Colluding through Suppliers*

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Abstract

In a dynamic game of competing supply chains where the bargaining power is on the retailers’ side and wholesale contracts are observable, I show that inefficient contracting emerges as a mechanism to implement collusion among retailers. When full collusion is not sustainable with efficient contracts, \(N\) retailers competing à la Bertrand on the final market might rely on wholesale contracts entailing positive wholesale prices and negative franchise fees to squeeze the wedge between collusive and deviation profits. The paper also offers insights about the role of communication on the equilibrium outcomes of games where retailers have the initiative. It turns out that communication is fundamental to sustain downstream collusion, although it may generate efficiency losses — i.e., profits lower than the monopoly level.

Keywords. Collusion, double marginalization, retail competition, vertical contracting.

JEL Classification. D21, D43, L42.

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1 Introduction

Manufacturer-retailer relationships have been widely studied by the IO literature. Existing models have underscored several aspects of these games. For instance, by studying the rationale behind alternative forms of vertical restraints\(^1\), the link between pre-commitment effects and renegotiation\(^2\), or by emphasizing the welfare effects of non-exclusive deals.\(^3\) But, this work has mainly taken a static approach, and has often neglected the strategic aspects stemming from the intertemporal dimension of vertical contracting. One issue that certainly deserves more attention is the link between wholesale contracting, ‘buyer power’ and the outcome of the repeated interaction between upstream and downstream firms.

The rise in many developed countries of big box retailers — e.g., Wal-Mart in the US and Ikea in Europe — and the widespread diffusion of large supermarket chains,\(^4\) has led competition authorities to renew their focus on buyer power. Yet, more analytical effort is required to better understand the potential harms of those practices that appear to be correlated with buyer power.\(^5\) What is the link between (implicit) collusion and vertical contracting in markets where the bargaining power is on the retailers’ side? Can the strategic design of wholesale contracts soften competition in these games? Do contracts that help sustaining collusion exhibit specific, easy to spot features?

To tackle these issues, I consider an industry where \(N\) retailers sell a homogenous good and compete by setting prices. The final good must be recovered from an intermediate input, which is supplied by upstream firms (suppliers) each being in an exclusive relationship with a single retailer. Contracts are public, the interaction is repeated over an infinite horizon and, in each period, retailers make take-it or leave-it offers to suppliers. Within this framework, I show that, whereas the stage game features a unique (zero-profit) competitive equilibrium with wholesale prices and franchise fees equal to zero, in the repeated game retailers might prefer to pay a positive wholesale price and receive a fixed payment from suppliers for collusive purposes. Interestingly, deals with these features allow to sustain positive profits even when the discount factor \(\delta\) falls short of the critical threshold \((N - 1)/N\).\(^6\)

One key trade-off drives the result. On the one hand, excessively high wholesale prices reduce the difference between deviation and collusive profits. To understand this effect remember that when retailers face zero (or very low) marginal costs, by undercutting the monopoly price a deviant retailer grabs a

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\(^3\)See, e.g., Bernheim and Whinston (1986) and (1998), Gal-Or (1991b), Martimort and Stole (2009) and Miklós-Thal et al. (2010) among others.
\(^4\)As noted in Miklós-Thal et al. (2010), large supermarket chains often account for a high share of a manufacturer’s production: in the UK, even large manufacturers typically rely on their main buyer for more than 30 percent of domestic sales. In contrast, the business of a leading manufacturer usually represents a very small proportion of business for each of the major multiples.
\(^5\)Antitrust authorities in the United States have been investigating ‘slotting fees’ and other related retail practices. At the same time, UK and EU authorities have commenced a number of inquiries into the competitiveness of the supermarket grocery retail sector.
\(^6\)Below this threshold collusion would not arise in the standard repeated Bertrand game, where retailers do not rely on suppliers to produce the final good.
spot gain close to the monopoly profit. This is no longer true when retailers are committed to pay large wholesale prices: undercutting would then secure lower profits to the deviant. On the other hand, a too large wholesale price — i.e., low downstream margins — might induce a retailer not only to undercut the collusive price, but also to change its wholesale contract in order to gain a competitive advantage over rivals and grab a higher profit from deviation: a ‘public deviation’.

When retailers punish deviations with grim-trigger strategies, the collusive wholesale price is chosen so as to balance these two effects. Of course, the (efficient) monopoly profit is sustainable for large values of the discount factor — i.e., $\delta \geq (N - 1)/N$. In this parameter region downstream firms charge the monopoly price, set the wholesale price and the franchise fee equal to zero and uniformly share final demand. However, unlike in the standard repeated Bertrand analysis, for $\delta < (N - 1)/N$ the monopoly profit is still sustainable, but only via vertical contracts entailing positive wholesale prices and negative franchise fees. These contracts allow to sustain collusion even for lower values of $\delta$; but, in this region of parameters, profits fall short of the monopoly level. Clearly, for $\delta$ close to zero only the competitive equilibrium is sustainable.

The paper offers two novel insights to the literature on dynamic competition between competing supply chains. First, it emphasizes the coordination role that suppliers play in dynamic games where retailers jointly gain by fixing downstream prices. The analysis shows that there exists a mechanism allowing to sustain collusion even in the region of parameters where self-enforceability would not hold in the standard (repeated) Bertrand game. Second, the paper provides a novel rationale for payments made by suppliers to retailers — e.g., slotting allowances — as well as for excessively high wholesale prices (double marginalization). While earlier models have discussed different reasons for double marginalization to be welfare detrimental, less research has been done on negative franchise fees: a contractual practice that, as a matter of fact, can be spot quite easily by antitrust authorities. This practice is common in many markets: according to the US Federal Trade Commission, since 1998 manufacturers’ expenses in slotting allowances have increased sharply from a share of 28% of their total expenses in promotional activities up to 50%. State and federal agencies conducted numerous investigations, but none have resulted in a conclusion against slotting allowances. On February 2001, the Federal Trade Commission released a staff-report addressing slotting allowances and other related practices in the supermarket industry. The report notes that such arrangements have the potential to facilitate anti-competitive horizontal collusion among groups of suppliers or retailers. My objective is precisely to formalize this point. While existing models mainly focus on suppliers’ incentives to use slotting allowances and similar practices, the evidence corroborates the view that negative franchise fees are positively correlated with the exercise of buyer power. In this respect, this is the first paper to emphasize that, in repeated games, negative fees can

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7 That is, up-front payments made by manufacturers to retailers, such as listing fees and slotting allowances.
8 For instance, according to a former FCT Chairman (Robert Pitofsky) there was still little theoretical work on the topic to issue guidelines on slotting allowances; this line was reaffirmed by the FTC staff in 2002, when it was claimed that more studies need to be conducted to learn about this practice before intervening.
10 See, e.g., Miklós-Thal et al. (2010).
be used for (implicit) collusive purposes. Moreover, while in the U.S. courts generally regard positive (and high) franchise fees as a signal of price fixing — see, e.g., Briley et al. (1994) — I show that this is not necessarily true when the contracting power is on the retailers’ side.

The rest of the paper is organized as follows. Section 2 reviews the existing related literature. The model is introduced in Section 3. The equilibrium characterization is provided in Section 4. In Section 5 I analyze the case of private contracts, study the effect of optimal penal codes and discuss why the predictions of the model are compelling for the case of buyer power. Section 6 concludes. Proofs are in the Appendix.

2 Related literature

The analysis overlaps with the literature on ‘buyer power’. Shaffer (1991) analyzes a static duopoly model with differentiated products and buyer power where slotting allowances and resale price maintenance (RPM) are substitutes. He shows that the equilibrium features slotting allowances with public contracts. This outcome cannot occur in my stage game because of Bertrand competition. Marx and Shaffer (2008) consider a (static) model with buyer power where strong retailers can exclude competitors by offering ‘three-part tariffs’ that include slotting allowances. In contrast to their model, where negative fees create negative externalities (exclusion) between downstream firms, in my framework they bring out positive externalities (cooperation). Miklós-Thal et al. (2010) also analyze the competitive effects of up-front payments, but they consider a contracting situation where rival retailers offer contracts to a single manufacturer. In contrast to Bernheim and Whinston (1986, 1998), both these papers show that two-part tariffs do not suffice to implement the monopoly outcome in a static game. Miklós-Thal et al. (2010) argue that more complex (contingent) arrangements, which combine slotting allowances and standard two-part tariffs, are necessary to internalize all contractual externalities stemming from common agency, and bring back the monopoly outcome. My model departs from Miklós-Thal et al. (2010) in two main respects. On one side, I study a dynamic framework while they consider a static game: in this sense my paper is a complement to them.11 On the other side, while I abstract from common agency issues, their results rely on the externalities that these games feature and do not hold in a model with Bertrand competition, which is instead my building block.12

Given its dynamic perspective, the paper is also related to the literature studying the repeated interaction between upstream and downstream firms — e.g., Jullien and Rey (2007), Nocke and White (2007), Norman (2009), Schinkel et al. (2008) and Piccolo and Reisinger (2010). The main difference with this literature is that, while I am interested in downstream collusion, all these papers study upstream collusion.

11 This is consistent with the legal approach taken in Carstensen (2000 and 2004) who argues that antitrust treatment of buyer power should be sensitive to the differences in the economic incentives to collude or unilaterally exercise monopsony power between buyers and sellers.

12 My analysis extends to product differentiation, see, e.g., Shaffer (1991).
Finally, the paper also relates to the existing literature on vertical contracting, which often assumes that wholesale contracts, or some of their dimensions, are public (Jullien and Rey, 2007, Nocke and White, 2007, and Rey and Stiglitz, 1995, among many others). According to Briley et al. (1994) there exists substantial evidence showing that strategic alliances and trade associations facilitate information sharing about wholesale contracts in several U.S. retail industries. For instance, this seems to be the established praxis in business format franchising where the mandatory disclosure of franchising contracts required by the Federal Trade Commission since 1979 allows firms to have free and almost instantaneous access to their rivals’ past and current contracts. Another important trend in product distribution is the growth of information-intensive channels. These are usually characterized by channel partners who invest in bundles of sophisticated information technology like telecommunication and satellite linkages, bar coding and electronic scanning systems, database management systems etc., to not only disseminate information within a given organization, but also among competitors — see, e.g., Stern et al. (1996).

3 The model

Players. Consider \( N \geq 2 \) independent and identical downstream firms (retailers), each denoted by \( R_i \) \((i = 1, \ldots, N)\), selling a homogenous good and competing by setting prices. The demand for the final good is \( D(p) \). Hence, given a vector of retail prices \( p = (p_1, \ldots, p_N) \), each downstream firm \( i \) faces the individual demand

\[
D_i(p) = \begin{cases} 
0 & \text{if } p_i > p_j \text{ for some } j \neq i, \\
\frac{D(p)}{\#\{j : p_j = \min\{p_1, \ldots, p_N\}\}} & \text{if } p_i = p = \min\{p_1, \ldots, p_N\}.
\end{cases}
\]  

(1)

Retailers’ production technologies are linear and marginal costs are normalized to zero. Nevertheless, the final output must be recovered from an intermediate input that is produced by upstream firms (suppliers). I assume that suppliers, each denoted by \( S_i \) \((i = 1, \ldots, N)\), are in exclusive relationships with retailers. The intermediate input is transformed into the final output according to a one-to-one technology and, for simplicity, upstream production technologies are linear, with zero marginal costs.

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13 The Entrepreneur’s Magazine collects yearly data about franchise fees that are published in the Franchise 500 survey. As noted by Lofantaine and Shaw (1999), these contracts are very stable over time — i.e., around 75% of franchisors never changed their royalty rate or franchise fee over a 13-year time period.

14 As noted by Niraj and Narasimhan (2004), major retailers such as Sainsbury and Marks & Spencers in U.K. as well as A&P grocery stores, Super Valu Stores and Von’s supermarket in U.S. have made substantial investments in these technologies. Similarly, leading manufacturers such as Procter and Gamble have responded to the availability of greater information by developing tracking and information systems at the retail store level.

15 A similar approach is taken in Jullien and Rey (2007) and Schinkel et al. (2008). One interpretation (certainly among others) of this exclusivity hypothesis is that those delegated suppliers are just spin-off units and that there are (un-modeled) fixed-costs of setting up those units. I will implicitly assume that replicating these costs would be too costly.

16 In a previous version of the paper I show that results do not change whenever the number of suppliers is larger than \( N \) and retailers have full bargaining power. In the opposite case where the number of suppliers is lower than that of retailers, assuming buyer power seems less compelling: in these instances suppliers should have the bargaining power.
Wholesale contracts. Retailers make take-it or leave-it offers to suppliers. A wholesale contract between \( R_i \) and \( S_i \) is a two-part tariff, \( C_i \equiv (T_i, w_i) \), specifying a wholesale price \( w_i \) for each unit of intermediate input ordered by \( R_i \) and a fixed franchise fee \( T_i \). Franchise fees are paid up-front and are thus sunk when downstream firms set retail prices. Once final demand materializes, \( R_i \) buys inputs from \( S_i \) and pays the negotiated unit price \( w_i \).

Information. Wholesale contracts are public — i.e., the contract signed between \( R_i \) and \( S_i \) is observed by all other players before downstream price competition takes place.\(^{17}\)

Timing. Consider an infinitely repeated game with discrete time \( \tau = 0, ..., +\infty \). The timing of the stage game, thereafter \( G \), is as follows:

\( (T=1) \) Retailers simultaneously offer contracts to suppliers.

\( (T=2) \) Suppliers accept or refuse the received offers. If contract \( C_i \) is finalized, \( T_i \) is paid.

\( (T=3) \) Contractual offers become public information. Downstream firms choose retail prices, final demands materialize and input orders are placed.

Each firm has an infinite life-time horizon and its objective is to maximize the discounted sum of profits. The common discount factor is \( \delta \in (0, 1) \). Retailers can only commit to spot distribution contracts and all players are risk neutral with reservation utility normalized to zero.

Histories. The game is one of perfect monitoring: all past actions become common knowledge at the end of each play. Before retailers compete in stage \( \tau \), history \( h^\tau \) is equal to \( (p^\tau, C^\tau) \) and contains the sequence of retail prices charged in previous stages \( p^\tau \equiv (p^\tau_1, ..., p^\tau_N) \), with \( p^\tau_t \equiv (p^\tau_0, ..., p^\tau_t-1) \), along with the sequence of wholesale contracts \( C^\tau \equiv (C^\tau_1, ..., C^\tau_N) \), with \( C^\tau_i \equiv (C^\tau_{i0}, ..., C^\tau_{i\tau}) \), offered up to \( \tau \).

Collusion. I focus on symmetric and stationary pure strategy equilibria. Hence, a collusive strategy, thereafter \( \hat{\sigma} \), requires all downstream firms to offer \( C^c \equiv (T^c, w^c) \) and charge \( p^c \) in the collusive phase, and to offer \( C^p \equiv (T^p, w^p) \) and charge \( p^p \) in the punishment phase. I focus on punishment codes requiring infinite Nash reversion — i.e., following a deviation by one retailer, rivals will offer the competitive and efficient contract \( C^* \equiv (0, 0) \) and price at marginal costs for the rest of the game.

Assumptions. The equilibrium concept is Subgame Perfect Nash Equilibrium (SPNE). The analysis will be developed under the following standard assumptions:

\[ A1 \] \( D(p) \) is strictly decreasing and twice continuously differentiable. It satisfies the Inada conditions: (i) \( D(0) > 0 \); and (ii) there exists an upper-bound \( \bar{p} \) such that \( D(p) > 0 \) for all \( p < \bar{p} \) and \( D(p) = 0 \) for all \( p \geq \bar{p} \).

\(^{17}\) Nothing would change if this information can be observed only by retailers and not by suppliers.
Let \( \phi(p) \equiv D(p)p \), \textbf{A2} below guarantees the existence of the monopoly price:

\textbf{A2} \( \phi(.) \) is single peaked: it features a unique internal maximum \( p^m \) identified by the first-order necessary and sufficient condition:

\[
\phi'(p^m) = D'(p^m) p^m + D(p^m) \equiv 0.
\]

\textbf{A3} If suppliers are indifferent between accepting a contract and remaining inactive, they prefer to secure input supply.

This hypothesis allows to restrict attention to the class of equilibria with positive sales. Finally, following the literature studying collusion in Bertrand models with cost asymmetries, and in particular Miklós-Thal (2010), I will make the following assumption:

\textbf{A4} Suppose that \( p_j = p \geq 0 \) for all \( j = 1, \ldots, N \) and \( w_i < w_j \) for all \( j \neq i \), then \( D_i(p) = D(p) \) and \( D_j(p) = 0 \) for all \( j \neq i \).

This is a standard tie breaking rule: when retailers charge the same final price, but have different marginal costs, the most efficient retailer gets the whole market.

4 Equilibrium analysis

In this section I provide the equilibrium characterization. I first derive the static outcome and then move to the repeated game.

4.1 The stage game

Game \( \mathcal{G} \) is a two-stage game. In the first stage, retailers offer wholesale contracts; in the second stage, those who finalized a deal, set retail prices to attract consumers. I focus on symmetric pure strategy equilibria where all retailers offer \( C^e \equiv (w^e, T^e) \), charge \( p^e \) and uniformly share demand.

Given the opponents’ prices \( p_{-i} = (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_N) \), retailer \( R_i \) solves:

\[
\max_{p_i \in \mathbb{R}} D_i(p_i, p_{-i})(p_i - w_i),
\]

Hence, given a symmetric equilibrium candidate \( (C^e, p^e) \), ‘off-equilibrium’ histories — i.e., those situations where one or more unexpected offers are observed — might lead to multiple Nash equilibria in the corresponding (downstream) subgame. A refinement criterion must then be chosen to select an equilibrium. To this purpose, I posit that, in every subgame featuring multiple equilibria, downstream firms coordinate on that satisfying weak Pareto-dominance. This is standard in the literature studying Bertrand models with cost asymmetry, which usually selects the equilibrium where the firm with the
lowest marginal cost makes all the sales at a price equal to the second-lowest cost — see, e.g., Deneckere and Kovenock (1996), Blume (2003) and Miklós-Thal (2010). Under this hypothesis, the static game has a unique competitive equilibrium.

**Proposition 1** Assume A1-A4. Game \( G \) features a unique SPNE satisfying the added weak Pareto-dominance refinement. In this equilibrium all players make zero profits, retailers offer the competitive and efficient contract \( (C^e = C^*) \) and set retail prices equal to marginal costs \( (p^e = 0) \).

The intuition for this result is straightforward. First, because retailers are in Bertrand competition, they must make zero profits in a symmetric equilibrium of the one-shot game. Second, since contracts with positive wholesale prices limit retailers’ ability to undercut one another and franchise fees are sunk, the unique equilibrium of the game must feature efficient wholesale contracts — i.e., \( w^* = T^* = 0 \).

### 4.2 Repeated interactions

Consider now the infinitely repeated game. In the following I will identify the conditions under which downstream collusion is sustainable, and then characterize the properties of the implicit agreement that supports such cooperative outcome.

I will focus on symmetric equilibria where, in the collusive phase, all retailers offer \( C^c \), charge \( p^c > 0 \) and share final demand evenly. To gain insights about the key forces shaping the equilibrium of the repeated game note that there are two types of deviations that a retailer may envision. First, it may stick to contract \( C^c \) and cheat its rivals only by undercutting \( p^c \): a secret (or unobservable) deviation. Second, a retailer may ‘announce’ its forthcoming deviation by offering a contract different than \( C^c \), and then charge a final price that maximizes its spot profits given the rivals’ reaction triggered by such a deviation: a public (or observable) deviation.\(^\text{18}\)

With public (observable) contracts these alternative types of deviations can be punished in different manners. Secret deviations are observed only after demand has materialized, so they can be punished from the next period onwards. A public deviation is instantaneously spot, and it therefore triggers a reaction already in the very same period where it occurred.

Before turning to the cartel’s optimization program it is important to note that, if at each stage of the game downstream firms symmetrically offer the competitive and efficient contract \( C^* \), the unique equilibrium features collusion supported by Nash reversion trigger strategies if and only if the discount factor \( \delta \) is larger than the standard threshold \( (N - 1) / N \).

**Lemma 1** Suppose that the equilibrium outcome path has the competitive and efficient contract \( C^* \) offered in each period. Then, every retail price level between monopoly \( (p^m) \) and perfect competition \( (0) \) can be supported by Nash reversion trigger strategies if and only if \( \delta \geq (N - 1) / N \). Otherwise, for \( \delta < (N - 1) / N \), there exists a unique SPNE featuring perfect (efficient) competition — i.e., retail and wholesale prices are both equal to zero.

\(^{18}\) ‘Within periods’ punishments are analyzed in broader perspective in Mailath et al. (2004), who also discuss why simple penal codes may fail in repeated extensive form games.
This lemma will be helpful to understand the case where, for \( \delta \) below the threshold \( (N - 1) / N \), collusion can be sustained through a contract different from the efficient one.

I focus on punishments that are repetitions of the static equilibrium. This implies that retailers never price below marginal costs — i.e., wholesale prices — when punishing public deviations.\(^{19}\) Hence, a stationary, symmetric collusion strategy \( \hat{\sigma} \) enforcing the non-competitive retail price \( p^c > 0 \) and the wholesale contract \( C^c \), requires each retailer \( i \):

(i) To keep offering \( C^c \) and charging \( p^c \) as long as none of its competitors has deviated;

(ii) To play the equilibrium of the stage game at any date \( \tau' > \tau \) if a deviation occurred before \( \tau \);

(iii) To charge the punishment price \( p^{P} = w^c \) at any date \( \tau \) where there is a public deviation such that the deviant, say \( R_j \), offers \( w^c_j \neq w^c \).\(^{20}\)

Of course, \( \hat{\sigma} \) entails a penal code that is not optimal. This is because retailers cannot price below marginal costs when punishing a public deviation. However, in Section 5.2, I will argue that the qualitative insights of the model do not change, and get actually strengthened, when retailers use a penal code based on the stick-and-carrot logic. It is important to note, though, that a minmaxing behavior occurs following a secret deviation. This is because when retailers spot a secret price cut, they will revert for the rest of the game to the equilibrium of the stage game that leads each player to get the lowest possible (individually rational) payoff that can be achieved in the stage game.

For \( \hat{\sigma} \) to be self-enforceable downstream and upstream firms must not have profitable deviations. First, suppliers must make non-negative profits when accepting \( C^c \) — i.e.,

\[
\frac{1}{N} D (p^c) w^c + T^c \geq 0. \tag{4}
\]

That is, in the collusive phase, the sum of the franchise fee and sales revenue must be non-negative.

Two types of deviations must be considered for downstream firms. Each retailer must not find it worthwhile to offer \( C^c \), and then undercut \( p^c \): a secret deviation. Under strategy \( \hat{\sigma} \), this behavior cannot be profitable if:

\[
V^c \equiv \frac{1}{N} D (p^c) (p^c - w^c) - T^c \geq D (p^c) (p^c - w^c) - T^c. \tag{5}
\]

Moreover, retailers must also not find it convenient to deviate ‘publicly’ — i.e., they must not gain by offering both a contract and a retail price different than \( C^c \) and \( p^c \), respectively. Given strategy \( \hat{\sigma} \), it is easy to show that the best public deviation is always to propose a zero wholesale price and set a final price slightly below \( w^c \). Formally, a public deviation is not profitable as long as the following

\(^{19}\)Arguably, this behavior rules out cases where, following a public deviation, least efficient retailers — i.e., those who are cheated — charge a price so low that they would make losses were the most efficient competitor — i.e., the deviant — mistakenly charging a too high price and retailers have limited liability constraints.

\(^{20}\)I am considering here single-player deviations for simplicity. Of course, it is easy to construct punishment codes based on the same logic in case of multilateral deviations.
self-enforceability constraint is met:

\[
V^c \geq \max_{C_i \in \mathbb{R}^2} \{ D(w^c)(w^c - w_i) - T_i : T_i = -D(w^c)w_i \} \equiv \phi(w^c). \tag{6}
\]

That is, the intertemporal profit that each retailer earns on the cooperative path must exceed the profit that it would make by cutting the price slightly below the punishment price charged by its competitors when spotting a public deviation — i.e., \(w^c\).

Hence, the retail price \(p^c\) and the wholesale contract \(C^c\) will be chosen so as to maximize retailers’ joint profits subject to the self-enforceability and participation constraints — i.e.,

\[
P \left\{ \begin{array}{c}
\max_{(p,C) \in \mathbb{R}^3} D(p)(p - w) - NT, \\
\text{s.t.} (4), (5) \text{ and } (6).
\end{array} \right.
\]

The economic forces shaping the solution of program \(P\) hinge on the effect of the collusive wholesale price \(w^c\) on the incentive constraints (5) and (6). To see why, note that suppliers’ participation constraint binds. Substituting \(T^c = -(1/N) D(p^c) w^c\) into (5) and (6) one gets:

\[
\frac{\phi(p^c)}{N(1 - \delta)} \geq \psi(w^c; p^c) \equiv \max \left\{ D(p^c) \left( p^c - \frac{w^c(N - 1)}{N} \right), \phi(w^c) \right\}, \tag{8}
\]

which leads to the next lemma:

**Lemma 2** Assume A1-A3. The following properties hold:

(i) For any \(p^c \in [0, p^m]\), there exits a threshold \(w^c(p^c) \in (0, p^m)\) such that:

\[
\psi(w^c; p^c) = \begin{cases} 
D(p^c) \left( p^c - \frac{w^c(N - 1)}{N} \right) & \forall \ w^c \leq w^c(p^c) , \\
\phi(w^c) & \forall \ w^c \in (w^c(p^c), p^m].
\end{cases}
\]

(ii) When collusion is sustainable for values of \(\delta\) below \((N - 1)/N\), contract \(C^c\) must require positive wholesale prices \((w^c > 0)\) and negative franchise fees \(T^c < 0\).

The trade-off shaping the self-enforceability constraint (8) is as follows. A higher \(w^c\) makes, ceteris paribus, public deviations more attractive — i.e., \(\phi(w^c)\) is increasing in \(w^c\) for all \(w^c \leq p^m\). This is because a high wholesale price — i.e., low downstream margin — makes the within-period punishment less severe. However, a low \(w^c\) — i.e., high downstream margin — makes secret deviations more attractive. This is because, undercutting the collusive price yields a larger profit when retailers face zero (or very low) marginal costs than when they are committed to pay large wholesale prices.

Building on these insights, the next theorem identifies the conditions that the collusive agreement between downstream firms must satisfy.
Theorem 1 Assume A1-A4. The solution of $P$ is such that:

(i) For $\delta \geq (N-1)/N$ full collusion is compatible with the efficient contract. Retailers share uniformly the monopoly profit and offer the efficient contract $C^*$. 

(ii) There exists a threshold $\delta^1(N) < (N-1)/N$, such that for $\delta \in [\delta^1(N), (N-1)/N]$ full collusion is viable. In this region of parameters retailers share uniformly the monopoly profit and offer contract $C^m = (w^m, T^m)$, with $w^m > 0$ solving:

$$\frac{p^m - w^m}{p^m} = \frac{\phi(w^m)}{\phi(p^m)} - \frac{w^m}{p^mN}, \quad (9)$$

the franchise fee is $T^m = -(1/N)D(p^m)w^m$.

(iii) There exists a threshold $\delta^0(N) < \delta^1(N)$ such that for all $\delta \in [\delta^0(N), \delta^1(N)]$ collusion is sustainable but it implies (retail) prices lower than the monopoly level — i.e., $p^c < p^m$. In this range, the collusive retail price, $p(\delta, N)$, and the wholesale price, $w(\delta, N)$, solve the system of equations:

$$\frac{p(\delta, N) - w(\delta, N)}{p(\delta, N)} = \frac{1}{N(1-\delta)} - \frac{w(\delta, N)}{p(\delta, N)N}, \quad (10)$$

$$\frac{p(\delta, N) - w(\delta, N)}{p(\delta, N)} = \frac{\phi(w(\delta, N))}{\phi(p(\delta, N))} - \frac{w(\delta, N)}{p(\delta, N)N}, \quad (11)$$

with $p(\delta, N) \in [0, p^m)$ and $w(\delta, N) \in [0, p(\delta, N))$. The franchise fee is negative — i.e., $\overline{T}(\delta, N) = -(1/N)D(p(\delta, N))w(\delta, N) < 0$.

(iv) The threshold $\delta^0(N)$ solves $\underline{p}(\delta, N) = 0$, whereas

$$\delta^1(N) = 1 - \frac{\phi(p^m)}{N\phi(w^m)}. \quad (12)$$

The intuition for this result is simple and relies on the trade-off analyzed in Lemma 2. When collusion is not sustainable with the efficient contract, retailers increase the wholesale price in order to squeeze the wedge between collusive and deviation profits. This increase in the wholesale price, however, is limited by the fact that an excessively high wholesale price could induce retailers to deviate publicly.

Summing up, when the penal code entails the repetition of the static equilibrium, a collusive strategy that maximizes downstream (aggregate) profits specifies a retail price $p^c$ and a wholesale price $w^c$ having the following features:

Insert Figure 1 here
Of course, if retailers are patient enough, collusion is efficient exactly as in the infinitely repeated Bertrand game with integrated firms. However, while in such a model, retailers’ temptation to undercut each other would be so high for $\delta < (N - 1)/N$ to frustrate any attempt of cooperation, with vertical separation this is not always true. A careful design of wholesale contracts can still make cooperation viable in this region of parameters. When the discount factor is not large, $\delta < \hat{\delta}^1(N)$, full collusion is not sustainable, yet a sufficiently large wholesale price can suffice to sustain equilibria with positive profits. In this region of parameters the monopoly profit is not sustainable because private deviations secure very high profits. This leads to reduce the collusive profit below the monopoly level to prevent such deviations. Finally, for very small discount factors, $\delta < \hat{\delta}^0(N)$, only the competitive outcome can be sustained.

Note that, whilst Schinkel et al. (2008) show that upstream collusion requires low wholesale prices when the bargaining power is in the suppliers’ hands, I find the opposite for downstream collusion. Moreover, note that besides the aforementioned static work on buyer power, there might be other stories that could square inefficient wholesale deals and public contracting — see, e.g., the large body of work on double marginalization, Motta (2000, Ch. 6). But, in this literature, where the initiative is on the suppliers’ hands, franchise fees are typically positive and have beneficial welfare effects insofar as they prevent double marginalization.

**The linear example.** I now construct a simple example putting Theorem 1 at work. Consider the linear demand function $D(p) = \max \{0, 1 - p\}$, such that $p^m = 1/2$ and $\bar{p} = 1$. First, solving (9) one gets:

$$w^m = \frac{3}{4} - \frac{1}{4N} - \frac{1}{4N} \sqrt{5N^2 - 6N + 1},$$

one can check that $w^m$ is decreasing in $N$ and that $w^m \in (0, 1/2)$. This expression together with (12) yields

$$\hat{\delta}^1(N) = 1 - \frac{1/4}{N \left(1 - w^m(N)\right) w^m(N)}.$$

Simple algebra allows to verify that $\hat{\delta}^1(N) \in (0, (N - 1)/N)$ and that $\hat{\delta}^1(N)$ is increasing in $N$.

Consider now the region of parameters where collusion is inefficient — i.e., $\delta < \hat{\delta}^1(N)$. Solving the system of equations (10)-(11) we have:

$$p(\delta, N) = \frac{(N - 1)(1 - \delta) \left(N^2 (1 - \delta) - 2N + 1\right)}{3N + \delta - 2N\delta - 3N^2 + N^3 + 3N^2\delta - 2N^3\delta + N^3\delta^2 - 1},$$

and

$$w(\delta, N) = \frac{(N (1 - \delta) - 1) \left(N^2 (1 - \delta) - 2N + 1\right)}{3N + \delta - 2N\delta - 3N^2 + N^3 + 3N^2\delta - 2N^3\delta + N^3\delta^2 - 1},$$

---

21 A paper related to Schinkel et al. (2008), which deals with collusion in oligopolistic markets, is Maskimovic (1988). He studies the effect of firms’ capital structure on collusion, and shows that high levels of debt may prevent a firm from credibly committing itself not to engage in disruptive competitive practices — i.e., the firm may be prevented from reaching an optimal tacit collusive agreement with its rivals.
which imply $0 \leq w(.) \leq p(.)$ for $\delta < (N - 1)/N$ and $p(\delta, N) = w(\delta, N) = 0$ for

$$\delta^0(N) = \left[\frac{N - 1}{N}\right]^2.$$  

Moreover, $\delta^0(N)$ is increasing in $N$, $\Delta\delta(N) = \delta^1(N) - \delta^0(N) \geq 0$ and $\Delta\delta(N)$ is inverted-U shaped with respect to $N$, with $\lim_{N \to +\infty} \Delta\delta(N) = 0$.

In the duopoly case ($N = 2$), it is easy to check that $w^m = 1/4$ and $\delta^1(2) = 1/3$. So, in the region of parameters where $\delta \in [1/3, 1/2]$ full collusion is sustained by a contract with a positive wholesale price, $w^m = 1/4$, and a negative fee, $T^m = -1/16$. Differently, below $\delta^1(2) = 1/3$, one has:

$$\frac{1}{2} \geq p(.) = \frac{(1 - \delta)(1 - 4\delta)}{1 + 8\delta^2 - 7\delta} \geq w(.) = \frac{(1 - 2\delta)(1 - 4\delta)}{1 + 8\delta^2 - 7\delta},$$

where $\delta^0(2) = 1/4$. Graphically:

Insert Figure 2 here

Both the retail and the wholesale prices are increasing in $\delta$ in the relevant range of parameters: more patient players can enforce implicit agreement sustaining larger retail prices. Clearly, the larger is the collusive price the higher is the wholesale price that refrains retailers from deviating.

5 Extensions

A few simplifying assumptions have been imposed so far. First, contracts were assumed to be public. Second, the punishment following a public deviation is not an optimal one. Here I show how the insights of the model change when these hypotheses are weakened each in turn. Finally, I also discuss some results for the case where the bargaining power lies on the suppliers’ side.

5.1 Private contracts

Suppose that contracts are unobservable (private). The key difference with the analysis developed above is that, here, a deviation in contracts can no longer be detected instantaneously. The punishment phase must then start with one period delay under all circumstances. The objective of the section is to show that this limit on retailers’ communication increases the lowest discount factor above which positive profits can be sustained in equilibrium. Since contracts are unobservable, the equilibrium concept is PBE with the added passive beliefs refinement: that is, when a supplier is offered a contract different from the one he expects in equilibrium, he does not revise his beliefs about the contract offered to the other suppliers.
The static outcome: It is straightforward to show that the static game still features a unique competitive and efficient equilibrium. This is simply because, absent public deviations, any candidate equilibrium where all retailers are expected to make profits can be successfully undercut by a deviant who can steal all customers from its rivals exactly as in the textbook Bertrand logic.\textsuperscript{22}

Repeated interactions: As before, I focus on a symmetric and stationary collusive strategy. The penal code follows Nash reversion trigger strategies, which here also imply minimaxing. Hence:

Proposition 2 Assume A1-A4. With private contracts, collusion can be sustained only in the region of parameters where $\delta \geq (N - 1)/N$. In this range, downstream firms are able to sustain monopoly profits and offer the competitive and efficient contract. For $\delta < (N - 1)/N$, there is a unique PBE which yields the competitive outcome.

Limits on retailers’ communication ability reduce collusion possibilities. When contracts are private, spot gains from deviation become so large to prevent cooperation below the critical discount factor $(N - 1)/N$. With public contracts, instead, retailers can limit this moral hazard problem by changing instantaneously their retail price in response to a public deviation.

Corollary 1 Public contracts are more likely to be anticompetitive the lower is $\delta$ and the larger is $N$.

This result offers simple testable predictions on the link between the anticompetitive use of wholesale contracts on the one hand, and the downstream market structure and retailers’ time preferences on the other. Information sharing agreements between retailers, which may help to enforce contracts observability, should be more likely to harm consumers in environments featuring a larger number of competitors in the downstream market and/or more shortsighted firms.

5.2 Harsher punishments after a public deviation

So far, I assumed that, following a public deviation, retailers never set final prices below marginal costs. This particular penal code implies that if a retailer deviates publicly by offering a wholesale price lower than $w_c$, its rivals will charge a final price equal to $w_c$ in the corresponding subgame. The deviant retailer then charges a final price slightly below $w_c$, gets the whole market and makes positive (spot) profits. This argument clearly rules out optimal penal codes à la Abreu (1986) and (1988). It turns out that, in the model at hand, it is easy to construct a penal code that punishes public deviations according to the simple stick-and-carrot logic.

\textsuperscript{22}Note that passive beliefs play an important role in constructing the equilibrium of the stage game under private contracts. Under passive beliefs when a supplier is offered an unexpected contract — i.e., undercutting the equilibrium candidate in the most interesting case — he believes that rival retailers are sticking to the equilibrium offer. And, on the basis of this conjecture he accepts any such undercutting that meets his participation constraint. This easily allows to destroy equilibria with positive prices exactly as in the Bertrand logic. When beliefs are arbitrary, such argument might fail because deviations must be tailored to beliefs. Positive price equilibria might then exist simply because the deviations that ruled them out with passive beliefs may now be no longer profitable. See, e.g., Pagnozzi and Piccolo (2010).
Proposition 3 Assume A1-A4. The downstream cartel achieves the monopoly profit regardless of $\delta$. For $\delta < (N-1)/N$, this equilibrium is supported by a contract $C^m = (w^m, T^m)$ with $w^m = p^m$ and $T^m = -(1/N)D(p^m)p^m < 0$.

The economic intuition of this result is simple. Under A4 a penal code based on the stick-and-carrot logic can be easily constructed to punish public deviations. To see why, consider the following penal code: when a deviant offers a wholesale price below $w^c$, its rivals charge a final price equal to zero in that period, and revert to collusion if all adhere to the punishment. Otherwise, in the next period, they offer the efficient contract and charge a final price equal to zero; and return to collusion afterwards if there was no deviation from the punishment. Of course, given this strategy there is no spot gain from offering a wholesale price below $w^c$. Moreover, the punishment is credible: cheated retailers do not make sales in the punishment phase because the deviant is more efficient and serves the whole market when they all charge the same final price.\textsuperscript{23}

Interestingly, Proposition 3 strengthens the conclusions of the baseline model. Not only full collusion can be achieved regardless of the discount factor when retailers use stick-and-carrot penal codes to punish public deviations, but negative fees are still necessary to get such outcome in the region of parameters where $\delta$ falls short of $(N-1)/N$. Another implication of the result is that there is no need for non-linear contracts more sophisticated than simple two-part tariffs to sustain monopoly profit.

5.3 Bargaining power on the suppliers’ hands

In this last extension I discuss how results change when suppliers have the bargaining power. Consider a game where each supplier makes a take-it or leave-it offer to its exclusive retailer. The timing is the same as that in game $G$, with the difference that here suppliers propose two-part tariffs. Contracts are public.\textsuperscript{24}

Following the literature dealing with upstream collusion — see, e.g., Jullien and Rey (2007) — I consider the case where suppliers are infinitely lived and discount future at the rate $\delta \in (0,1)$, whereas retailers are short-lived and just maximize their spot profit — i.e., they are too short-sighted to collude at their level. In this game, because of Bertrand competition, every symmetric profile of wholesale contracts leads each downstream firm to set its retail price equal to the wholesale price — i.e., $p_i = w^c$ for all $i$ as long as all suppliers offer $C^c = (w^c, T^c)$. Hence, upstream firms can sustain collusion only by offering contract $C^c$, specifying a positive wholesale price ($w^c > 0$). As before, following a deviation, suppliers offer the competitive and efficient contract $C^*$ from the next stage onwards: so that all players make zero profits in the punishment phase.

In this setting the concepts of public and secret deviations are equivalent: anticipating its retailer’s behavior, a supplier can deviate from a collusive agreement only by charging a wholesale price slightly lower than $w^c$. So, in order for collusion to be viable, the following conditions must be satisfied:

\textsuperscript{23}Remember that it is never convenient to deviate by raising the wholesale price above $w^c$.

\textsuperscript{24}The case of private contracts is uninteresting for obvious reasons.
(i) As in the collusive phase \( p^c = w^c \), the retailer’s participation constraint requires:

\[
-T^c \geq 0. \tag{13}
\]

(ii) Suppliers must not gain by undercutting \( w^c \). For this behavior not to be convenient, the following incentive constraint must hold:

\[
\frac{\phi(p^c)}{N} + T^c \geq \phi(p^c) + T^c \implies \phi(p^c) \left[ \frac{1 - N(1 - \delta)}{N} \right] \geq -\delta T^c. \tag{14}
\]

Combining equations (13) and (14), it follows that any price between competition and monopoly can be sustained for \( \delta \geq (N - 1)/N \). However, since retailers will never accept to pay a positive franchise fee, as implied by (13), there cannot be collusion below the threshold \( (N - 1)/N \). This leads to the following result:

**Proposition 4** If the bargaining power is on the suppliers’ side and retailers are shortsighted, even with public contracts, collusion can only be sustained for \( \delta \geq (N - 1)/N \). Otherwise, there is only one equilibrium entailing the competitive outcome.

Hence, the emergence of collusion is more likely when the bargaining power is on the retailers’ hand. Precisely in these instances, negative franchise fees might actually reflect some form of implicit collusion. What is interesting, though, is that while with buyer power collusion can be sustained by the efficient contract \( C^* \) in the region where \( \delta \geq (N - 1)/N \), a positive wholesale price is always needed when the bargaining power is on the other side of the market.

Clearly, the result of Proposition 4 may dramatically change when retailers are long-lived and behave strategically vis-à-vis suppliers. In this case, the equilibrium outcome could exhibit features similar to those described in Theorem 1 in the range of parameters where \( \delta < (N - 1)/N \). But, the way things work in this more complex game heavily rely on how the collusive surplus \( \phi(p^c)/N \) is shared within each supply chain, and it also depends on whether the sharing rule that would sustain collusion is itself self-enforceable. More precisely, to achieve cooperative outcomes in this environment, suppliers might want to reward retailers — i.e., by promising not to extract a fraction larger than \( \alpha > 0 \) of the total surplus \( \phi(p^c)/N \) — as long as they ‘behave well’ by keeping retail prices above marginal costs in the collusive phase, and punish them by extracting the whole downstream surplus otherwise. Were this possible, the same logic developed above would enforce collusion even below the critical value \( (N - 1)/N \), as it can be seen from the constraint (14). However, in this scenario, one needs to be careful: retailers might hold up suppliers by grabbing the ex ante side payment \( (1 - \alpha) \phi(p^c)/N \), and then undercut rivals ex post by increasing their individual demand up to \( D(p^c) \), so as to enjoy larger sales profits at the expense both of suppliers and rivals. This extra moral hazard problem adds a source of complexity to my model that is certainly worth studying, but that goes behind the scope of the current paper, whose main focus is on buyer power. I plan to address this and related issues in future research.
6 Concluding remarks

The analysis has achieved two main results. First, the model throws new light on the role that a careful design of wholesale arrangements plays in softening competition in a dynamic framework with buyer power. I argued that inefficient vertical contracting emerges as a mechanism to implement collusion among retailers when cooperation is not sustainable with efficient wholesale deals. In this case, collusion must be supported by wholesale contracts featuring negative franchise fees, a practice intensely debated by antitrust and competition policy authorities. The second main insight of the paper is about the effect of communication between competing retailers on the set of collusive outcomes. Communication turns out to be fundamental to strengthen cartels’ sustainability, although generating efficiency losses.
Appendix

Proof of Proposition 1. Some preliminary notation is useful before proving the result. Given the profile of contracts $C$, throughout I will defined with $p^e(C) = (p_1^e(C), \ldots, p_N^e(C))$ the Nash equilibrium of the retail stage game following $C$. Moreover, $p^e(C_j^e, C_i) \equiv (p_j^e(C_j^e, C_i), p_i^e(C_i, C^e))$, where $p_j^e(C_j^e, C_i)$ for all $j \neq i$, is the price vector chosen in the selected Nash equilibrium of the subgame corresponding to the history where $C_j = C_j^e$ for all $j \neq i$ and $C_i \neq C^e$. While $p^e \equiv p_i^e(C_i)$ is the retail price obtained in the symmetric equilibrium of game $G$.

Consider the symmetric equilibrium where downstream firms share evenly final demand, offer $C^e \equiv (w^e, T^e)$ and charge $p^e$. In the following steps I will show that there exists a unique SPNE such that $C^e = C^*$ and $p^e = 0$ consistent with weak Pareto-dominance.

Step 1. There cannot exist a symmetric SPNE where $p^e > w^e > 0$.

The proof is by contradiction. Suppose that there exists a SPNE where $p^e > w^e > 0$. In this equilibrium candidate each retailer earns a profit of

$$\pi^i (p_j^e | C^e) = \frac{D(p_j^e)}{N} (p_j^e - w^e) - T^e, \quad \forall i = 1, \ldots, N.$$  

Consider the following deviation: $R_i$ offers $C^e$, but charges a final price $p_i$ slightly below $p^e$. Given the rivals’ equilibrium strategies, this deviation yields to $R_i$ a profit of

$$\pi^i (p_i | C^e) = D(p_i) (p_i - w^e) - T^e.$$  

For $p^e > w^e$, this implies

$$\pi^i (p_i | C^e) \approx D(p^e) (p^e - w^e) - T^e > \pi^i (p^e | C^e) = \frac{D(p^e)}{N} (p^e - w^e) - T^e.$$  

Hence, $R_i$ can profitably undercut $p^e$ and steal the entire market from its rivals. This provides the desired contradiction. A symmetric SPNE must then necessarily entail $p^e \leq w^e$.

Step 2. There cannot exist a symmetric SPNE where $p^e = w^e > 0$.

The proof is again by contradiction. Suppose that there exists a symmetric equilibrium such that $p^e = w^e > 0$. Consider the following public deviation: $R_i$ offers $C^* \equiv (0, 0)$. Given such unexpected offer and the fact that franchise fees are paid up-front, in the corresponding competitive subgame each retailer $R_j$ (with $j \neq i$) makes a total profit of

$$\pi^j (p | C^e) = \begin{cases} D_j (p) (p_j - p^e) & \text{if } D_j (p) > 0, \\ 0 & \text{otherwise}, \end{cases}$$  

(A.1)
given the vector of prices $p = (p_1, \ldots, p_N)$. Hence, it is straightforward to see that the subgame following such a public deviation features a continuum of Nash equilibria. Each of those equilibria is identified
by a pair of prices \((p_{-i}, p_i)\) such that: \(p_j = p^e(C_i, C_i) \in (0, p^e]\) for all \(j \neq i\), and \(0 \leq p_i \leq p^e_i(C_i, C^e)\). However, the unique Nash equilibrium that survives to weak Pareto-dominance is the one where \(p_j = p^e\) for all \(j \neq i\) and \(p_i = p^e_i\). Focusing on such continuation equilibrium, under A4 \(R_i\)'s deviation profit is:

\[
\pi^i(p^e|C^e) = \phi(p^e) > \pi^i(p^e|C^e) = \frac{\phi(p^e)}{N},
\]

which yields the desired contradiction.

**Step 3.** There cannot exist a symmetric SPNE where \(p^e < w^e\) and retailers share evenly final demand.

The proof of this result is straightforward. Suppose that there exists a SPNE where \(p^e < w^e\) and all retailers sell the same positive amount of final good. Given its rivals’ strategies, \(R_i\) would then be better-off by not selling at all — i.e., by setting \(p_i > p^e\), instead of \(p_i = p^e\). This is immediate because

\[
-T^e > -T^e + \frac{D(p^e)}{N} (p^e - w^e) \implies 0 > \frac{D(p^e)}{N} (p^e - w^e),
\]

for \(p^e < w^e\). A contradiction.

**Step 4.** There cannot exist a symmetric SPNE where all downstream firms offer \(C^e \equiv (w^e, T^e)\), with \(T^e > 0\), and charge a positive retail price \(p^e\).

The argument is by contradiction. Suppose that such an equilibrium exists. Since suppliers’ participation constraint must bind at equilibrium, \(T^e > 0\) requires \(w^e < 0\) — i.e.,

\[
T^e = -\frac{D(p^e) w^e}{N} > 0 \implies w^e < 0.
\]

For any positive price \(p^e\) this implies

\[
\pi^i(p^e|C^e) = \frac{D(p^e)}{N} (p^e - w^e) < D(p^e) (p^e - w^e).
\]

Hence, a profitable deviation for \(R_i\) would be to announce \(C^e\) according to the equilibrium strategy, but then undercut \(p^e\). A contradiction.

**Step 5.** There exists a symmetric SPNE where all downstream firms offer \(C^* \equiv (0, 0)\) and set a retail price equal to 0.

Consider the following profile strategy:

(i) Each retailer \(i\) offers \(C_i = C^*\);

(ii) Each retailer \(i\) charges \(p_i = 0\) as long as there is at least another retailer who offered \(C^*\) or if \(\min_{j \neq i} w_j < 0\); while it charges \(p_i = \min_{j \neq i} w_j\) if \(w_j > 0\) for all \(j \neq i\);

(iii) Supplier \(i\) accepts \(C_i\) if and only if \(T_i \geq 0\) and

\[
T_i + D_i (\tilde{p}^e(C^e, C_i)) w_i \geq 0,
\]
where \( \hat{p}^e (C^e, C_i) = (p^e (C^e, C_i), p_i^e (C_i, C^e)) \) is the retail price vector in the Nash equilibrium satisfying weak Pareto-domination of the subgame triggered by the deviation \( C_i \).

Showing that \((i) - (iii)\) identifies a SPNE is immediate. First, note that, given \((ii)\), if all retailers offer \( C^* \), the unique Nash equilibrium of the retail game is such that \( p_i = 0 \) for all \( i = 1, \ldots, N \). Hence, no downstream firm can profitably deviate by changing the retail price only.

I now show that no retailer can also profitably deviate by offering a contract \( C_i \neq C^* \). Given \((ii)\), no retailer can gain by offering \( C_i \) with \( T_i > 0 \). This is because \( S_i \)'s participation constraint would require \( w_i < 0 \) and in the corresponding subgame \( p_j = 0 \) for all \( j \neq i \) according to \((ii)\). Indeed, the maximal profit that \( i \) can expect to make from the retail price game is \( -D(0) w_i > 0 \), but then to accept \( C_i \) supplier \( S_i \) would require a fee

\[
T_i \geq -D(0) w_i,
\]

so that \( R_i \)'s (ex ante) profit would be

\[
-D(0) w_i - T_i \leq -D(0) w_i + D(0) w_i = 0,
\]

implying that \( C_i \) is not profitable. Note that charging \( p_j = 0 \) for all \( j \neq i \) satisfies perfection because this outcome is a Nash equilibrium of the game where at least two downstream firms have zero wholesale prices.

Next, suppose that \( R_i \) offers \( C_i = (w_i, T_i) \), with \( T_i < 0 \). This is also not profitable because \( S_i \) would refuse such an offer according to \((iii)\). This action is sequentially rational for \( S_i \). Given \((ii)\), \( S_i \)'s participation constraint implies \( T_i \geq -w_i D(0) \) and thus \( w_i \geq 0 \). Hence, if \( S_i \) would accept a negative fee, given \((ii)\), all retailers \( j \neq i \) will set \( p_j = 0 \) and \( R_i \) will then gain by not selling. Indeed, if \( R_i \) wishes to sell, it must charge a price \( p_i = 0 \), in this case its total profit would be \( -p_i D(0) = 0 \) and \( T_i < -T_i \). As a consequence, in the subgame following such a deviation \( R_i \) would prefer not to sell, so that \( S_i \) would make losses by accepting \( C_i \). So it is rational for him to refuse the offer. Showing that for all retailers \( j \neq i \) setting \( p_j = 0 \) satisfies perfection is again immediate since this outcome is a Nash equilibrium of the game where at least two downstream firms have zero wholesale prices.

Perfection must be checked also in all other contractual histories — i.e., for all possible off-equilibrium wholesale offers. Assume that \( C_i = C^* \). First, if \( \min_{j \neq i} \{ w_j \} < 0 \), retailer's \( i \) strategy satisfies perfection from the argument above, and the same is true in all histories such that \( \min_{j \neq i} \{ w_j \} \geq 0 \) and there exists at least another downstream firm offering \( C^* \). Finally, consider the case where all retailers but \( i \) offer a wholesale contract such that \( w_j > 0 \) for all \( j \neq i \). In this case \( p_i = \min_{j \neq i} w_j \) also satisfies perfection because weak Pareto-domination implies that no retailer with a wholesale price \( w_j > 0 \) will charge a price lower that \( w_j \).

Gathering steps 1-5 yields the result. ■

**Proof of Lemma 1.** Suppose that all retailers offer \( C^* \) along the equilibrium path. A collusive equilibrium can then be sustained as long as the self-enforceability conditions \((5)\) and \((6)\) hold. Since \( w^c = 0 \), it can be easily verified that these two inequalities are met if and only if \( \delta \geq (N - 1) / N \). Finally, since the self-enforceability is not met when all retailers offer \( C^* \) and \( \delta < (N - 1) / N \), it is immediate to show that, in this region of parameters, the unique SPNE of the game where retailers offer \( C^* \) at every stage entails zero retail prices. ■
Proof of Lemma 2. I will first show part (i) and then turn to (ii).

Part (i): The argument is straightforward. Consider the function $\psi(w^c; p^c)$. First, for $w^c = 0$:

$$\psi(0; p^c) = \max \left\{ D(p^c) \left( p^c - \frac{w^c(N-1)}{N} \right), \phi(w^c) \right\} = \phi(p^c) > 0,$$

moreover, for $w^c = p^c$:

$$\psi(p^c; p^c) = \max \left\{ D(p^c) \left( p^c - \frac{w^c(N-1)}{N} \right), \phi(w^c) \right\} = \phi(p^c) > 0.$$

Next, note that $D(p^c)(p^c - w^c(N-1)/N)$ is strictly decreasing in $w^c$ and positive at $w^c = 0$ for any $p^c > 0$. Moreover, $\phi(w^c)$ is strictly concave, it is equal to zero at $w^c = 0$ and has a unique maximum at $p^m$. Since $\phi(x)$ is single peaked, it must then be the case that, for any $p^c > 0$, there exists a unique threshold $w^c = \underline{w}(p^c) < p^c$ which equalizes $D(p^c)(p^c - w^c(N-1)/N)$ and $\phi(w^c)$, and thus minimizes

$$\max \left\{ D(p^c) \left( p^c - w^c(N-1)/N \right), \phi(w^c) \right\}.$$  

Note also that strict concavity of $\phi(x)$ implies $D'(w^c)w^c + D(w^c) > 0$ when $p^c \leq p^m$. Hence:

$$-\frac{N-1}{N} - \frac{D'(w^c)w^c + D(w^c)}{D(p^c)} \neq 0 \quad \forall \ w^c < p^c.$$

Hence, by the Implicit Function Theorem $\underline{w}(p^c)$ is continuous and differentiable around any $p^c < p^m$.

Part (ii): In order to show this result one first needs to argue that, as long as there exists a collusive equilibrium featuring a (strictly) positive retail price in the region of parameters where $\delta < (N-1)/N$, the collusive wholesale contract cannot be the efficient one. The argument is by contradiction. Suppose that there exists a collusive symmetric equilibrium where retailers charge a strictly positive retail price and offer $C^*$. Then (5) implies that secret deviations are always profitable. As a consequence, if a collusive equilibrium exists in this region of parameters, it must be the case that the wholesale price is strictly positive (with a negative franchise fee) or strictly negative (with a positive franchise fee). I now show that the latter case is impossible. Suppose that $w^c < 0$, then $T^c = -(1/N)D(p^c)w^c > 0$. Equation (5) implies:

$$p^c \frac{1 - N(1 - \delta)}{N(1 - \delta)} + \frac{w^c(N-1)}{N} \geq 0,$$

but this cannot be true in the relevant range of parameters. In fact, for $p^c \geq 0$ and $w^c < 0$ the above condition requires $\delta > (N - 1)/1$: a contradiction. Therefore, if a collusive equilibrium exists it must feature $w^c > 0$ and $T^c < 0$. □
Proof of Theorem 1. Given \( \hat{\sigma} \), the cartel’s program is:

\[
P: \begin{cases} 
    \max_{(p,C) \in \mathbb{R}^3} D(p) (p - w) - NT \\
    \text{s.t.} \\
    \frac{1}{N} D(p) w + T \geq 0, \\
    D(p) (p - w) \geq N \max \{(1 - \delta) D(p) (p - w) + \delta T, (1 - \delta) \phi(w) + T\}, 
\end{cases}
\]

Clearly, suppliers’ participation constraint must be binding in the optimum — i.e., \( T = -(1/N) D(p) w \). Hence, \( P \) becomes:

\[
P : \begin{cases} 
    \max_{(p,w) \in \mathbb{R}_+^2} \phi(p) \\
    \text{s.t.} \\
    \frac{\phi(p)}{N(1-\delta)} \geq \psi(w;p) \equiv \max \left\{ D(p) \left( p - \frac{w(N-1)}{N} \right), \phi(w) \right\} . 
\end{cases}
\]

Showing that for \( \delta \geq (N - 1)/N \) the monopoly price is self-enforceable and is supported by the efficient contract \( C^* \) is immediate and will thus be omitted. Then, consider \( \delta < (N - 1)/N \). The proof is developed in the steps below.

**Step 1.** For \( \delta < (N - 1)/N \), \( p^c > 0 \) implies \( w^c > 0 \).

This fact can be easily shown by contradiction. Suppose that \( w^c = 0 \) and \( p^c > 0 \). From the self-enforceability constraints one has:

\[ \phi(p^c) \geq N (1 - \delta) \max \{ \phi(p^c), 0 \} = N (1 - \delta) \phi(p^c), \]

which cannot hold for \( \delta < (N - 1)/N \).

**Step 2.** For \( \delta < (N - 1)/N \), \( w^c > 0 \) implies \( w^c < p^m \).

The argument is again by contradiction. Suppose that \( w^c \geq p^m \) and that \( p^c \geq w^c \) (a condition that I will check later). For \( \delta < (N - 1)/N \) strict concavity of \( \phi(x) \) implies:

\[ \phi(p^c) \leq \phi(w^c) < N (1 - \delta) \phi(w^c), \]

which is incompatible with (6).

**Step 3.** For \( \delta < (N - 1)/N \) and \( p^c \in (0, p^m] \), there exists a unique wholesale price \( w(p^c) < p^c \) such that:

\[ w(p^c) \equiv \arg \min_{w^c \geq 0} \psi(w^c; p^c). \]

See the proof of Lemma 2.

**Step 4.** For \( \delta < (N - 1)/N \), the monopoly profit can be sustained for \( \delta \geq \delta^1(N) \), with

\[ \delta^1(N) = 1 - \frac{\phi(p^m)}{N \phi(w^m)} < \frac{N - 1}{N}, \]

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as long as the equilibrium contract specifies a wholesale price \( w^m > 0 \) solving

\[
\frac{p^m - w^m}{p^m} = \frac{\phi (w^m)}{\phi (p^m)} = \frac{w^m}{p^m N}.
\]

The proof of this result requires to use step 3. Consider an equilibrium such that the monopoly price is self enforceable even below the critical value \((N - 1)/N\). For this to be true one must have:

\[
\phi (p^m) \geq N (1 - \delta) \max \left\{ D \left( p^m \right) \left( p^m - \frac{w^c (N - 1)}{N} \right), \phi (w^c) \right\}, \tag{A.2}
\]

implying that:

\[
\delta \geq 1 - \frac{\phi (p^m)}{N \max \left\{ D \left( p^m \right) \left( p^m - \frac{w^c (N - 1)}{N} \right), \phi (w^c) \right\}}.
\]

By step 3 there exists a unique value \( w^m = w (p^m) \) minimizing the right hand side of (A.3) and this value solves

\[
D \left( p^m \right) \left( p^m - \frac{w^m (N - 1)}{N} \right) = \phi (w^m),
\]

so that the wholesale price that maximizes the region of parameters where (A.2) is met is precisely \( w^m \). It then follows that the monopoly price can be sustained by way of the wholesale price \( w^m \) in the region of parameters where

\[
\delta \geq \delta^1 (N) \equiv 1 - \frac{\phi (p^m)}{N \phi (w^m)}.
\]

Clearly, by construction, below this threshold the monopoly price cannot be sustained. Moreover, it is easy to show that \( \delta^1 (N) < (N - 1)/N \). Indeed, for \( w^c = p^m \) it follows that

\[
\delta^1 (N) < 1 - \frac{\phi (p^m)}{N \max \left\{ D \left( p^m \right) \left( p^m - \frac{w (N - 1)}{N} \right), \phi (w) \right\}} \bigg|_{w = p^m} = \frac{N - 1}{N}.
\]

**Step 5.** For \( \delta < \delta^1 (N) \) there exists a unique solution \((p (\delta, N), w (\delta, N))\) of \( P \) which satisfies the following system of equations:

\[
D \left( p \right) \left( p - \frac{w (N - 1)}{N} \right) = \phi (w), \tag{A.3}
\]

\[
\frac{p}{N (1 - \delta)} = p - \frac{w (N - 1)}{N}. \tag{A.4}
\]

The argument follows directly from the fact that for any \( p^c > 0 \), the cartel’s optimal strategy implies \( w^c = w (p^c) \) in order to minimize the upper bound imposed by incentive compatibility on the strictly concave objective function \( \phi (p) \). It then follows that the unique positive solution of \( P \) must solve the system (A.3)-(A.4), which is equivalent to (10) and (11) in the statement of the theorem.
It remains to verify that \( p^c \geq w^c \). The proof of this fact follows immediately from \( (A.4) \). Indeed, it is easy to check that for \( \delta < (N - 1)/N \) it must be:

\[
\frac{(N - 1) (1 - \delta)}{(1 - \delta) N - 1} \geq 1,
\]

so that \( p^c \geq w^c \).

**Step 6.** For \( \delta \) close to zero, the unique solution of program \( \mathcal{P} \) that is compatible with positive sales must entail \( p^c = w^c = 0 \).

Consider first \( \delta = 0 \). In this case, it is immediate to verify that the solution of \( (A.3)-(A.4) \) yields \( p^c = w^c \) and \( p^c \in \Pi \equiv \{ p \geq 0 : \phi (p) = 0 \} \). Given \( A1 \) and \( A2 \), it is easy to verify that \( \Pi \equiv \{ 0, \bar{p} \} \). Hence, the only price level compatible with positive sales is \( p^c = w^c = 0 \).

Next, I show that in a neighborhood of \( \delta = 0 \) there cannot exist a collusive equilibrium with a positive price, so that for sales to be positive one must have \( p^c = 0 \). To prove this fact one needs to show how the implicit function \( p^c (\delta, N) \), which solves the system of equations \( (A.3)-(A.4) \), varies around the point \( \delta = 0 \). Differentiating \( (A.3)-(A.4) \):

\[
\frac{\partial p^c (\delta, N)}{\partial \delta} = \frac{-p^c (\delta, N) (D' (w^c (\delta, N)) w^c (\delta, N) + D (w^c (\delta, N)))}{(1 - \delta) \Delta (p^c (\delta, N), w^c (\delta, N), \delta, N)},
\]

where \( \Delta (.) \) is the determinant of the Jacobian corresponding to system \( (A.3)-(A.4) \),

\[
\Delta (.) = \left( \frac{D' (p^c (\delta, N)) p^c (\delta, N)}{(1 - \delta) N} + D (p^c (\delta, N)) \right) (1 - \delta) (N - 1) +
\]

\[
+ \left( D' (w^c (\delta, N)) w^c (\delta, N) + D (w^c (\delta, N)) \right) (1 - N (1 - \delta)) .
\]

Taking the limit for \( \delta \to 0 \) and selecting the solution \( \bar{p} \) such that \( D (\bar{p}) = 0 \) one gets:

\[
\lim_{\delta \to 0} \frac{\partial p^c (\delta, N)}{\partial \delta} \bigg|_{w^c (0, N) = p^c (0, N) = \bar{p}} = \frac{ND' (\bar{p}) \bar{p}}{D (\bar{p}) (N - 1)^2} > 0,
\]

hence, around \( \delta = 0 \) there will be no sales as long as one selects the price \( p^c (0, N) = \bar{p} \). As a consequence, positive sales around \( \delta = 0 \) are compatible only with \( p^c (0, N) = 0 \), where \( D (0) > 0 \) by definition. This concludes the step.

**Step 7.** There exists a lower-bound \( \delta^0 (N) \leq \delta^1 (N) \) such that for all \( \delta \leq \delta^0 (N) \), the unique solution of \( \mathcal{P} \) compatible with positive sales entails \( w^c (\delta, N) = p^c (\delta, N) = 0 \).

This result can be easily shown by noting that at \( \delta = 0 \) the solution of the system \( (A.3)-(A.4) \) entails \( p^c (\delta, N) > \bar{p} \). Hence, for all \( \delta \) close to 0 one must have \( p^c (\delta, N) = 0 \). Now, observe that for \( \delta \to (N - 1)/N \) the system \( (A.3)-(A.4) \) yields \( w^c = 0 \) and \( \phi (p^c) = 0 \), implying once again \( p^c \in \Pi \). Taking the solution with the highest price \( p^c = \bar{p} \), substituting \( \delta = (N - 1)/N \) and \( w^c = 0 \) into \( (A.3) \) one has:

\[
\Delta (\bar{p}) = D' (\bar{p}) \frac{N - 1}{N} .
\]
and thus:

\[
\frac{\partial p^c(\delta, N)}{\partial \delta} \bigg|_{\delta = \left(\frac{N-1}{N}\right)^-, p^c(.):=\overline{p}} = \frac{-D(0) N^2}{D'(\overline{p}) (N - 1)} > 0,
\]

implying that \(p^c(\delta, N) < \overline{p}\) for all \(\delta\) close to \((N - 1)/N\).

Observe that for any \(\delta \in (0, (N - 1)/N)\) the solution of the system \((A.3)-(A.4)\) is continuous since the demand function \(D(p)\) is continuous. From step 6 it then follows that there must exist a lower bound \(\overline{\delta}^0(N) < \overline{\delta}^1(N) < (N - 1)/N\) such that \(p^c(\overline{\delta}^0(N), N) \equiv 0\) for all \(\delta < \overline{\delta}^0(N)\). Moreover, step 6 also implies that \(0 < p^c(\delta, N) < \overline{p}\) for all \(\delta > \overline{\delta}^0(N)\) and \(\delta < (N - 1)/N\). Finally, showing that \(p^c(\overline{\delta}(N), N) \equiv 0\) also implies \(w^c(\overline{\delta}(N), N) \equiv 0\) is immediate from equations \((A.3)-(A.4)\).

**Step 8:** The price \(\overline{p}(\delta, N)\) is lower than \(p^m\) for all \(\delta < \overline{\delta}^1(N)\).

The proof follows from step 4: there cannot exist a zero-profit contract \((w', T')\) — i.e., with \(w'D(p^m) = -T'\) — which allows to sustain the monopoly price \(p^m\) in the region of parameters where \(\delta < \overline{\delta}^1(N)\).

Finally, the theorem follows from gathering the results demonstrated in steps 1-8. ■

**Proof of Proposition 2.** With private contracts only secret deviations matter. The collusive strategy \(\hat{\sigma}\) is now as follows:

- Each retailer keeps offering \(C^c\) and charging \(p^c\) in period \(\tau\) if no deviation has occurred before \(\tau\);
- Each retailer offers \(C^*\) and charges \(p^* = 0\) at any date \(\tau' > \tau\) if a deviation occurred before \(\tau\).

Of course, \(\hat{\sigma}\) must satisfy suppliers’ participation constraint, which is again \((4)\). Given \((p^c, C^c)\), self-enforceability here requires that no downstream firm must find it profitable to deviate by undercutting its rivals with a retail price slightly below \(p^c\) and by issuing the best wholesale contract given that rivals stick to \(C^c\). Formally:

\[
V^c \equiv \frac{1}{1 - \delta} \left[ \frac{1}{N} D(p^c) (p^c - w_i^c) - T^c \right] \geq \max_{C_i \in \mathbb{R}^2} \left\{ D(p^c) (p^c - w_i) - T_i : D(p^c) w_i + T_i \geq 0 \right\} \equiv \phi(p^c).
\]

It can be easily verified that any price between monopoly and perfect competition can be sustained for \(\delta \geq (N - 1)/N\) and that for \(\delta < (N - 1)/N\) there exists a unique competitive equilibrium. ■

**Proof of Corollary 1.** The proof of this result simply follows from the fact that with public contracting inefficient collusion emerges in the region of parameters where \(\delta < (N - 1)/N\). It is then immediate to show that the critical value \((N - 1)/N\) is increasing in \(N\). ■

**Proof of Proposition 3.** I first show that there exists a penal code based on the stick-and-carrot logic which allows to sustain full collusion for every \(\delta\). Consider the following strategy:

- On the equilibrium path, all retailers charge the monopoly price and offer the contract \(C^m = (T^m, w^m)\) with \(w^m = p^m\) and \(T^m = -(1/N)D(p^m)p^m\).
- Following a secret deviation in period \(\tau\), retailers offer the competitive contract \(C^*\) and charge the competitive price \(p^* = 0\) for the rest of the game.
When a retailer, say $R_i$, publicly deviates in stage $\tau$ by offering contract $C_i \neq C^m$ then:

- if $w_i > p^m$, all retailers $R_j$ ($j \neq i$) keep charging $p^m$ and offering $C^m$ for the rest of the game;
- if $w_i < p^m$, all retailers $R_j$ ($j \neq i$) charge a punishment price $p^p = 0$ in stage $\tau$, and return to offer $C^m$ and charge $p^m$ if $p^p_j = 0$ for all $j \neq i$. Otherwise, if $p^p_j \neq 0$ for some $j \neq i$, then all retailers offer $C^*$ and charge $p^* = 0$ at $\tau + 1$, and return to offer $C^m$ and charge $p^m$ if $C_j = C^*$ and $p^p_j + 1 = 0$ and for all $j$.

The incentive constraint associated to a secret price cut is then:

$$\frac{\phi(p^m)}{N(1-\delta)} \geq \frac{\phi(p^m)}{N} \implies \delta \leq 1.$$ 

Hence, it will never be convenient to deviate secretly. Consider now a public deviation such that $C_i \neq C^m$ and $w_i < p^m$, $R_i$’s incentive constraint is therefore:

$$\frac{\phi(p^m)}{N(1-\delta)} \geq -D(0)w_i - T_i + \frac{\phi(p^m)}{N(1-\delta)} \implies \frac{\phi(p^m)}{N(1-\delta)} \geq \frac{\delta}{1-\delta} \frac{\phi(p^m)}{N} \implies \delta \leq 1.$$ 

Hence, also a public deviation such that $w_i < p^m$ is not profitable. One must then check that the punishment following such a public deviation is actually credible. Under $A4$ this requires the following incentive-constraint

$$\frac{\delta}{1-\delta} \frac{\phi(p^m)}{N} \geq \frac{\delta^2}{1-\delta} \frac{\phi(p^m)}{N} \implies \delta \leq 1,$$

which is clearly always satisfied.

Finally, consider a public deviation with $w_i > p^m$. Because lump sum fees are sunk when retail prices are chosen, sequential rationality implies that $w_i$ will never exceed $p_i$, therefore $S_i$ will never accept such a contract $C_i$ (otherwise it will make losses).

To conclude the proof one must show that for $\delta < (N - 1)/N$ positive profits can be sustained only if $w^c > 0$ and $T^c < 0$. This is immediate by Lemma 1. Moreover, since a higher $w^c$ reduces the gain from a secret deviation — see equation (8) — it is immediate to conclude that the optimal penal code requires $w^c = w^m > 0$. ■

**Proof of Proposition 4.** The proof is in the text. ■
Figure 1: Retail and wholesale prices

Figure 2: Linear duopoly ($N = 2$)
References


