Are Kaldor and Kuznets facts theoretically compatible? *

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Abstract  
Growth literature has shown that the Kaldor and Kuznets facts can only be simultaneously explained in a growth model when knife-edge conditions on parameters are introduced. In particular, Kongsamut, Rebelo and Xie (2001) show that in a growth model with Stone-Geary preferences the two sets of facts are compatible when the value of the minimum consumptions is equal to zero. In this paper, by interpreting this value as a state variable, we show that the Kaldor and Kuznets facts can be explained even when the value of the minimum consumption is different from zero. Thus, we show that a growth model with non-homothetic preferences can explain these growth facts even though strong knife-edge conditions are not introduced. Using numerical examples, we also show that the fit of the simulations increases when we assume that the value of the minimum consumptions is positive and when differences across sectoral labor income shares are introduced.

JEL classification codes: O41, O47.

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1. Introduction

Two well-known sets of facts that developed economies satisfy are the Kaldor and Kuznets facts. The Kuznets facts are defined by the change in the sectoral shares of labor and consumption. The Kaldor facts are characterized by an almost constant interest rate and an almost constant value of the ratio of capital to GDP. These two sets of facts cannot be generally explained by multisector-growth models where structural change is driven only by the imbalances between production factors. In these models, the equilibrium exhibits structural change and unbalanced growth during the transition, whereas it exhibits a constant sectoral composition and balanced growth in the long run. Recently, the growth literature has studied growth models that can explain simultaneously both sets of facts because they introduce other sources of structural change. This literature has distinguished between models where structural change is driven by demand factors and models where it is driven by supply factors.

On the one hand, supply factors are studied by Ngai and Pissaradies (2007), Acemoglu and Guerrierie (2008), Melck (2002), among many others. Ngai and Pissaradies (2007) assume that there is biased technological change and thus the change in relative prices drives structural change in this model. They show that reconciling Kaldor and Kuznets facts only occurs when knife-edge conditions on both preferences and technology are introduced. The strong condition on preferences is introduced to make the saving decisions independent of relative prices. Regarding the condition on technology, they assume that all sectors have the same capital intensity. This assumption implies that the ratio between capital and GDP does not depend on the sectoral composition, which is necessary to make both sets of facts compatible.

On the other hand, demand factors are studied by Kongsamunt, Rebelo and Xie (2001) (KRX). These authors consider a growth model where consumption decisions are driven by Stone-Geary preferences. These preferences introduce minimum consumption requirements for every consumption good which makes the utility function be non-homothetic. Obviously, this non-homothetic preferences drive structural change in a growing economy. These authors show that this model can explain balanced growth with structural change when the value at market prices of the minimum consumption requirements is zero. Obviously, this is a knife-edge condition that requires strong assumptions on both preference parameters and technology. As shown by KRX, when this condition is satisfied, the intertemporal decision on consumption expenditures is driven by homothetic preferences, which implies that aggregate variables converge to a BGP. Therefore, when this knife-edge condition is introduced, the model can explain simultaneously structural change and balanced growth of the aggregate variables.

Related literature has explained the Kuznets and Kaldor facts by introducing strong knife-edge conditions. In this paper, we show that these facts can be explained in a growth model without imposing knife-edge conditions. In particular, we analyze a growth model with different sectoral capital intensities, where growth in the long run is driven by exogenous TFP growth. Structural change is driven by non-homothetic preferences and by the change in relative prices due to capital deepening (see Acemoglu and Guerrierie, 2008). The utility function is non-homothetic because minimum-consumption requirements are introduced, as in KRX.

We show that the equilibrium of this model is characterized by a two-dimensional
manifold. Therefore, there is a continuum of equilibrium paths that can be selected using the initial conditions on two different state variables: capital intensity and the intensity of the minimum consumption requirements. We analyze the equilibrium of this model when the sectors have the same capital intensities and thus the model is directly comparable with the one analyzed by KRX. We show that these authors, by imposing a knife-edge condition, select one particular equilibrium. Along this equilibrium, convergence in the aggregate variables is faster than convergence in the sectoral composition. This implies that eventually aggregate variables exhibit balanced growth, while there is structural change. Obviously, this implies that this equilibrium explains both the Kuznets and the Kaldor facts. By using a continuity argument, we conjecture that these facts can also be explained by other equilibrium paths that are close enough to this equilibrium. We numerically simulate the economy and we show that there is a continuum of equilibrium paths that explain both Kuznets and Kaldor facts, which implies that these two facts can be explained without introducing the knife-edge condition. This is the main contribution of this paper.

We also show that the fit of the numerical simulations increases if we assume that the intensity of the minimum consumption requirements is initially positive and large. This implies that demand factors can explain sectoral change when we assume that the intensity of the minimum consumption requirements is initially large, it is decreasing and it converges to zero. However, this model of structural change driven only by demand factors cannot explain simultaneously the sectoral change in labor and expenditure shares. We then introduce differences across sectors in the labor income shares and, thus, capital intensities are different across sectors. As a consequence, prices change along the transition implying that supply factors also drive structural change. From numerical simulations, we show that this model of sectoral change driven by both supply and demand factors explains the sectoral change in the labor shares and the main trends in the expenditure shares.

2. The Model

2.1. Consumers

Let us consider a representative consumer that obtains income from capital, $K$, and labor, $L$. This income is devoted to either consumption or investment. Therefore, the budget constraint is

$$rK + wL = \sum_{i=1}^{m} p^i c^i + \dot{K} + \delta K,$$

where $r$ is the interest rate, $w$ is the wage, $c^i$ is the amount consumed of good $i$, $p^i$ is the price of good $i$ in units of the investment good, $m$ is the number of sectors and $\delta$ is the depreciation rate of capital. The representative consumer maximizes the following utility function:

$$U(c^i) = \left[ \sum_{i=1}^{m} \theta^i (c^i - \bar{c}^i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \left( \frac{1}{1 - \sigma} \right),$$

\[ \text{(2.1)} \]

\[ ^{1}\text{For the sake of simplicity, time subindexes are not introduced.} \]
where $\theta^i$ provides the weights of the different consumption goods in the utility function, $\tilde{c}^i$ is the minimum consumption of good $i$, $\varepsilon > 0$ is the elasticity of substitution between the different consumption goods and $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution.\footnote{This utility function includes as particular cases the non-homothetic Cobb-Douglas utility function considered by Kongsamunt, Rebelo and Xie (2001) (assume that $\varepsilon = 1$) and the homothetic constant elasticity of substitution utility function studied by Ngai and Pissariadis (2007) (assume that $\tilde{c}^i = 0$).}

The representative consumer maximizes the discounted sum of utilities subject to the budget constraint. The first order conditions are

$$U_i(c^i) = p^i \lambda, \quad (2.2)$$

and

$$\frac{\dot{\lambda}}{\lambda} = \delta + \rho - r, \quad (2.3)$$

where $\rho > 0$ is the subjective discount rate. Using (2.2), we obtain $U_i = p^i U_m$, for all $i$. Note that this implies that

$$(c^i - \tilde{c}^i) = \left( \frac{p^i \theta^m}{\theta^i} \right)^{-\varepsilon} (c^m - \tilde{c}^m) \quad (2.4)$$

and log-differentiating with respect to time we obtain

$$\frac{\dot{c}^i}{c^i - \tilde{c}^i} = -\varepsilon \left( \frac{p^i}{p^m} \right) + \frac{\dot{c}^m}{c^m - \tilde{c}^m}. \quad (2.5)$$

We define $n^i = p^i (c^i - \tilde{c}^i) / (c^m - \tilde{c}^m)$ as the fraction between the value of effective consumption in sector $i$ and value of effective consumption in sector $m$.\footnote{This variable was first introduced by Ngai and Pissariadis (2007).} Using (2.4), we obtain

$$n^i = (p^i)^{1-\varepsilon} \left( \frac{\theta^m}{\theta^i} \right)^{-\varepsilon}, \quad (2.6)$$

and $n^m = 1$.

Note that using (2.2) and (2.3), we obtain

$$\delta + \rho - r = \frac{\sum_{i=1}^m U_{mi} \dot{c}^i}{U_m} = \frac{U_{mm} \dot{c}^m}{U_m} + \frac{\sum_{i=1}^{m-1} U_{mi} \dot{c}^i}{U_m}, \quad (2.7)$$

In Appendix A, we use (2.1) to rewrite (2.7) as follows

$$\frac{\dot{E}}{E - \bar{E}} = \frac{r - \delta - \rho}{\sigma} + \left( \frac{\sigma - 1}{\sigma (1 - \varepsilon)} \right) \frac{\dot{x}}{x}, \quad (2.8)$$

where $x = \sum_{i=1}^m n_i$ is the ratio between the value of total effective consumption and effective consumption in sector $m$, $E = \sum_{i=1}^m p^i c^i_t$ is the value of consumption expenditures and $\bar{E} = \sum_{i=1}^m p^i \tilde{c}^i$ is the value of the minimum consumption requirements.
2.2. Firms

We assume that there are $m$ sectors characterized by the following Cobb-Douglas technology:

$$Y^i = (s^i K)^{\alpha^i} (A^i u^i L)^{1-\alpha^i} = A^i u^i L (z^i)^{\alpha^i},$$

where $s^i$ is the share of total capital employed in sector $i$, $u^i$ is the share of labor in sector $i$, $A^i$ is the total factor productivity in sector $i$, $\alpha^i$ is the capital income share in sector $i$ and $z^i = s^i K / A^i u^i L$. We assume that $A^i$ grows at the exogenous growth rate $\gamma$, which is identical across sectors. This assumption implies that technological change is unbiased. We also assume perfect competition and perfect mobility across sectors of the production factors. Then,

$$w = A^i p^i (1 - \alpha^i) (z^i)^{\alpha^i},$$

and

$$r = p^i \alpha^i (z^i)^{1-\alpha^i}.$$

Let sector $m$ be the manufacturing sector which we consider as the numeraire, $p^m = 1$. Using the first order conditions from the firms’ problem, we obtain

$$A^i p^i (1 - \alpha^i) (z^i)^{\alpha^i} = A^m (1 - \alpha^m) (z^m)^{\alpha^m},$$

and

$$p^i \alpha^i (z^i)^{1-\alpha^i} = \alpha^m (z^m)^{\alpha^m-1}.$$

We solve this system of equations to obtain that $\forall i \neq m$

$$z^i = \frac{\alpha^i A^m (1 - \alpha^m)}{A^i (1 - \alpha^i) \alpha^m} z^m,$$

(2.9)

and

$$p^i = \pi^i (z^m)^{\alpha^m-\alpha^i},$$

(2.10)

where

$$\pi^i = \left( \frac{\alpha^m}{\alpha^i} \right)^{\alpha^i} \left( \frac{1 - \alpha^m}{1 - \alpha^i} \right)^{1-\alpha^i} \left( \frac{A^m}{A^i} \right)^{1-\alpha^i}.$$

Note that prices are the product of two terms. First, $\pi^i$ measures the effect of technological progress on relative prices. This term is constant if technological change is unbiased, whereas it is a time-varying variable if there is biased technological change. The second term measures the effect of capital deepening on relative prices: as capital becomes more abundant, prices of those goods with a larger capital output elasticity decrease. Obviously, capital deepening does not affect relative prices when $\alpha^m = \alpha^i$. We conclude that prices are constant when technological progress is unbiased and the capital income shares are identical across sectors. In this case, sectoral change is driven only by demand factors.
2.3. Market clearing

Sector $m$ produces a commodity that can be used as a consumption good or as capital good

$$Y^m = c^m + \dot{K} + \delta K.$$  

The other sectors only produce consumption goods and thus the market clearing conditions in these sectors are $c^i = Y^i$, for all $i \neq m$, which can be rewritten as

$$u^i = \frac{c^i}{A^i L (z^i)^{\alpha^i}}. \quad (2.11)$$

Market clearing in the labor market implies that

$$\sum_{i=1}^{m} u^i = 1, \quad (2.12)$$

and in the capital market implies that $\sum_{i=1}^{m} s^i = 1$, where $s^i$ is obtained from the definition of $z^i$ as follows

$$s^i = \left( \frac{A^i}{A^m} \right) \left( \frac{z^i}{z} \right) u^i. \quad (2.13)$$

Finally, from the budget constraint, we obtain that

$$Q = E + \dot{K} + \delta K, \quad (2.14)$$

where $Q$ measures gross domestic product (GDP).

3. The equilibrium

In order to characterize the equilibrium, we define the following transformed variables: $z$, $z^m$, $e$ and $\tilde{e}$. Firstly, $z = K/A^m L$ is the stock of aggregate capital per efficiency unit of labor. Using the definition of $z$, we obtain that $z^m = s^m z / u^m$ and $z^i = s^i A^m z / u^i A^i$. Note that $z^m$ is a measure of capital intensity in the manufacturing sector. Next, $e = E / w L$ is the ratio between consumption expenditures and labor income where $w = (1 - \alpha_m) A^m (z^m)^{\alpha_m}$ is the wage. Finally, $\tilde{e} = \tilde{E} / w L$ is the ratio between the value at market prices of the minimum consumption requirements and labor income. This variable is a measure of the intensity of the minimum consumption requirements.

The knife-edge condition imposed by KRX implies that $\tilde{E} = 0$ and thus $\tilde{e} = 0$. Therefore, by imposing this knife-edge condition from the beginning, the dimensionality of the equilibrium is reduced. In the analysis that follows, we characterize the equilibrium without imposing this condition.

3.1. Static equilibrium: sectoral composition

We proceed to characterize those variables determining the sectorial composition as functions of the transformed variables: $e$, $\tilde{e}$ and $z^m$. To this end, we use the market clearing conditions in the capital and labor markets. In Appendix B, we combine
(2.11), (2.13) and the equilibrium condition in the capital market to obtain the following relationship between \( u^m \) and \( z^m \):

\[
\frac{z}{z^m} \left( \frac{\alpha^m}{1 - \alpha^m} \right) = \frac{\tilde{e} - x \tilde{z}^m}{x} - e \left( 1 - x \right) + \frac{u^m}{1 - \alpha^m} - 1, \tag{3.1}
\]

where

\[
\tilde{v}^i = \frac{\tilde{c}^i}{uL} = \frac{\tilde{c}^i}{(1 - \alpha_m) A_m (z^m)^{\alpha_m} L} = \frac{\tilde{c}^i}{E}. \tag{3.2}
\]

In the same Appendix, we combine \( n^i \), (2.6), (2.10) and (2.11) to obtain the labor share in the consumption sectors

\[
u^i = (1 - \alpha^i) \left( \frac{e - \tilde{c}}{x} \right) n^i + (1 - \alpha^i) p^i \tilde{v}^i. \tag{3.3}
\]

Finally, using the equilibrium condition in the labor market, \( u_m = 1 - \sum_{i=1}^{m-1} u^i \), we obtain the following relationship between \( u_m \) and \( z^m \):

\[
u_m = 1 - \sum_{i=1}^{m-1} (1 - \alpha^i) \left( \frac{e - \tilde{c}}{x} \right) n^i - \sum_{i=1}^{m-1} (1 - \alpha^i) p^i \tilde{v}^i. \tag{3.4}
\]

From combining (3.1) and (3.4), we obtain

\[
\frac{z}{z^m} = \left( 1 - \frac{\Delta}{\alpha^m} \right) z^m, \tag{3.5}
\]

and

\[
u^m = 1 - e \Delta - \left( \frac{1 - \alpha^m}{x} \right) \left[ \tilde{e} - e \left( 1 - x \right) - x \tilde{v}^m \right], \tag{3.6}
\]

where \( \Delta = \tilde{\Delta} (e, \tilde{e}, z^m) \) is

\[
\Delta = \alpha^m - 1 + \left( 1 - \frac{\tilde{c}}{e} \right) \left( \frac{1 - \alpha^i}{x} \right) \left( \prod_{i=1}^{m-1} (1 - \alpha^i) \left[ \prod_{i=1}^{m-1} (z^{\alpha^i - 1}) \right]^{\frac{1 - \alpha^i}{x}} \right) + \left( \frac{\tilde{c}}{e} \right) \left( \frac{1 - \alpha^i}{x} \right) \left( \prod_{i=1}^{m-1} (z^{\alpha^i - 1}) \right). \]

The variable \( \Delta \) provides a measure of the capital intensity of sector \( z^m \) relative to the aggregate capital intensity. The following Lemma characterizes this relative factor intensity.

**Lemma 3.1.** If \( \alpha^i = \alpha \) for all \( i \) then \( \Delta = 0 \) and if \( \alpha^i \neq \alpha \) for some \( i \) then

\[
\tilde{\Delta}_e = - \left( \frac{1}{e} \right) \left[ \chi(z^m) - \omega(z^m) \right],
\]

\[
\tilde{\Delta}_c = \left( \frac{\tilde{c}}{e^2} \right) \left[ \chi(z^m) - \omega(z^m) \right] = - \left( \frac{\tilde{c}}{e} \right) \tilde{\Delta}_e,
\]

\[
\tilde{\Delta}_{zm} = \left( 1 - \frac{\tilde{c}}{e} \right) \chi_{zm} + \left( \frac{\tilde{c}}{e} \right) \omega_{zm},
\]

Note that equations (3.3) and (3.4) define the labor shares as functions of the transformed variables.
3.2. Dynamic equilibrium: aggregate variables

We proceed to obtain the system of differential equations driving the time path of the variables: $e$, $z^m$ and $\bar{e}$. The differential equation governing the time path of $e$ is obtained from log-differentiating the definition of the transformed variables $e$ and $\bar{e}$ and using (2.8). We obtain

$$\frac{\dot{e}}{e - \bar{e}} = \frac{r - \delta - \rho}{\sigma} + \left( \frac{\sigma - 1}{\sigma (1 - \varepsilon)} \right) \frac{\dot{x}}{x} - \left( \frac{e}{e - \bar{e}} \right) \frac{\dot{w}}{w}.$$  

Using the first order conditions from the firms’ problem, we obtain

$$\frac{\dot{e}}{e - \bar{e}} = \frac{\alpha^m (z^m)^{\alpha^m - 1}}{\sigma} - \delta - \rho + \left( \frac{\sigma - 1}{(1 - \varepsilon)} \right) \frac{\dot{x}}{x} - \left( \frac{\gamma_m + \alpha^m \left( \frac{\dot{z}^m}{z^m} \right)}{\sigma} \right) \left( \frac{e}{e - \bar{e}} \right). \quad (3.7)$$

From combining (2.6) and the definition of $x$, we obtain that $\frac{\dot{x}}{x} = (1 - \varepsilon) \tau (z^m) \frac{z^m}{z^m}$, where

$$\tau (z^m) = \frac{\sum_{i=1}^{m} (\pi^i(z^m)^{\alpha^m - \alpha^i})^{1 - \varepsilon} \left( \frac{\alpha^m - \alpha^i}{\alpha^m} \right)^{\varepsilon}}{\sum_{i=1}^{m} (\pi^i(z^m)^{\alpha^m - \alpha^i})^{1 - \varepsilon} \left( \frac{\alpha^m - \alpha^i}{\alpha^m} \right)^{\varepsilon}}.$$  

Therefore, we obtain

$$\frac{\dot{e}}{e - \bar{e}} = \frac{\alpha^m (z^m)^{\alpha^m - 1} - \delta - \rho + \left( \frac{\sigma - 1}{\sigma} \right) \tau - \left( \frac{\gamma_m + \alpha^m \left( \frac{\dot{z}^m}{z^m} \right)}{\sigma} \right) \left( \frac{e}{e - \bar{e}} \right) \alpha^m}{\sum_{i=1}^{m} \left( \frac{\alpha^m}{\alpha^m - \alpha^i} \right)^{1 - \varepsilon} \left( \frac{\alpha^m - \alpha^i}{\alpha^m} \right)^{\varepsilon}}. \quad (3.8)$$

The differential equation governing the path of capital is obtained from the budget constraint

$$\frac{\dot{K}}{K} = \frac{Q}{K} - \frac{E}{K} - \delta.$$  

From using the first order conditions of the firms’ problem, the aggregate GDP can be written as follows $Q = wL \sum_{i=1}^{m} u^i / (1 - \alpha^i)$, where $\sum_{i=1}^{m} u^i / (1 - \alpha^i)$ is the inverse of the labor income share. From combining these two equations, we obtain

$$\frac{\dot{K}}{K} = \left( \sum_{i=1}^{m} \frac{u^i}{1 - \alpha^i} - \frac{e}{K} \right) \frac{wL}{K} - \delta.$$  

Using the market clearing condition in the capital market, (2.13), (2.9) and the first order condition from the firms’ problem, we obtain

$$\frac{\dot{K}}{K} = \left( \frac{1 - \alpha^m}{\alpha^m} \right) \left( \frac{\alpha^m (1 - \frac{\Delta e}{\alpha^m})}{1 - \alpha^m} + 1 - e \right) - \delta.$$  

Using (3.5) and the definition of $z$, we obtain

$$\frac{\dot{z}^m}{z^m} = \left( \frac{(1 - \alpha^m)(z^m)^{\alpha^m - 1}}{1 - \alpha^m} \right) \left( \frac{\alpha^m (1 - \frac{\Delta e}{\alpha^m})}{1 - \alpha^m} + 1 - e \right) - \delta - \gamma + \left( \frac{\Delta e}{\alpha^m - \Delta e} \right) \left( \frac{\frac{\Delta e}{\Delta} \dot{z}^m}{\Delta} + \left( \frac{\Delta e}{\Delta} + 1 \right) \frac{\dot{z}^m}{\Delta} \right).$$  

\begin{equation} \label{3.9} \end{equation}
From the definition of \( \tilde{e} \), we obtain

\[
\frac{\dot{\tilde{e}}}{\tilde{e}} = -\gamma - \alpha_m \frac{\dot{z}^m}{z^m}.
\]  

Finally, we substitute (3.8) and (3.10) into (3.9) to obtain

\[
\frac{\dot{z}^m}{z^m} = \kappa \left( e, z^m, \tilde{e} \right),
\]

where

\[
\kappa = \frac{\left( \alpha^m - \Delta e \right) (1 - \alpha^m) (1 - \epsilon)}{\Delta e} \alpha^m \left( z^m \right)^{\alpha^m - 1} - \frac{\alpha^m \gamma_m + (\alpha^m - \Delta e) \delta}{\Delta e} + \frac{\epsilon - \tilde{e}}{\epsilon - e} \left( \frac{\alpha^m (z^m)^{\alpha^m - 1} - \delta - \rho}{\sigma} \right) - \left( \frac{\epsilon - \tilde{e}}{\epsilon - e} \right) \kappa - \left( \frac{\epsilon - \tilde{e}}{\epsilon - e} \right) \gamma_m \left( \frac{e - \tilde{e}}{e - e} \right).
\]

**Definition 3.2.** Given \( \{ z^m_0, \tilde{e}_0 \} \), the dynamic equilibrium is a path of \( \{ e, z^m, \tilde{e} \}_{t=0}^\infty \) such that satisfies the following system of equations:

\[
\frac{\dot{z}^m}{z^m} = \kappa \left( \tilde{e}, e, z^m \right),
\]

\[
\frac{\dot{e}}{e} = \left[ \frac{\alpha^m (z^m)^{\alpha^m - 1} - \delta - \rho}{\sigma} \right] - \left( \frac{\epsilon - \tilde{e}}{\epsilon - e} \right) \kappa - \left( \frac{\epsilon - \tilde{e}}{\epsilon - e} \right) \gamma_m \left( \frac{e - \tilde{e}}{e - e} \right),
\]

\[
\frac{\dot{\tilde{e}}}{\tilde{e}} = -\gamma - \alpha_m \kappa.
\]

**Remark 1.** The control variable is \( e \) and \( z^m \) and \( \tilde{e} \) are state variables. Note that \( \tilde{e}_0 = \frac{E}{A_0^m(1-\alpha^m)(z^m_0)^{\alpha^m}} \) and thus \( z^m_0 \) and \( \tilde{e}_0 \) can be chosen independently.

**Proposition 3.3.** There is a unique steady state and the value of the variables is

\[
\begin{align*}
\tilde{e}^* &= 0 \\
z^m_* &= \left( \frac{\sigma \gamma_m + \delta + \rho}{\alpha^m} \right)^{\frac{1}{\alpha^m - 1}} \\
e^* &= \left( \frac{\sigma (1 - \alpha^m) }{\sigma (1 - \alpha^m) - \Delta^* (1 - \sigma)} \right) \gamma_m + (1 - \alpha^m) (1 - \alpha^m) \Delta^* + (1 - \alpha^m + \Delta^*) \rho \\
\Delta^* &= \begin{cases} 
\alpha^m - 1 + \chi (z^m) & \text{if } \alpha_i \neq \alpha \text{ for some } i \\
0 & \text{if } \alpha_i = \alpha \text{ for all } i
\end{cases}
\end{align*}
\]

**Proposition 3.4.** Assume that \( \alpha^i = \alpha \) for all \( i \). The unique steady state is saddle path stable.\footnote{Stability when \( \alpha^i \neq \alpha \) for some \( i \) is not proven. However, from numerical exercises, we conclude that the equilibrium is also saddle path stable in this case.}

Saddle path stability implies that the equilibrium is characterized by a two-dimensional stable manifold and the particular equilibrium path will depend on the initial values of the two state variables. Given initial conditions on both state variables, there is a unique equilibrium path converging towards the steady state. However, given initial condition on relative capital intensity, \( z^m_0 \), there is a continuum of equilibrium paths indexed by the initial value of the relative intensity of the minimum consumption.
requirements, $\tilde{c}_0$. The knife-edge condition in KRX, $\tilde{E} = 0$, implies that $\tilde{c}_0 = 0$. Therefore, this knife-edge condition is equivalent to select a particular equilibrium path of the two dimensional manifold. Along this equilibrium path, the Kuznets and Kaldor facts can be simultaneously explained. By a continuity argument, we conjecture that other equilibrium paths close enough will also satisfy these two sets of facts. These equilibrium paths can be selected using the initial value of the minimum consumption requirement. In the following section, we numerically show that these equilibrium paths still satisfy the Kaldor and Kuznets facts. We conclude that these two sets of facts can be satisfied even though the knife-edge condition is not imposed.

4. Numerical simulations

The value of the parameters is set as follows. We assume, as KRX, that $\alpha^i = \alpha$ for all $i$ and $\alpha = 0.35$ implying that the labor income share equals 65%. This assumption implies that prices are constant and, therefore, in this numerical example only demand factors drive structural change. The long run growth rate of GDP is $\gamma = 2\%$. We set $\delta = 5.6\%$ to replicate the fact that the ratio of investment to capital equals 7.6%. We set $\sigma = 2$ implying an intertemporal elasticity of substitution equal to 0.5 and $\rho = 0.014$ implying a long run interest rate equal to 5.4%. We assume that $z_0 = 0.75 z^*$. Finally, the ratios of total factor productivities, $A^m$ and $A^s$, and the minimum consumptions, $c^1$ and $c^2$, are set to match the labor shares in the three sectors in the years 1869 and 2000.

Figure 1 shows the time path of the aggregate variables of economies that are differentiated only by the initial value of the minimum consumption requirements. The economy $\tilde{c}_0 = 0$ corresponds to the economy studied by KRX. This figure shows that the different economies converge to the same long run BGP. This is a consequence of the fact that in a growing economy the intensity of the minimum consumption converges to zero, regardless of the initial condition, as shown in Figure 2. Therefore, the relevant differences among these economies are in the transition. Those economies with an initially large intensity of the consumption requirement devote a large fraction of GDP to consumption expenditures. As a consequence, investment in these economies is small in the initial periods, implying that both capital per unit of efficiency labor and the ratio of capital to GDP decrease. This smaller capital accumulation implies that the speed of convergence of aggregate variables is smaller in those economies with a large value of the minimum consumption requirement. This suggests that these economies may not be consistent with the Kaldor and Kuznets facts, as these two facts require that aggregate variables should converge faster than the labor shares. Table 1 shows half-life as a measure of the speed of convergence in these different economies. As follows from this table, when the initial intensity of the minimum consumption requirements is zero or below 0.75, half life is smaller for aggregate variables than for the labor shares. These economies then satisfy the Kaldor and Kuznets facts. In contrast, when $\tilde{c}_0 = 1$, half life is larger for aggregate variables and then this economy does not explain the Kaldor and Kuznets facts. Note that the results in this table provide numerical support

---

5 Using Historical statistics of the United States from colonial times, we obtain that the labor shares in the year 1869 (2000) in agriculture, services and manufactures are 49% (3%), 25% (72%) and 26% (25%), respectively.
to our conjecture that economies where the initial value of the minimum consumption requirement is not far from zero are consistent with the Kaldor and Kuznets facts. We then conclude that Kaldor and Kuznets facts can be explained in a model of structural change driven by demand factors even though the knife-edge condition is not introduced.

Figure 1 shows that capital per efficiency unit of labor initially decreases in economies where the intensity of the minimum consumption requirement is initially large. Diminishing returns to capital imply that the interest rate increases in these economies, which causes the acceleration of capital accumulation in the future. This larger amount of investment, after 30-40 periods, implies that more resources must be devoted to the manufacturing sector. This explains the time path of the labor shares shown in Figure 3. This figure shows that in the economies with an initially large value of the minimum consumption requirement the labor share in the manufacturing sector increases during the nineteenth century and first years of the twenty century and after decreases. This time path of the labor share in the manufacturing sector is consistent with data and it is explained only when a large and positive initial intensity of the minimum consumption requirements is assumed. In fact, as follows from Figure 3, numerical simulations fit better actual data when it is assumed that the initial value of the minimum consumption requirements is large and positive. Table 1 provides three measures of the fit of the numerical simulations to actual data; means square errors, the ratio of areas and the mean absolute percentage error. The three measures show that the fit of the numerical simulations increases with the initial intensity of the minimum consumption requirement. Therefore, this result implies that explaining sectoral change only with demand factors requires assuming that the intensity of the minimum consumption requirement in the nineteenth century was large, decreasing and converging towards zero.

5. Labor and consumption shares

The model of structural change driven only by demand factors analyzed in the previous section cannot explain the sectoral change in the consumption shares, shown in Figure 4. Indeed this model is not able to explain simultaneously the sectoral change in labor and consumption shares. In order to see this, we use the first order conditions from the firms problem and the market clearing condition in the consumption sectors to obtain the following result:

**Proposition 5.1.** If there is perfect competition in the product and labor markets, labor is mobile across sectors and the production functions of the consumption sectors exhibit constant returns to scale then for every consumption sector the following equation holds:

\[
LIS_i = \left( \frac{u^i}{p^i c^i} \right) \left( \frac{LIS}{E} \right),
\]

According to Herrendorf (2013), using consumption added value in the years 1947 (2000), the ratio of consumption of manufactures to total expenditures is 30% (18%), the ratio of consumption of agriculture goods to total expenditures is 9% (2%) and the ratio of consumption of services to total expenditures is 61% (80%).
where \( LIS_i \) is the sectoral labor income share and \( LIS \) is the economy labor income share.

This result shows that the labor income share (LIS) in every consumption sector can be obtained as the product of two ratios: the ratio between the share of labor and consumption in the sector and the ratio between aggregate LIS and the fraction of GDP devoted to consumption expenditures. Using data on these two ratios, we obtain that the LIS in agriculture should be 164% and 70% in the years 1947 and 2000. In the same years, the LIS in the service sector should be, respectively, 61% and 81%. These are the values of the LIS that, according to the assumptions of the model, are consistent with the sectoral change in labor and consumption shares. While the LIS in the service sector are within the values of this variable obtained in the literature (see Herrendorf and Valentinyi, 2007), the values of the LIS in agriculture are clearly larger. From these numbers, we obtain three conclusions. First, growth models with the standard assumptions (perfect competition, constant returns to scale, agriculture is a consumption sector) cannot explain the sectoral change in consumption and labor shares in the agriculture sector. Second, growth models of structural change should consider that sectoral labor income shares do not remain constant and thus the production function cannot be Cobb-Douglas. Third, growth models of structural change should consider that sectoral labor income shares are different across sectors.

In what follows, we assume that sectoral labor income shares are different across sectors and identical to the sectoral LIS in the year 2000 that, according to the assumptions of the model, are consistent with the sectoral consumption and labor shares. This assumption is introduced in the numerical example of the previous section by assuming that \( \alpha_1 = 0.3, \alpha_2 = 0.19 \) and, finally, \( \alpha_m = 0.61 \) so that the aggregate LIS equals 65%. Obviously, the differences in the capital output elasticities imply that prices do not remain constant and thus supply factors also drive structural change. The results from this numerical example are displayed in Figures 5, 6 and 7. Figure 5 shows the time path of aggregate variables and Figure 6 shows the time path of the labor shares. The results obtained from these two figures are similar to those obtained when only demand factors drive structural change. Table 3 shows that the performance of the model in explaining the labor shares increases when the minimum consumption is initially positive and large and, from the comparison between Tables 2 and 3, we conclude that the introduction of supply factors improves the fit of the model. Finally, Figure 7 shows the time path of expenditure shares and compares it with available data. We conclude that this model of structural change driven by both demand and supply factors explains the main trends of expenditure shares. Table 4 shows the performance of the numerical simulation in explaining expenditure shares. The fit of the expenditure shares in manufactures and services increases with the initial intensity of the minimum consumption requirement, whereas the best fit of the expenditure share in agriculture is obtained when the initial intensity of the minimum consumption requirements is close to zero.
6. Concluding remarks

We have analyzed the transitional dynamics of the equilibrium of a growth model where the introduction of minimum consumptions requirements makes preferences be non-homothetic. Obviously, this non-homothetic preferences drive structural change. The equilibrium is characterized by a two dimensional manifold and, therefore, there is a continuum of equilibrium paths. We show that a particular equilibrium path is selected when the knife-edge condition in KRX is introduced. This equilibrium path satisfies the Kaldor and Kuznets facts. By a continuity argument, we conjecture that any equilibrium path close enough to this path will also satisfy these two facts. Using numerical examples, we prove this conjecture by showing that there is a continuum of equilibrium paths that satisfy these two sets of facts. We conclude that the introduction of a knife-edge condition is not necessary to make these two facts compatible. Moreover, we also show that the fit of the numerical simulations increases if we assume that the minimum consumption requirements are initially positive and large.

In this paper, we also show that if the labor income shares are assumed to be identical across sectors then the model cannot explain the change in both expenditures and labor shares observed in the data. We then introduce differences across sectors in the labor income shares and we show that the model explains both the sectoral change in labor and the main trends of the expenditure shares.
References


A. Derivation of Equation (2.8)

Combining (2.7) and (2.5), we obtain

\[ \delta + \rho - r = c^m \sum_{i=1}^{m} \left[ \left( \frac{U_{mi}}{U_m} \right) \left( \frac{c^i - \bar{c}^i}{c^m - \bar{c}^m} \right) \right] \frac{c^m}{c^m} - \sum_{i=1}^{m-1} \varepsilon \left( \frac{U_{mi}}{U_m} \right) \frac{c^i - \bar{c}^i}{\Phi^i} \left( \frac{\tilde{p}^i}{p^i} \right). \]  

We next use the utility function, (2.1), to obtain the following analytical expressions of both \( \Sigma \) and \( \Phi \):

\[ \Sigma = -c^m \sum_{i=1}^{m} \left[ \left( \frac{U_{mi}}{U_m} \right) \left( \frac{c^i - \bar{c}^i}{c^m - \bar{c}^m} \right) \right] = -c^m \frac{U_{mm}}{U_m} - c^m \sum_{i=1}^{m-1} \left[ \left( \frac{U_{mi}}{U_m} \right) \left( \frac{c^i - \bar{c}^i}{c^m - \bar{c}^m} \right) \right]. \]

This equation implies that

\[ \Sigma = \sigma \left( \frac{c^m}{c^m - \bar{c}^m} \right). \]  

We combine the definition of \( \Phi^i \) and the derivatives to obtain

\[ \Phi^i = \varepsilon \left( \frac{U_{mi}}{U_m} \right) \left( c^i - \bar{c}^i \right) = \frac{(1 - \sigma \varepsilon) \bar{\theta}^i \left( c^i - \bar{c}^i \right)^{\frac{\varepsilon - 1}{\varepsilon}}}{\sum_{j=1}^{m} \theta^j \left( c^j - \bar{c}^j \right)^{\frac{\varepsilon - 1}{\varepsilon}}} = \frac{(1 - \sigma \varepsilon) \bar{\theta}^i \left( c^i - \bar{c}^i \right)^{\frac{\varepsilon - 1}{\varepsilon}}}{\sum_{j=1}^{m} \theta^j \left( c^j - \bar{c}^j \right)^{\frac{\varepsilon - 1}{\varepsilon}}}. \]

We use (2.4) to obtain

\[ n^i = \left( \frac{p^i}{\tilde{p}^i} \right)^{1-\varepsilon} \left( \frac{\bar{\theta}^m}{\theta^m} \right)^{-\varepsilon} \frac{\theta^i}{\bar{\theta}^m} \left( \frac{c^i - \bar{c}^i}{c^m - \bar{c}^m} \right)^{\frac{\varepsilon - 1}{\varepsilon}} \]

and

\[ \Phi^i = \frac{(1 - \sigma \varepsilon) n^i}{\sum_{j=1}^{m} n^j} = \frac{(1 - \sigma \varepsilon) n^i}{1 + \sum_{j=1}^{m-1} n^j}. \]  

From combining (A.1), (A.2) and (A.3), we obtain

\[ \delta + \rho - r = -\sigma \left( \frac{c^m}{c^m - \bar{c}^m} \right) \frac{c^m}{c^m} - \frac{(1 - \sigma \varepsilon) n^i \left( \frac{\tilde{p}^i}{p^i} \right)}{1 + \sum_{j=1}^{m-1} n^j}. \]  

Let us define \( x = \sum_{i=1}^{m} n^i \). Then,

\[ \frac{\dot{x}}{x} = \frac{m-1}{1 + \sum_{j=1}^{m-1} n^j} \left( \frac{1 - \varepsilon}{1 - \varepsilon} \right) \frac{\tilde{p}^i}{p^i}. \]

From combining (A.4) and (A.5), we obtain

\[ \frac{c^m}{c^m - \bar{c}^m} = \frac{r - \delta - \rho}{\sigma} - \left( \frac{1 - \sigma \varepsilon}{\sigma (1 - \varepsilon)} \right) \frac{\dot{x}}{x}. \]
We rewrite the definition of \( n_i \) as \( E - \tilde{E} = (c_i^m - \tilde{c}) x \), where \( E = \sum_{i=1}^m p^i c_i \) and \( \tilde{E} = \sum_{i=1}^m p^i \tilde{c} \). We log-differentiate with respect to time to obtain
\[
\frac{\dot{E} - \dot{\tilde{E}}}{E - \tilde{E}} = \frac{\dot{c}_i^m}{c_i^m - \tilde{c}} + \frac{\dot{x}}{x}.
\]
Combining with (A.6), we obtain (2.8) in the main text.

**B. Static equilibrium**

The purpose of this Appendix is to obtain equations (3.1) and (3.3) in the main text. We first use (2.13) and the equilibrium condition in the capital market to obtain
\[
\sum_{i=1}^m A \alpha_i u^i \zeta_i = \sum_{i=1}^m s^i z = z
\]
and combining with (2.9) we obtain
\[
\sum_{i=1}^m u^i = \frac{p^i c_i^m}{p^i A^i L(z^i)^{\alpha_i}} = \frac{(1 - \alpha^i) p^i c_i^m}{Lw}, \quad \forall i \neq m. \tag{B.1}
\]
and aggregating across sectors
\[
\sum_{i=1}^{m-1} u^i = \sum_{i=1}^{m-1} p^i c_i^m = E - c^m. \tag{B.3}
\]
From the definition of \( n^i \) we obtain \( x = \sum_{i=1}^m n^i = \left( E - \tilde{E} \right) / \left( c_i^m - \tilde{c} \right) \) and using the definitions of the transformed variables we obtain
\[
c_i^m - E = \frac{E (1 - x) - \tilde{E}}{x} + \tilde{c}. \tag{B.4}
\]
Combining (B.3) and (B.4), we obtain
\[
\sum_{i=1}^{m-1} u^i = \frac{E (x - 1) + \tilde{E}}{Lwx} - \frac{\tilde{c}}{Lw}. \tag{B.5}
\]
Finally, we combine (B.5) and (B.1) we obtain (3.1) in the main text.

We proceed to obtain (3.3). First, from the definition of \( n^i \), (2.6) and (2.10) we obtain
\[
c_i^t = (c_i^m - \tilde{c}) \left( \pi^i (z^m)^{\alpha^m - \alpha} \right)^{-\varepsilon} \left( \frac{\theta^m}{\theta^i} \right)^{-\varepsilon} + \tilde{c}^i
\]
and using (B.4) we obtain
\[
c_i^t = \left( \frac{wL}{x} \right) (e - \tilde{e}) \left( \pi^i (z^m)^{\alpha^m - \alpha} \right)^{-\varepsilon} \left( \frac{\theta^m}{\theta^i} \right)^{-\varepsilon} + \tilde{c}. \tag{B.6}
\]
Next, from (B.2), we obtain that \( \forall i \neq m \)

\[
u^i = (1 - \alpha^i) \pi^i (z^m)^{\alpha^m - \alpha^i} \left[ \left( \frac{1}{x^i} \right) (e - \varepsilon) (\varepsilon_i (z^m)^{\alpha^m - \alpha^i})^{-e} \left( \frac{\theta^m}{x^i} \right)^{-e} \varepsilon^i + \frac{x^i}{L \omega} \right].
\]

Using the transformed variables, this equation can be rewritten as equation (3.3) in the main text.

C. Proof of Proposition 4.4

From equation (3.10), we obtain

\[
\begin{align*}
\frac{\partial \dot{z}}{\partial z} &= z \kappa_z = (\alpha - 1)(\delta + \gamma_m) < 0, \\
\frac{\partial \dot{z}}{\partial e} &= z \kappa_e = -\dot{z} (1 - \alpha) z^{\alpha - 1} = -(1 - \alpha) z^{\alpha} < 0, \\
\frac{\partial \dot{z}}{\partial \varepsilon} &= z \kappa_\varepsilon = 0.
\end{align*}
\]

From equation (3.8), we obtain

\[
\begin{align*}
\frac{\partial \dot{\varepsilon}}{\partial z} &= e \left( \frac{(\alpha - 1) z^{\alpha - 2}}{\sigma} - \alpha \kappa_z \right), \\
\frac{\partial \dot{\varepsilon}}{\partial e} &= -e \alpha \kappa_e = e \alpha (1 - \alpha) z^{\alpha} > 0, \\
\frac{\partial \dot{\varepsilon}}{\partial \varepsilon} &= -\gamma_m.
\end{align*}
\]

From equation (3.9), we obtain

\[
\begin{align*}
\frac{\partial \ddot{\varepsilon}}{\partial z} &= 0, \\
\frac{\partial \ddot{\varepsilon}}{\partial e} &= 0 \\
\frac{\partial \ddot{\varepsilon}}{\partial \varepsilon} &= -\gamma_m - \alpha \kappa_\varepsilon = -\gamma_m < 0.
\end{align*}
\]

Thus, the Jacobian matrix is

\[
J = \begin{pmatrix}
\frac{\partial \dot{z}}{\partial z} - \lambda & \frac{\partial \dot{z}}{\partial e} & \frac{\partial \dot{z}}{\partial \varepsilon} & 0 \\
\frac{\partial \dot{e}}{\partial z} & \frac{\partial \dot{e}}{\partial e} - \lambda & 0 & \frac{\partial \dot{e}}{\partial \varepsilon} \\
0 & 0 & \frac{\partial \dot{\varepsilon}}{\partial \varepsilon} - \lambda
\end{pmatrix}
\]

and the characteristic polynomial is

\[
P(J) = \left( \frac{\partial \dot{\varepsilon}}{\partial \varepsilon} - \lambda \right) \left[ \left( \frac{\partial \dot{\varepsilon}}{\partial e} - \lambda \right) \left( \frac{\partial \dot{z}}{\partial z} - \lambda \right) - \frac{\partial \dot{z}}{\partial e} \frac{\partial \dot{e}}{\partial z} \right].
\]
The roots are
\[ \lambda_1 = \frac{\partial \tilde{c}}{\partial \tilde{c}} = -\gamma_m < 0, \]
and the solutions of
\[ \lambda^2 - \lambda \left( \frac{\partial \tilde{z}}{\partial \tilde{z}} + \frac{\partial \tilde{e}}{\partial \tilde{e}} \right) + \left( \frac{\partial \tilde{z}}{\partial \tilde{e}} \frac{\partial \tilde{e}}{\partial \tilde{e}} \frac{\partial \tilde{z}}{\partial \tilde{z}} \frac{\partial \tilde{e}}{\partial \tilde{e}} \right) = 0, \]
where
\[
\begin{align*}
\frac{\partial \tilde{z}}{\partial \tilde{z}} \frac{\partial \tilde{e}}{\partial \tilde{e}} - \frac{\partial \tilde{z}}{\partial \tilde{e}} \frac{\partial \tilde{e}}{\partial \tilde{z}} &= -e\alpha \kappa \kappa_z - z\kappa \varepsilon \left( \frac{\alpha (\alpha - 1) z^{\alpha - 2}}{\sigma} - \alpha \kappa_z \right) = \\
&= -e\kappa \varepsilon \frac{\alpha (\alpha - 1) z^{\alpha - 1}}{\sigma} = -e \frac{\alpha (1 - \alpha)^2 z^{(\alpha - 1)^2}}{\sigma} < 0.
\end{align*}
\]

Note that the determinant of the Jacobian matrix being negative implies that \( \lambda_2 > 0 \) and that \( \lambda_3 < 0 \).
D. Tables and Figures

### Table 1. Half life

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### Table 2. Performance of the simulations

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### Table 3. Performance of the simulated labor shares

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Table 4. Performance of the simulated expenditure shares

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<td>Agricult.</td>
<td>0.11%</td>
<td>0.10%</td>
<td>0.07%</td>
<td>0.23%</td>
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<td>2.97%</td>
<td>2.70%</td>
<td>2.55%</td>
<td>2.18%</td>
<td>1.74%</td>
<td>1.09%</td>
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<td>Servic.</td>
<td>2.01%</td>
<td>1.82%</td>
<td>1.85%</td>
<td>1.01%</td>
<td>0.68%</td>
<td>0.20%</td>
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<td>Ratio of Areas</td>
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<td>Agricult.</td>
<td>41.3%</td>
<td>40.6%</td>
<td>36.4%</td>
<td>50.5%</td>
<td>51.4%</td>
<td>58.1%</td>
</tr>
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<td>Manufact.</td>
<td>67.6%</td>
<td>64.5%</td>
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<td>52.5%</td>
<td>42.5%</td>
</tr>
<tr>
<td>Servic.</td>
<td>19.4%</td>
<td>18.5%</td>
<td>18.6%</td>
<td>13.9%</td>
<td>11.5%</td>
<td>5.58%</td>
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<td>MAPE</td>
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<td>Agricult.</td>
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<td>14.4%</td>
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<td>31.4%</td>
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<tr>
<td>Manufact.</td>
<td>16.8%</td>
<td>16.1%</td>
<td>15.6%</td>
<td>14.5%</td>
<td>12.9%</td>
<td>10.3%</td>
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<tr>
<td>Servic.</td>
<td>4.91%</td>
<td>4.66%</td>
<td>4.70%</td>
<td>3.51%</td>
<td>2.90%</td>
<td>0.12%</td>
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Figure 1. Time path of aggregate data.
Figure 2. Time path of the minimum consumption requirement.

Figure 3. Labor shares
Figure 4. Data on consumption shares. Source. Herrendorf, et al. (2013).

Figure 5. Aggregate variables.
Figure 6. Labor shares

Figure 7. Expenditure shares.