Inflation targets, endogenous mark-ups and the non-vertical Phillips curve

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Abstract

Search-theoretic models of monetary exchange resurrect the view that welfare costs of inflation arise because the latter acts as a tax on money balances. In contrast with standard Calvo pricing models, empirical work on institutional features of the labor market shows that wage negotiations take place while expiring contracts are still in place, implying that wages are predetermined to future consumption and money holdings decisions. Bringing these seemingly unrelated aspects together in a stylized general equilibrium model, we identify a long-term trade-off between inflation and output. Model simulations suggest that a moderate long-run inflation rate generates non-negligible output gains.

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1 Introduction.

Recent developments in macroeconomics contradict the widely held belief that permanently higher inflation cannot affect unemployment. A long-run relationship between inflation and real activity is obtained in New Keynesian models based on price staggering, where inflation has adverse effects due to relative price dispersion and to the effect of expectations on mark-ups (Goodfriend and

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King, 1997; Woodford, 2003; Schmitt-Grohè and Uribe, 2004). Recent contributions point in the opposite direction. For instance, Graham and Snower (2008) show that the interaction of staggered nominal contracts with hyperbolic discounting leads to a positive long-run effect of inflation on real variables. Benigno and Ricci (2007) have recently resurrected the “grease in the wheels” argument, showing that downward nominal wage rigidity generates a long-run inflation-unemployment trade-off at low inflation rates.

We share the view that modern monetary models may underestimate the benefits of inflation on wage markups, but we highlight a different disciplining channel. A positive inflation rate is typically associated with higher nominal interest rates, which increase the opportunity cost of holding money. Thus inflation is a tax on money balances. To model this effect, we introduce money in the utility function, as in Christiano et al. (2005). Lagos and Wright (2005) point out that this is a reduced form monetary model, where assumptions about money holdings are meant to stand in for some role of money that is not made explicit, i.e. that it helps overcome spatial, temporal, or informational frictions.

The next step in our analysis is to identify a channel such that the inflation-tax effect on money balances disciplines wage markups. We are critical of the way nominal rigidities are modelled in new Keynesian models, i.e. by a mechanical transposition of the Calvo pricing formalism originally designed to characterize goods price rigidities. In fact, Calvo pricing implies that new wage contracts are set simultaneously to households decisions concerning consumption, labor supply and money demand. In line with evidence reported in a recent analysis of institutional features of wage bargaining in 22 European Union countries, the United States and Japan (Du Caju et al. 2008, p. 25), we emphasize that wage renegotiations take place while expiring contracts are still in place, enabling wage setters to internalize their consequences for household choices.

In our stylized model, this is captured by the assumption that within each period wages are predetermined to macroeconomic variables (see Corsetti and Pesenti, 2001, for a similar assumption). As a result wage setters internalize the effect of their wage choice on their own real money holdings. In the paper we show that such an effect is negative and becomes stronger with the expected inflation rate, inducing wage setters to limit their wage claims. We therefore obtain a new justification for the existence of a non-vertical Phillips curve. Model simulations show that a moderate inflation rate can generate substantial output gains relative to both the Friedman rule and to commitment to price stability, popularized in standard New Keynesian models. A central tenet of the New Keynesian literature is that nominal rigidities determine socially inefficient outcomes. Our paper reverses this view: properly designed monetary policies may take advantage of predetermined nominal wages to discipline wage setters. This, in turn, requires a positive inflation rate.

The rest of the paper is organized as follows. The next section outlines our model. Section 3 discusses the benchmark case of flexible nominal wages. Section 4 introduces pre-determined wages and explains why a positive inflation target disciplines wage setters. Section 5 concludes.
2 The model.

We build on Neiss (1999), where a staggered timing structure in the acquisition of nominal money balances within a money-in-the-utility function framework generates a discretionary inflation equilibrium when the economy is plagued by monopolistic distortions. To simplify the analysis, we impose full price flexibility in the goods market, whereas wages are pre-determined. In addition, we assume that the central bank pre-commits to a money growth rate.

2.1 Households

The representative household \((i)\) maximizes the following utility function

\[
U = \sum_{t=0}^{\infty} \beta^t \left( \ln C_{t,i} - \frac{\eta}{1 + \phi} t_{t,i}^{1+\phi} + \frac{\gamma}{1 - \varepsilon} \left( \frac{M_{t,i}}{P_t} \right)^{1-\varepsilon} \right)
\]

(1)

where \(\beta \in (0, 1)\) is the intertemporal discount rate, \(C_{t,i}\) is a consumption bundle, \(t_{t,i}\) is a differentiated labor type that is supplied to all firms, \(\frac{M_{t,i}}{P_t}\) denotes real money holdings.

The flow budget constraint is:

\[
C_{t,i} + \frac{M_{t+1,i}}{P_{t,i}} + \frac{B_{t+1,i}}{P_t} = w_{t,i} t_{t,i} + \frac{M_{t,i}}{P_t} + \tau_t + \theta_t + R_t \frac{B_{t,i}}{P_t}
\]

(2)

where \(B_{t,i}\) denotes holdings of one-period bonds; \(w_{t,i}\) is the nominal wage; \(\tau_t\) is a lump-sum transfer from central bank profits; \(\theta_t\) denotes firms profits; \(R_t\) is the nominal interest rate. Note that \(M_{t+1,i}\) is chosen at \(t\).

Consumption basket and price index are defined as follows:

\[
C_t = \left( \int_0^1 c_t(j)^{\rho} d\pi \right)^{\frac{1}{\rho}}
\]

(3)

\[
P_t = \left( \int_0^1 p_t(i)^{\frac{\rho-1}{\rho}} d\pi \right)^{\frac{\rho}{\rho-1}}
\]

(4)

The standard first order conditions for consumption are:\(^1\)

\[
c_t(j) = C_t \left( \frac{p_t(j)}{P_t} \right)^{\frac{\rho-1}{\rho}}
\]

(5)

\[
C_t = \frac{1}{\beta} \frac{1}{R_{t+1}} \frac{P_{t+1}}{P_t} C_{t+1}
\]

(6)

The money demand equation is

\(^1\)Index \(i\) is dropped for simplicity.
\[ \frac{M_{t+1}}{P_t} = \left( \frac{P_t}{P_{t+1}} \right)^{\frac{1-\sigma}{\sigma}} \left( \frac{\gamma \beta C_t}{1 - R_{t+1}} \right)^{\frac{1}{\sigma}} \]  \hspace{1cm} (7)

As in Neiss (1999) the agent faces a trade-off between period \( t \) consumption and period \( t+1 \) holdings of nominal money balances. Observe that (7) can also be interpreted as a demand function: when the central bank increases next period nominal money balances, *coeteris paribus* current consumption increases. Straightforward manipulations would show that \( \frac{1}{\epsilon} \) denotes the income elasticity of money demand. As the size of this parameter is crucial for our results, it is worth noting that money demand studies typically report income elasticities which are below 1.\(^2\)

The optimal labor supply condition will be introduced at a later stage, when we consider different wage-setting regimes.

### 2.2 Monetary Policy

By assumption, the central bank directly controls the money growth rate \( m_t \)

\[ M_{t+1} = M_t (1 + m_t) \]  \hspace{1cm} (8)

### 2.3 Firms

There is a continuum of monopolistically competitive firms uniformly distributed over the interval \([0, 1]\). Each firm \((j)\) produces a differentiated good using a Cobb-Douglas production function:

\[ y_t(j) = l_t(j)^{\alpha} \]  \hspace{1cm} (9)

where

\[ l_{t,j} = \left[ \int_0^1 l_{t,j}(i) \frac{\sigma-1}{\sigma} di \right]^{\frac{\sigma}{\sigma-1}} \]  \hspace{1cm} (10)

denotes a labor bundle and \( \sigma \) is the intra-temporal elasticity of substitution across different labor inputs.

The price is set as a markup, \( \mu^p = \frac{\sigma}{\sigma-1} = \frac{1}{\rho} \), over real marginal costs. For any given level of its labor demand, \( l_{t,j} \), the firm must decide the optimal allocation across labor inputs, subject to aggregation technology (10). Firm \((j)\) demand for labor type \((i)\) is

\[ l_{t,j}(i) = \left( \frac{w_t(i)}{w_t} \right)^{-\sigma} l_{t,j} \]  \hspace{1cm} (11)

\(^2\)See e.g. Choi and Oh (2003), Dib (2004), Knell and Stix (2005) and references therein. Christiano *et al.* (2005) obtain an estimate of 0.1.

\(^3\)This is equivalent to assume that the central bank implements a constant nominal interest rate rule \( R_t = \frac{1 + m_t}{M_t} \), which implies that \( \frac{M_{t+1}}{M_t} = 1 + m_t \).

\(^4\)Capital is assumed fixed and normalized to unity.
where
\[ w_t = \left[ \int_0^1 w_t(i)^{1-\sigma} \, di \right]^{\frac{1}{1-\sigma}} \] (12)
is the wage index.

Aggregating across firms we obtain
\[ Y_t = C_t = l_t^{\alpha} \] (13)
\[ l_t(i) = \left( \frac{w_t(i)}{w_t} \right)^{-\sigma} l_t \] (14)
\[ l_t = \left( \frac{1}{\alpha \rho \rho_t} \right)^{-\frac{1}{1-\sigma}} \] (15)

3 Flexible wages

The flexible wages solution provides the benchmark case for the evaluation of our results. Each household maximizes (1) subject to the (2), given (14) and (15). This amounts to
\[ w_t = \eta \mu w l_t C_t \] (16)
where \( \mu w = \sigma (\sigma - 1)^{-1} \) denotes the wage mark-up under flexible wages. As shown in Neiss (1999) the model has a time-independent solution. The nominal interest rate interest rate is
\[ R_{t+1} = \frac{1}{\beta} P_{t+1} \] (17)

By using (15) and (13) equilibrium employment is:
\[ l_t^{\mu} = \left( \frac{\alpha - 1}{\eta \mu w} \right)^{\frac{1}{1-\sigma}} \] (18)

Observe that \( \mu \mu w \) denotes the labor and goods market wedge. The competitive (Pareto optimal) level of employment obtains if \( \mu \mu w = 1 \).

Using (7), (8), (13), (18), it is straightforward to show that
\[ \frac{P_{t+1}}{P_t} = 1 + \pi_t = 1 + m_t \]

Central bank commitment determines the inflation rate but has no effect on distortions. In this case the optimal monetary policy coincides with the Friedman rule \( m_t = \beta - 1 \).
4 Predetermined wages.

The timing of the game is as follows.

1. At the beginning of the period, the central bank commits to a fixed money growth rate consistent with a certain exogenous inflation target, $\bar{m}$.

2. Given the central bank rule, households set the nominal wage rate, $w_t$, that maximizes expected utility (1), conditional to the expected values for labor demand, for the money growth rule and for their own choices concerning money demand and consumption, i.e. conditions (11), (15), (8), (7), (6), (17). To simplify exposition, we characterize the nominal wage rate as $\bar{w}_t = \bar{w}_t P_t^e$, where $P_t^e$ is the rational expectation of the price level and $\bar{w}_t$ is the desired real wage rate.

3. Households choose consumption and next-period nominal money holdings. Simultaneously, full price flexibility ensures that markets clear.

Relative to the flex-wage solution, the key difference is that now households anticipate the effects of their wage choice on real money holdings. Imposing rational expectations ($\pi_t^* = \pi_t^e = \bar{m}$), the money demand equation becomes:

$$\frac{M_{t,i}}{P_t^e} = \left( \frac{\gamma \beta C_t^e}{1 + \bar{m} - \beta} \right)^{\frac{1}{\varepsilon}}$$

(19)

Imposing the symmetrical equilibrium we obtain the desired wage rate, which is lower than in the flex-wage case:

$$\bar{w}_t = \frac{w_t}{P_t^e} = \eta \frac{\mu^w}{1 + \delta_m} \frac{\mu^w}{1 + \bar{m}} C_t^e$$

(20)

where

$$\delta_m = \frac{\gamma}{\varepsilon} \left( \frac{M_{t,i}}{P_t^e} \right)^{1-\varepsilon} = \frac{1}{\varepsilon} \left[ \gamma \left( \frac{1 + \bar{m} - \beta}{\beta} \right)^{\varepsilon-1} C_t^1 \right]^\frac{1}{\varepsilon}$$

(21)

By comparing (20) to (16), it is clear that the combination of predetermined wages and positive inflation target has a disciplining effect on labour market distortions.

The rationale is as follows. Under flexible wages, the wage-setters’ optimization problem is solved by choosing a real wage such that consumption falls below the perfectly competitive rate. This loss of utility is more than compensated for by the corresponding reduction in labor effort. When wages are predetermined and the central bank adopts an inflation targeting strategy, households also anticipate that real money balances fall due to the adverse effect of the wage choice on consumption. The term $\delta_m$ captures the impact of a real wage increase on

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5Equation (19) has been obtained substituting (6), (8), (17) into (7) and imposing rational expectations.
expected real money holdings. The size of this adverse effect is unambiguously increasing in \( \tilde{m} \).

To support intuition, it is worth emphasizing the key difference relative to standard New Keynesian models incorporating nominal rigidities. In our framework the wage choice is antecedent to consumption, employment and money demand realizations, whereas under Calvo’s wage setting rule all these variables obtain simultaneously to the optimizing wage setters’ decision.

By substituting (21) (20) into (15) and imposing rational expectations we obtain an implicit solution for employment

\[
I_t = \left\{ \frac{\alpha}{\eta \mu \mu w} \left[ 1 + \frac{1}{\varepsilon} \left\{ \gamma \left( \frac{1 + \tilde{m} - \beta}{\beta} \right)^{\varepsilon - 1} I_t^{\alpha(1 - \varepsilon)} \right\} \right] \right\}^{\frac{1}{1 + \varepsilon}}
\]

(22)

Straightforward manipulations would show that the inflation target effect on employment (and thus on consumption) is always positive, i.e. \( \frac{\partial I_t}{\partial m} > 0 \) (see Appendix I for a formal proof).

5 The non-vertical Phillips curve.

By using (22), we obtain a non-vertical Phillips curve plotting the employment gap and the inflation rate (see Figure 1).\(^7\)

\(^6\)Recall that \( \frac{1}{\varepsilon} < 1 \).

\(^7\)The employment gap is defined as \( \frac{I_t^\alpha - I_t}{I_t^\alpha} \). See Appendix II for calibration details.
Observe that when $m = \beta - 1$ the central bank implements the Friedman rule, the disciplining effect is nil and the employment gap is maximum, just like the flex wage case, when (18) obtains. Our calibrations show that a 2% inflation target causes a non-negligible reduction in the employment gap. This result is robust to changes in key parameters such as the income elasticity of money demand and the inverse Frisch elasticity, measured by $\varepsilon$ and $\phi$ (see Figure 2).
below).

The obvious next step in our analysis is the identification of the optimal inflation rate. In this class of models the Friedman rule \((R_{t+1} = 1, \bar{m} = \beta - 1)\) is optimal when goods and labor markets distortions are inflation invariant (Neiss, 1999). In our framework one would expect that the optimal inflation rate should strike a balance between the benefits in terms of markup reduction and the inflation-induced distortion on real money balances. Our calibrations show instead that the disciplining effect of inflation on wage markups never compensates for the inflation-induced distortion on real money balances. The Friedman rule remains therefore optimal. This result is surprisingly robust and holds for a wide range of parameter values (see the Appendix II for details).

To support intuition, we use (21) to define \(I_t, C_t\) as functions of real money balances. The utility function (1) becomes

\[
U = \left\{ \frac{\alpha}{1 + \phi} \ln \left[ \frac{\alpha \left(1 + \frac{\gamma}{\varepsilon} \left(\frac{M}{P}\right)^{1-\varepsilon}\right)}{\eta(\mu P \mu w)} \right] - \eta \left[ \frac{\alpha \left(1 + \frac{\gamma}{\varepsilon} \left(\frac{M}{P}\right)^{1-\varepsilon}\right)}{\eta(\mu P \mu w)} \right]^{\frac{1}{1+\varepsilon}} \right\} + \frac{\gamma}{1 - \varepsilon} \left(\frac{M}{P}\right)^{1-\varepsilon}
\]

(23)

The term in curly brackets denotes the inflation-induced gain in the consumption/effort trade-off. In our simulations we always find that this gain is lower than the welfare loss deriving from the real money balances when these become lower as an effect of inflation: welfare is monotonically increasing in real money balances (Figure 3).
This apparent setback suggests that our result should be qualified. So far we have shown that a non-vertical Phillips curve obtains to the extent that two conditions are satisfied: 1) wage contract renegotiations take place while expiring contracts are still in place, enabling wage setters to internalize their consequences for subsequent households’ choices; 2) inflation adversely affects households’ welfare. Within the relatively narrow framework of our model, this is not sufficient to support the optimality of a positive inflation rate. Intuition suggests, however, that the disciplining effect on wage markups outlined here would still apply in different models that justify deviations from the Friedman rule. Consider for instance Keynesian models based on price staggering in the goods market, where full price stability is required in order to avoid relative price dispersion and the adverse effect of inflation expectations on mark-ups (Woodford, 2003; Schmitt-Grohè and Uribe, 2004). Simulations of our model show that, relative to the Friedman rule, commitment to full price stability should bring about a reduction in the employment gap of 0.4%. Moreover, several contributions suggest that the requirement of full price stability is unduly restrictive.8 In all these cases our results identify a further beneficial effect of the optimal, positive inflation rate.

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8 One possible argument in favour of commitment to a positive inflation target is that it softens the problems posed by a binding zero lower bound for interest rates in the face of
Even though the theoretical debate on the optimal inflation target is far from settled, empirical macro models have begun to incorporate an exogenous and positive long-run inflation rate. A growing literature has shown that New Keynesian models significantly improve their ability to replicate the business cycle facts if monetary policy rules are assumed to target time-varying, non-zero long-run inflation rates (see Cogley and Sbordone, 2008, and the references therein). Ireland (2007) estimates a New Keynesian model to draw inferences about the behavior of the Fed unobserved inflation target. His results indicate that the target soared from 1.25% in 1959 to over 8% percent in the mid-to-late 1970s before falling back to below 2.5% in 2004. He provides evidence which is consistent with the view that shifts in the secular trend in inflation, i.e. the expected long-term inflation rate, could be attributed to a systematic tendency for Federal Reserve policy either to limit the contractionary consequences of adverse shocks (Blinder, 1982; Hetzel, 1998; Mayer, 1998) or to exploit favorable economic conditions to eventually bring inflation down (Bomfim and Rudebusch, 2000; and Orphanides and Wilcox, 2002).

Our model is consistent with the view that changes in real macroeconomic factors induced the Fed to shift the inflation target. In Figure 4 the dashed line shows the inflation target adjustments necessary to stabilize the employment gap following a mark-up increase. For instance, an inflation target surge to 6% is required to sterilize the real effects of about 1% mark-up increase when the

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\(9\) We consider a shock to \(\sigma\), the intra-temporal elasticity of substitution across different labor inputs, that typically determines cost-push (wage mark-up) shocks in New Keynesian models. Another argument is that deflation subsidizes money holdings at the expense of deposits and loans, causing disintermediation and a weakening of financial markets, whereas a moderate inflation deepens financial markets and improves the mobility of asset-trading households to smooth consumption (Antinolfi et al., 2007). A third classical argument is that some positive inflation is necessary to “grease the wheels” of the labor market (Tobin, 1972).
initial inflation rate is 2%.

Figure 4 – Phillips Curve shift

6 Conclusions.

Search-theoretic models of monetary exchange resurrect the view that welfare costs of inflation arise because the latter acts as a tax on money balances. Recent empirical works on institutional features of the labor market show that wage negotiations take place while expiring contracts are still in place. Bringing these seemingly unrelated aspects together in a stylized general equilibrium model, we find a disciplining effect of a positive inflation target on the wage markup and identify a long-term trade-off between inflation and output. Model simulations suggest that a moderate long-run inflation rate generates non-negligible output gains.

The key assumption for our result is that, differently from standard New Keynesian models usually based Calvo pricing, all nominal wages are predetermined to both the individual households’ and the policymaker’s decisions. As a consequence, wage setters internalize the effects of their choice on money holdings for the representative household’s welfare because inflation is costly. This, in turn, paves the way for the disciplining effect of a positive inflation target and a non-vertical Phillips curve. Further research should integrate this mechanism into models where the Friedman rule is suboptimal.
Appendix I

Under preset wage employment is determined by the following implicit expression (see (22)):

\[
l_t(\bar{m}) = \left( \frac{\alpha 1 + \delta_m (l_t(\bar{m}))}{\mu\mu^w} \right)^{\frac{1}{1+\phi}} = l_{flex} (1 + \delta_m (l_t(\bar{m})))^{\frac{1}{1+\phi}}
\] (24)

Recall that the endogenous markup is a function of the consumption level, which is non-linearly related to the labor supply by the real wage.

Differentiating (24) we obtain

\[
\frac{\partial l_t(\bar{m})}{\partial \bar{m}} = \frac{l_{flex} (1 + \delta_m (l_t(\bar{m})))^{-\frac{\phi}{1+\phi}} \partial \delta_m (l_t(\bar{m}))}{1 + \phi}
\] (25)

By differentiating (21) we get:

\[
\frac{\partial \delta_m (l_t(\bar{m}))}{\partial \bar{m}} = \frac{\alpha - 1}{\varepsilon - 1} \frac{\delta_m (l_t(\bar{m}))}{\frac{\partial l_t(\bar{m})}{\partial \bar{m}}} + \frac{\varepsilon - 1}{\varepsilon} \frac{\delta_m (l_t(\bar{m}))}{1 + \delta_m (l_t(\bar{m}))}
\] (26)

By combining (25)-(26) and using (24) we find

\[
\frac{\partial l_t(\bar{m})}{\partial \bar{m}} = \left( 1 - \frac{\alpha - 1}{\varepsilon} \frac{\delta_m (l_t(\bar{m}))}{1 + \delta_m (l_t(\bar{m}))} \frac{1}{1 + \phi} \right)^{-1} \frac{\varepsilon - 1}{\varepsilon} \frac{\delta_m (l_t(\bar{m}))}{1 + \delta_m (l_t(\bar{m}))} > 0
\] (27)

Note that the expression in parenthesis is one minus a product of terms all smaller than one. Thus it is positive.

Appendix II

Our benchmark calibration follows Christiano et al. (2005), under the assumption of flexible prices (p. 15-17). We therefore set $\alpha = 0.64$, $\phi = 1$, $\varepsilon = 8.5$, $\beta = 0.97$, $\mu^p = 1.2$. Christiano et al. (2005) calibrate the wage markup at 1.05.

In our case the wage markup is endogenous, we therefore choose a value of $\mu^w$ such that the model solution yields $\frac{\mu^w}{1+\delta_m} = 1.05$ when inflation is 4%, i.e. the observed average post-war inflation in the US. Christiano et al. (2005) calibrate $\gamma$ at a level consistent with the observed money velocity ($0.44$ for $M2$). In our model income is endogenous; thus $\gamma$ is obtained by imposing $0.44 = \frac{\bar{Y}}{\bar{P}}$ when $\bar{m} = 0.04$.

References


