Gamma}(0.035, 0.01)$.

²⁰Notice that I cannot discriminate across filters on the basis of the Marginal Likelihood. This is
5.2 Impulse Response Functions

Figure 3 displays the impulse response functions to the two monetary policy shocks affecting the system, the ‘traditional’ shock to the nominal interest rate in the Taylor rule (3) and the shock to the trend inflation process (4). In all cases, the reactions have the expected sign. A monetary policy tightening induces an increase in the policy rate as well as the real interest rate, a decrease in the output gap, and a demand-driven deflation. A positive trend inflation shock triggers a take-off in inflation and calls for a monetary policy tightening. Given that policymakers react with gradualism, the real interest rate takes negative values in the short run, which leads to a temporary expansion. These reactions are qualitatively in line with those put forward by Ireland (2007) and Cogley, Primiceri, and Sargent (2009).

However, while the dynamics of the system are qualitatively clear, the situation is quite shaded when seen from the quantitative angle.\textsuperscript{21} In fact, the business cycle reaction to both shocks in both subsamples is very heterogeneous. To fix this concept, I compute the percentage deviations of each estimated reaction with respect to the MF filter, which I take as reference. Table 6 collects the figures regarding the 4 and 8-quarter ahead percentage deviations. A strikingly heterogeneity arises. As regards the standard policy shock, figures related to the 4-quarter horizon range from the zero deviation suggested by the CBO filter to the 50\% of FD, with HP and LIN associated to a deviation of about 30\% under the Great Inflation sample. Interestingly, when accounting for the break, the linear trend lines up (in terms of deviations) to the CBO trend. Figures are somewhat magnified under the Great Moderation sample, with FD’s deviation reading 75\%. 8-quarter ahead predictions suggest larger figures for all the filters but BP under the Great Moderation.

Filter-uncertainly clearly affects also the estimated reaction of inflation to a standard monetary policy shock. Again, CBO suggests milder deviations when contrasted to HP and LIN, and LBR somewhat dampens the effects induced by LIN. Notably, the widely employed HP filter is associated to a percentage deviation of about 40\% (8-quarter ahead), a very large percentage indeed. The growth rate, once more, turns out to be the filter departing the most with respect to the MF ’weighted average’, with figures over 80\%.

Also trend inflation shocks trigger quantitatively very different business cycle responses. Under Great Inflation, the 4-quarter ahead output gap reaction to a trend inflation shock inducing a 1\% on impact hike in the inflation rate reads 17\%, 34\%, due to the procedure at hand, which implies a different data set for each estimated model. Differently, Ferroni (2008) filters raw data and estimates the DSGE cyclical model jointly, i.e. in a single-step fashion.

\textsuperscript{21} Credible sets (confidence bands) are intentionally not displayed. The point here is that of assessing the heterogeneity due to the filtering choice, and not the sample uncertainty surrounding objects like impulse responses or forecast error variance decompositions.
and 44% when - respectively - HP, LIN, and FD filters are considered under the Great Inflation, and even larger under the Great Moderation, with FD’s departures peaking 75% in the 8-quarter ahead scenario. Interestingly, inflation reactions turn out to be much more homogeneous, with the highest deviation being 5.78% (8-quarter ahead). This might be due to the role played by the direct impact exerted by trend inflation on the inflation rate via NKPC (1).

5.3 Forecast Error Variance Decomposition

To gauge some sense on the quantitative relevance that the filtering choice may have when conducting variance decomposition analysis, Table 7 collects percentage deviations of the filter-specific contributions of the two monetary policy shocks on inflation and output with respect to the one associated to MF. Some common patterns with the previously analyzed filter-specific impulse response function arise. In fact, one may notice that HP, LIN, and FD suggest decompositions that are percentually very different with respect to the one proposed by MF, both when looking at 16-quarter ahead and when going for the ‘long run’ - 40-quarter ahead. Percentage deviations are relatively less important in the case of inflation under the Great Inflation period for most of the filters but HP and LIN. Again, accounting for the break in the linear trend lines remarkably dampens the departures from MF, which are anyhow still present. While the standard monetary policy shock is subject to a very large amount of filter-induced uncertainty, that surrounding the contribution of trend inflation shocks for the inflation process is much lower. Indeed, the highest departure is that of LIN under the Great Moderation - about 35%, i.e. a large figure but somewhat milder than other deviations, a chief example being the contribution of the standard monetary policy shock for output under the Great Inflation sample - 340%! To say that the large contribution assigned to trend inflation shocks by all filters as regards inflation volatility is a very robust fact. This finding lines up with recent research - e.g. Ireland (2007), Cogley, Primiceri, and Sargent (2009) pointing towards trend inflation shocks as one of the main inflation driver of the post-WWII U.S. period, and supports studies aiming at understanding the reasons behind trend inflation, one of the possible reasons being learning of the structure of the economy by the U.S. monetary policy authorities (Cogley and Sargent (2005b), Primiceri (2006), Sargent, Williams, and Zha (2006), and Carboni and Ellison (2008)).

Wrapping up, the evidence presented above clearly point towards a marked heterogeneity concerning the quantification of the one may certainly conclude that filtering is a very relevant empirical issue in the context of standard macroeconometric investigations.
5.4 Robustness checks

I ran some checks to verify the solidity of the previously discussed empirical findings. In particular,

- I re-estimated the model with multiple filters by dropping the ‘FD’ filter, which appears to be an outlier when contrasted to the other filters at hand. This is not ‘bad’ \textit{per se}. Indeed, the information content of the ‘FD’ business cycle proxy is weighted ‘endogenously’ via the estimated $\lambda_{FD}$ per each subsample, and its precision is assessed via its period-specific measurement error variance. However, to be sure such particular filter is not driving the results in any important manner, I undertook the estimation of the model with the remaining five filters and re-plotted IRFs and FEVDs. The results presented above are basically unaltered.

- I re-estimated all the models but ‘CBO’ by filtering inflation and federal funds rate along with the real GDP. E.g., the model ‘HP’ has been estimated with HP filtered log-real GDP, GDP deflator inflation, and federal funds rate (the same holds for the other filters). Figures 5-7 collect our results.\footnote{The LBR filter is not considered because of the multicollinearity induced by such trend in the estimation of the MF framework.} The main conclusion of this paper, i.e. the pervasive heterogeneity induced by different filterings as regards IRFs and FEVDs, is unaffected. Interestingly, the role played by trend inflation shocks is mildened according to some filters, but some evidence in favor or its role is still provided by e.g. the HP filter. However, one should take this last result with care. Indeed, the structural model already displays a filter for raw inflation (and, indirectly, the policy rate), which is the trend inflation process. Then, the remarkable reduction in the importance of trend inflation shocks may very well due to ‘over-detrending’ that is typically \textit{not} applied when assessing the role of trend inflation shocks for macroeconomic volatilities (see e.g. Ireland (2007) and Cogley, Primiceri, and Sargent (2009)).

- I checked the sensitivity of my results to the implemented demeaning strategy. When re-estimating the models with undemeaned data and constants allowed to take up the sample mean of the ‘observables’, results turned out to be unchanged.\footnote{These robustness checks, not shown for the sake of brevity, are available upon request.}

6 Contacts with the literature

This paper is closely related to some recent contributions regarding filtering and the estimation of DSGE models on the one hand, and the relevance of trend inflation on the other. As anticipated in the Introduction, Canova and Ferroni (2009) propose a
methodology to jointly deal with different contaminated proxies of the cyclical component of the variables of interest when taking the model to the data. They perform a Monte Carlo analysis in order to study the properties of their proposal, and show that the joint employment of different filters returns estimated parameters and impulse responses much more precise than those obtained with a standard single-filter approach. Then, they take a new-Keynesian business cycle model of the business cycle to the data, and show that money enters significantly both the inflation schedule and the aggregate demand equation. The Fed is also shown to have systematically reacted to oscillations in the growth rate of money. While dealing with multiple filters, my paper focuses on different objects, i.e. ultimately the filter-induced heterogeneity concerning the conditional reaction of inflation and output to two different monetary policy shocks and the participation of identified structural shocks to the U.S. macroeconomic volatility.

Cogley (2001) suggests to estimate the model with GMM techniques before solving the Euler equations for rational expectations, so to avoid to specify the driving processes at the estimation stage. Instead, I stick to the 'first solve, then estimate' sequence typically called for by likelihood-based estimation techniques, also in the light of Fuhrer and Rudebusch (2004), who show that likelihood-based estimations of the Euler equation for output are superior to GMM due to (typically) weak-instruments.

Related papers are Ferroni (2008) and Canova (2008). Ferroni (2008) contrasts the standard 'first filter, then estimate' two-stage approach with a novel 'jointly filter and estimate' one-step strategy. The novelty hinges upon the joint estimation of trend and structural parameters. Importantly, this strategy allows a researcher to exploit the cross-equation restrictions of the DSGE model when performing the trend-cycle decomposition, to compare the descriptive ability of different filters, and to employ the resulting information to construct robust estimates via Bayesian averaging. Ferroni's (2008) 'trend agnostic' methodology turns out to be more consistent than alternatives also in case of model misspecification. He also estimates a standard AD/AS model with U.S. data and show that different filters may indeed induce different estimates of the parameters/moments of interest. Differently, Canova (2008) also proposes a 'single step' methodology that allows for a flexible link between unfiltered raw data and the theoretical model at hand, and in which cyclical and non-cyclical components are allowed to have power in all the frequencies of the spectrum. Simulations performed by the author show that standard data transformations induce distortions in structural estimates and policy conclusions that are drastically reduced when applying his methodology. With respect to these contributions, I undertake a more conventional 'two-stage strategy' to highlight the consequences of detrending in the context of a modern monetary policy model of the business cycle that embeds, among others, trend inflation shocks, i.e. possibly one of the main drivers of the great moderation in inflation (Cogley, Primiceri, and Sargent (2009)). Moreover, in a subset of my estimations some flexibility of the
trending process is allowed, in that I jointly consider a variety of differently filtered business cycle representations and let the data speak about their relative weights.

From a more exquisitely economic standpoint, my contributions intersects those concerned with the modeling of the U.S. inflation and output. One of the main features of inflation is its persistence, which has often been modeled via somewhat ad hoc indexation mechanisms. Going against this tendency, Cogley and Sbordone (2008) and Benati (2008b) show that, once trend inflation is embedded in the new-Keynesian Phillips curve describing the evolution of the U.S. inflation, price indexation is statistically not significant. The remarkable evidence supporting the hypothesis of a time-varying inflation target pursued by the Fed (Cogley and Sargent (2001), Cogley and Sargent (2005a), Ireland (2007), Bjørnland, Leitemo, and Maih (2007), Stock and Watson (2007), Cogley, Primiceri, and Sargent (2009), Castelnuovo, Greco, and Raggi (2008), and the two previously mentioned papers) motivates my choice of working with a model in which trend inflation is allowed to play an active role in shaping the inflation process. Theoretical underpinnings for the model I employ are discussed in Woodford (2007) and Goodfriend and King (2008).

7 Conclusions

This paper has estimated a new-Keynesian model of the business cycle with single and - following Canova and Ferroni (2009) - multiple filters to assess the role that filtering choices may play as regards objects of interest such as posterior densities, impulse response functions, and forecast-error variance decompositions.

My findings read as follows. The different proxies of the 'output gap', widely employed in the applied macroeconomic literature, are remarkably heterogeneous in terms of turning points, volatility, and persistence, and share low-power when it comes to isolate business cycle frequencies. When employed to estimate the business cycle model I focus on to the data, I found that the filter-induced uncertainty surrounding the values of some key parameters - slope of the Phillips curve, degree of intertemporal elasticity of substitution, Taylor rule parameters, persistence and volatility of the structural shocks - is substantial. This uncertainty affects objects typically investigated by applied macroeconomists such as impulse response functions to a monetary policy shock and variance decompositions. These results, conceptually in line with those presented in Canova (2008), Ferroni (2008), and Canova and Ferroni (2009), open the issue of robustness to different filtering-choices as regards the drivers of the U.S. macroeconomic dynamics.

This uncertainty notwithstanding, a very robust result stand out. Shocks to trend inflation turn out to be the main driver of the post-WWII U.S. inflation. This result squares up with recent findings by Ireland (2007) and Cogley, Primiceri, and Sargent
2009), and lends support to research scrutinizing the reasons underlying such the time-variation of the inflation target. Among the candidate explanations, one deserving further attention is learning, i.e. imperfect knowledge of the economic structure and the evolution of the perceived inflation-output volatility trade-off by the Fed. Cogley and Sargent (2005b), Primiceri (2006), Sargent, Williams, and Zha (2006), and Carboni and Ellison (2008) have proposed interesting investigations along this dimension.

The employment of a variety of business cycle proxies to estimate macroeconomic models is a promising avenue to perform robust evaluations on the impact of macroeconomic shocks and systematic policies on the macroeconomic dynamics of interest. Other implementations of multiple filtering to study issues like the role of monetary aggregates in business cycle models and the inflation reaction to a monetary policy shock in a variety of different macroeconomic models are already in my agenda.

References


Figure 1: **Proxies of the Business Cycle: Multiple Filters.** Left column: U.S. real GDP filtered with different proxies of the low-frequency component (‘trend’). List of filters indicated in the text. Grey vertical bars identify recessions (from peak to through) as dates by the NBER. Middle column: AutoCorrelation Functions of the business cycle proxies. Right column: Log-Spectral Density of the business cycle proxies. Blue vertical bars identify the normalized business cycle frequencies in the range [1/16, 1/4] corresponding to 8-32 quarters.
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\[ \hat{\rho} \]
|       | 0.94  | 0.86  | 0.96  | 0.94  | 0.92  | 0.27  |

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Table 2: Structural Parameters, Prior Densities.
Harmonic Mean approach proposed by Geweke (1998).

Marginal-Likelihoods computed with the Modified Harmonic Mean approach proposed by Geweke (1998).

Table 3: Structural Parameters, Posterior Densities Conditional on Different Filters: Great Inflation. Figures reported in the Table refer to posterior medians and [5th,95th] posterior percentiles. Last two rows: Figures concerning log-Marginal Likelihoods of the benchmark model with trend inflation (BMK) and the standard model with price-indexation (IND).
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Table 4: Structural Parameters, Posterior Densities Conditional on Different Filters: Great Moderation. Figures reported in the Table refer to posterior medians and [5th,95th] posterior percentiles.
Figure 2: Structural Parameters, Posterior Densities. Filters described in the text.
Figure 3: Impulse Response Functions to Monetary Policy Shocks. First two rows: Responses to a standard monetary policy shock - normalized to induce a 1% on-impact increase in the policy rate. Last two rows: Responses to a trend inflation shock - normalized to induce a 1% on-impact increase in the inflation rate. Median Bayesian impulse responses based on 500 draws from the posterior densities.
Table 5: Impulse Response Functions to a Monetary Policy Shock: Percentage Deviations with respect to Multiple Filters Models. Figures computed by relying on median responses.
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<tr>
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<th>CBO</th>
<th>HP</th>
<th>LIN</th>
<th>LBR</th>
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<tr>
<td>Output</td>
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<td>-98.66</td>
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Table 6: **Forecast Error Variance Decomposition to Monetary Policy Shocks: Percentage Deviations with respect to Multiple Filters Models.** Figures computed by relying on posterior modes.